Which do we know?

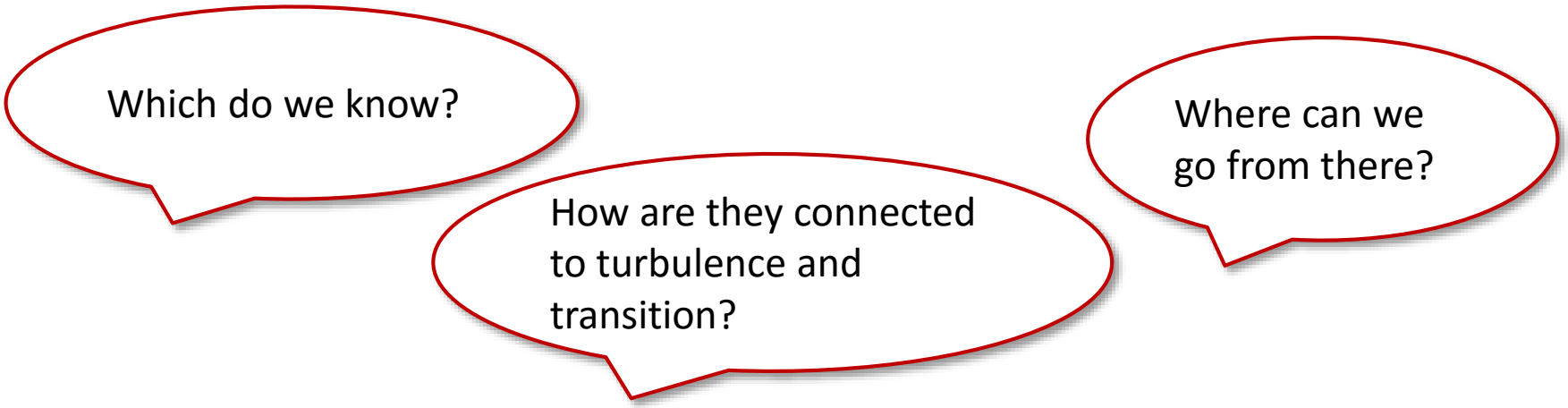
How are they connected to turbulence and transition?

Where can we go from there?

Exact coherent structures in boundary layers

Tobias Kreilos

Emergent Complexity in Physical Systems group
EPFL Lausanne, Switzerland



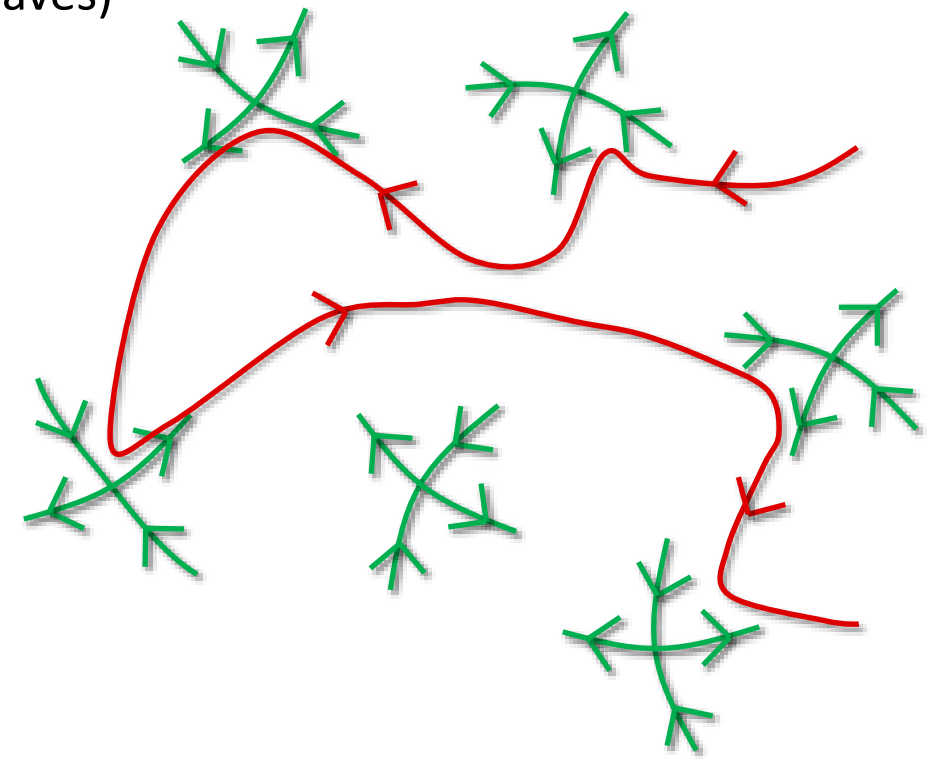
Outline

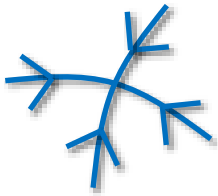
- Exact coherent structures in boundary layers?
- A simplifying case: the asymptotic suction boundary layer
- The edge state – a particular coherent structure
- Application 1: nucleating turbulent spots in a noisy environment
- Application 2: edge state and relaminarization (pCf)

Exact coherent structures in boundary layers?

Exact coherent structures (a.k.a. invariant solutions):

- Fixed points (travelling waves)
- (Relative) periodic orbits

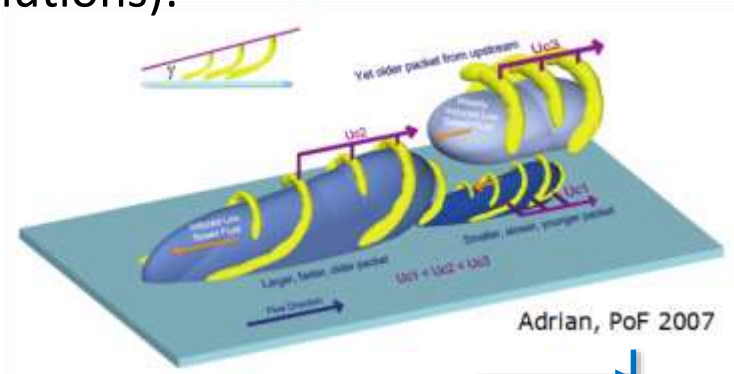


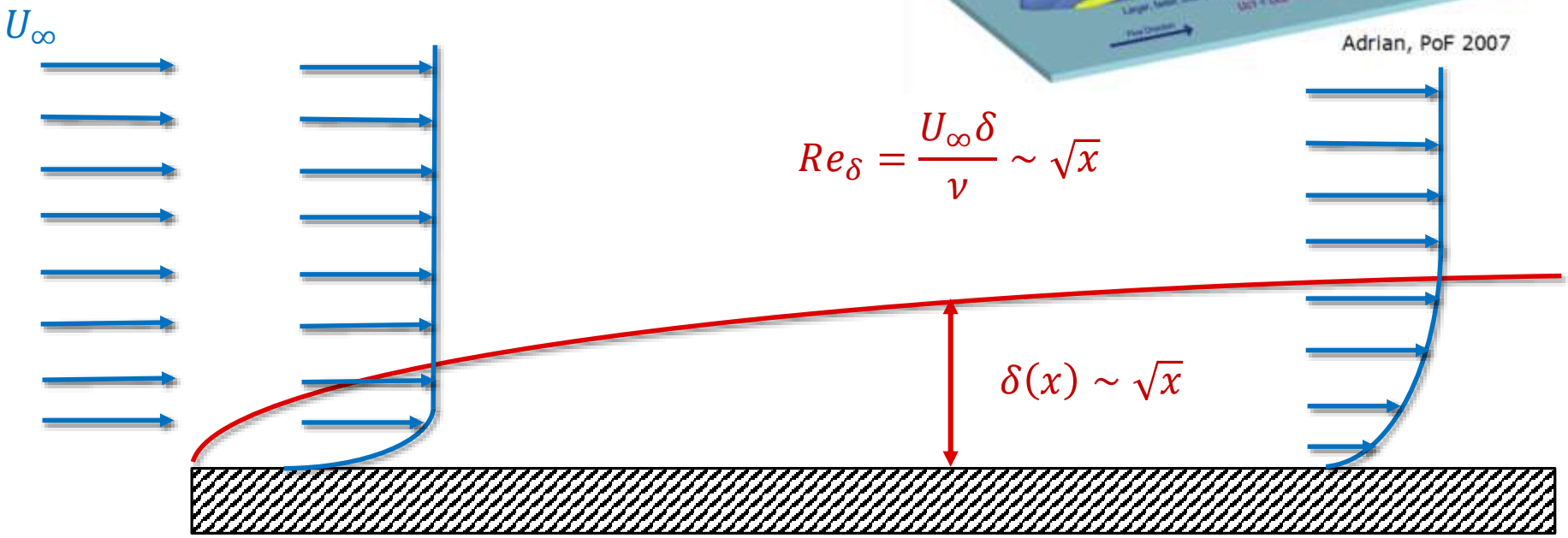


Exact coherent structures in boundary layers?

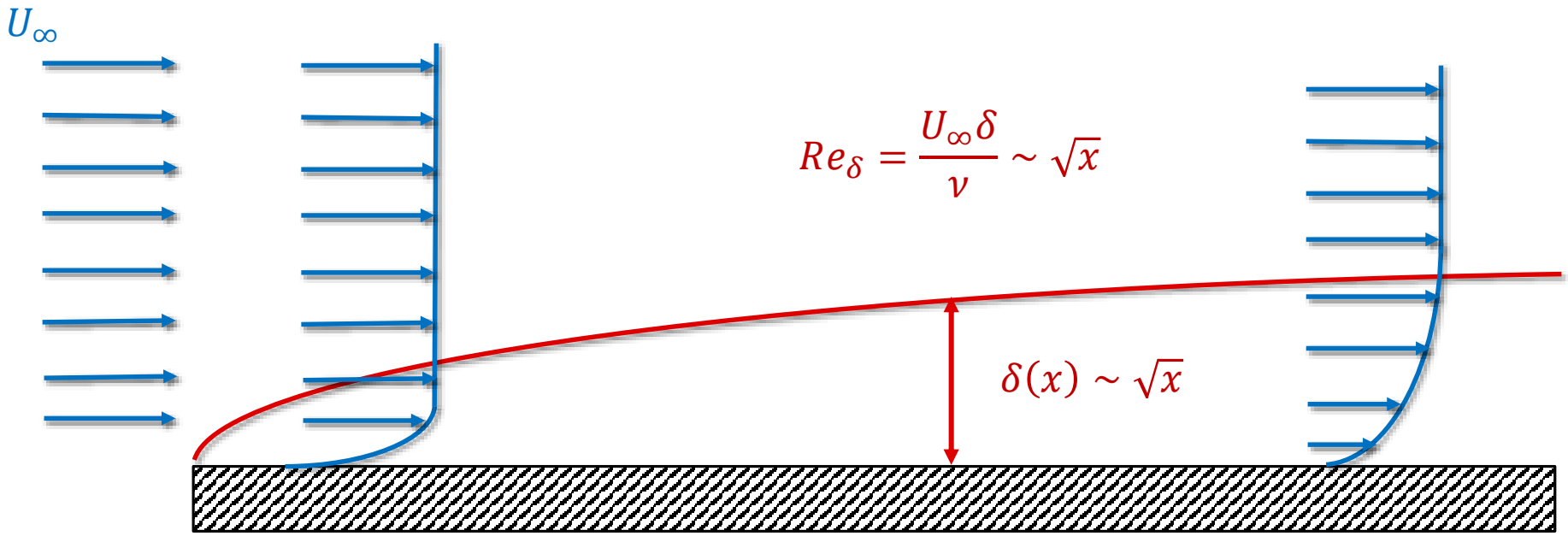
Exact coherent structures (a.k.a. invariant solutions):

- Fixed points (travelling waves)
- (Relative) periodic orbits



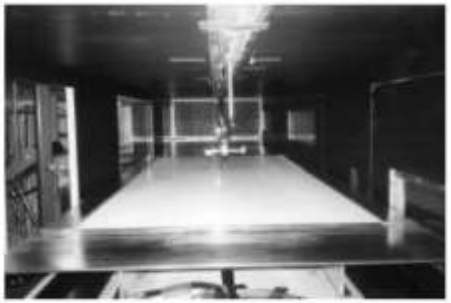


The asymptotic suction boundary layer



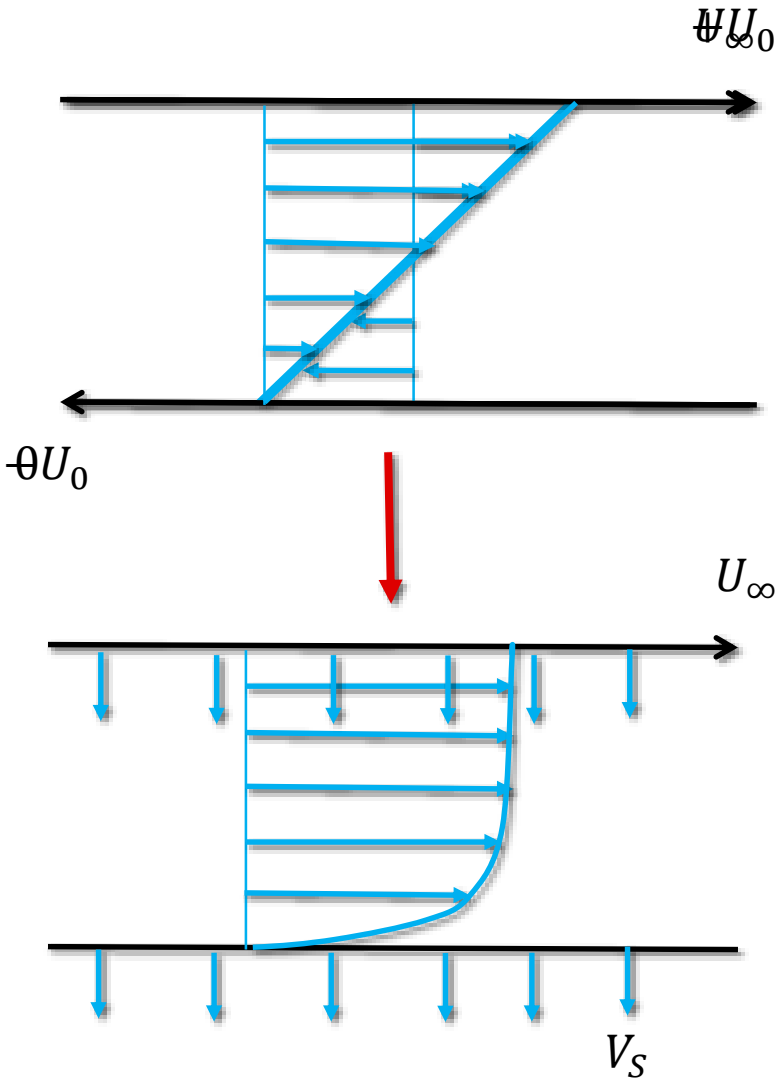
The asymptotic suction boundary layer

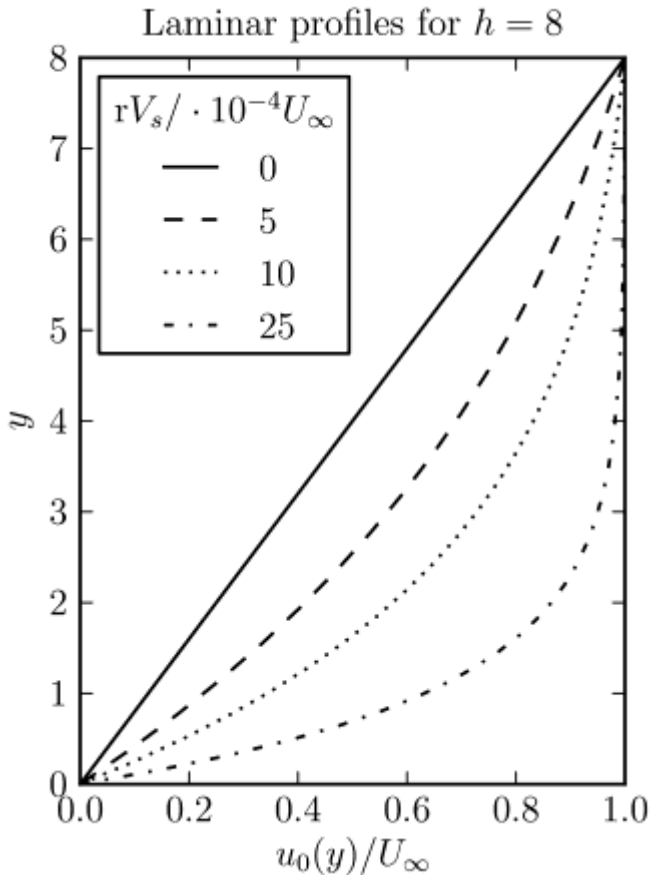
- True boundary layer flow, exact solution of Navier-Stokes
- Translational invariance
- Simple laminar profile: $\mathbf{u}(\boldsymbol{x}) = U_\infty\left(1 - e^{-y/\delta}\right)\hat{\boldsymbol{e}}_x - V_s\hat{\boldsymbol{e}}_y$
- Laminar displacement thickness: $\delta = \nu/V_S$
- Reynolds number: $Re = \frac{U_\infty \delta}{\nu} = \frac{\mathrm{U}_\infty}{V_s}$
- $\delta_{99\%} = 4.6\delta$



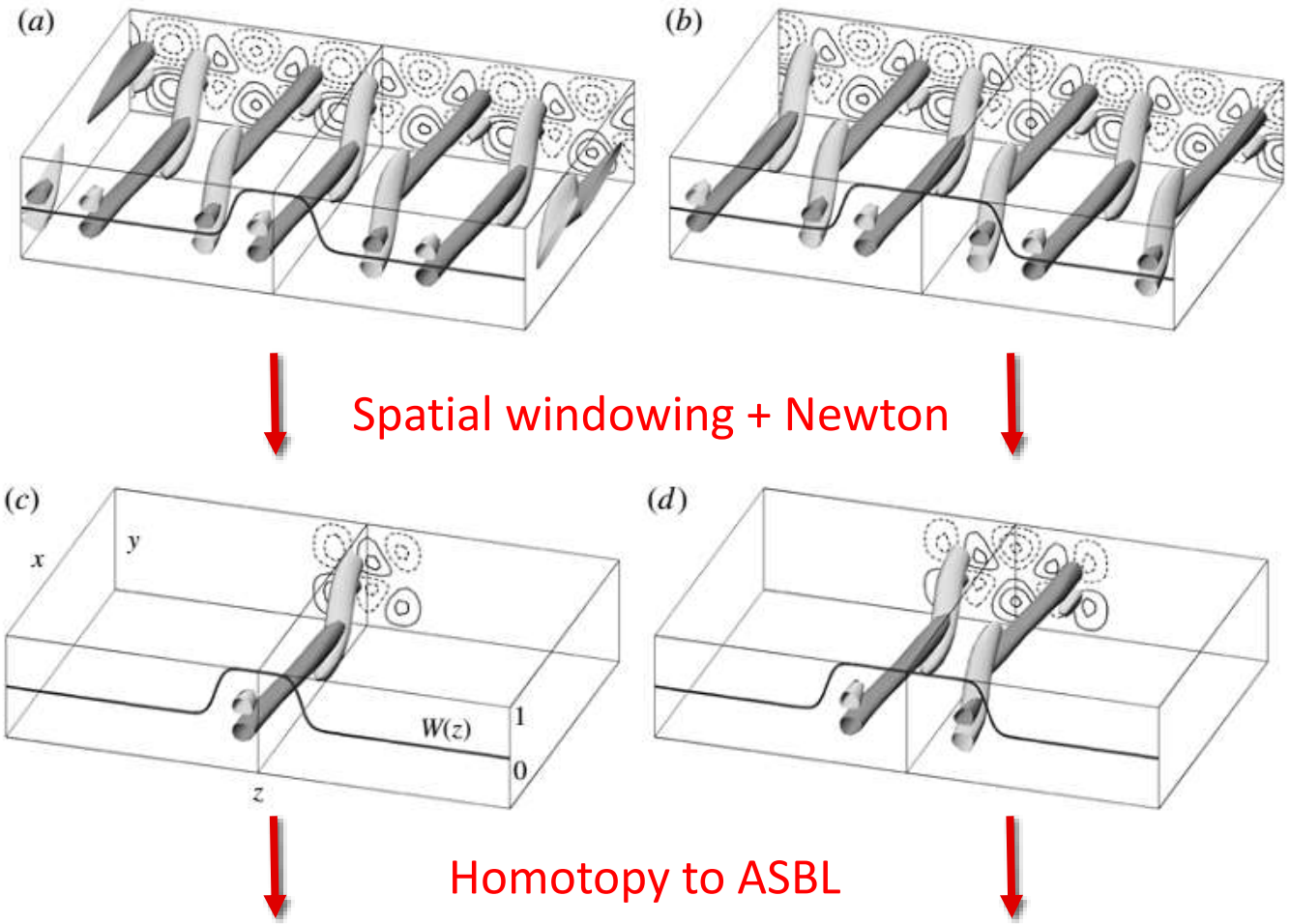
Fransson 2001, KTH

Homotopy from plane Couette





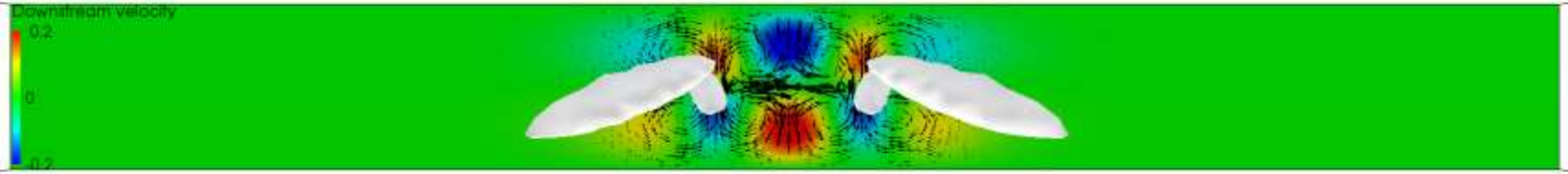
Solutions in plane Couette



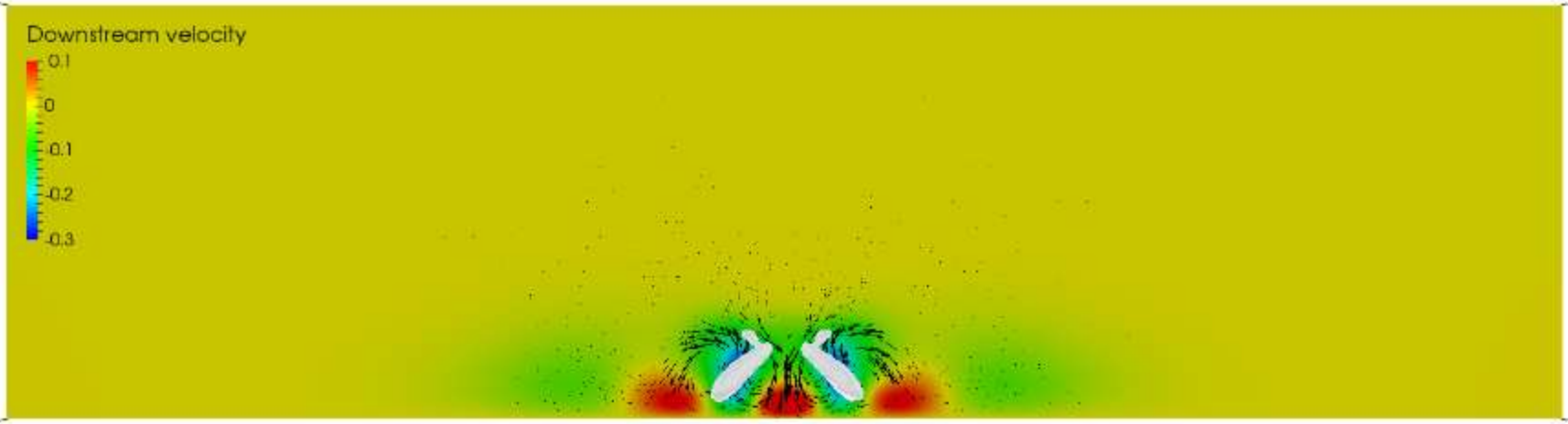
Gibson & Brand, JFM 2014

EQ7-2

Which do we know?



ASBL



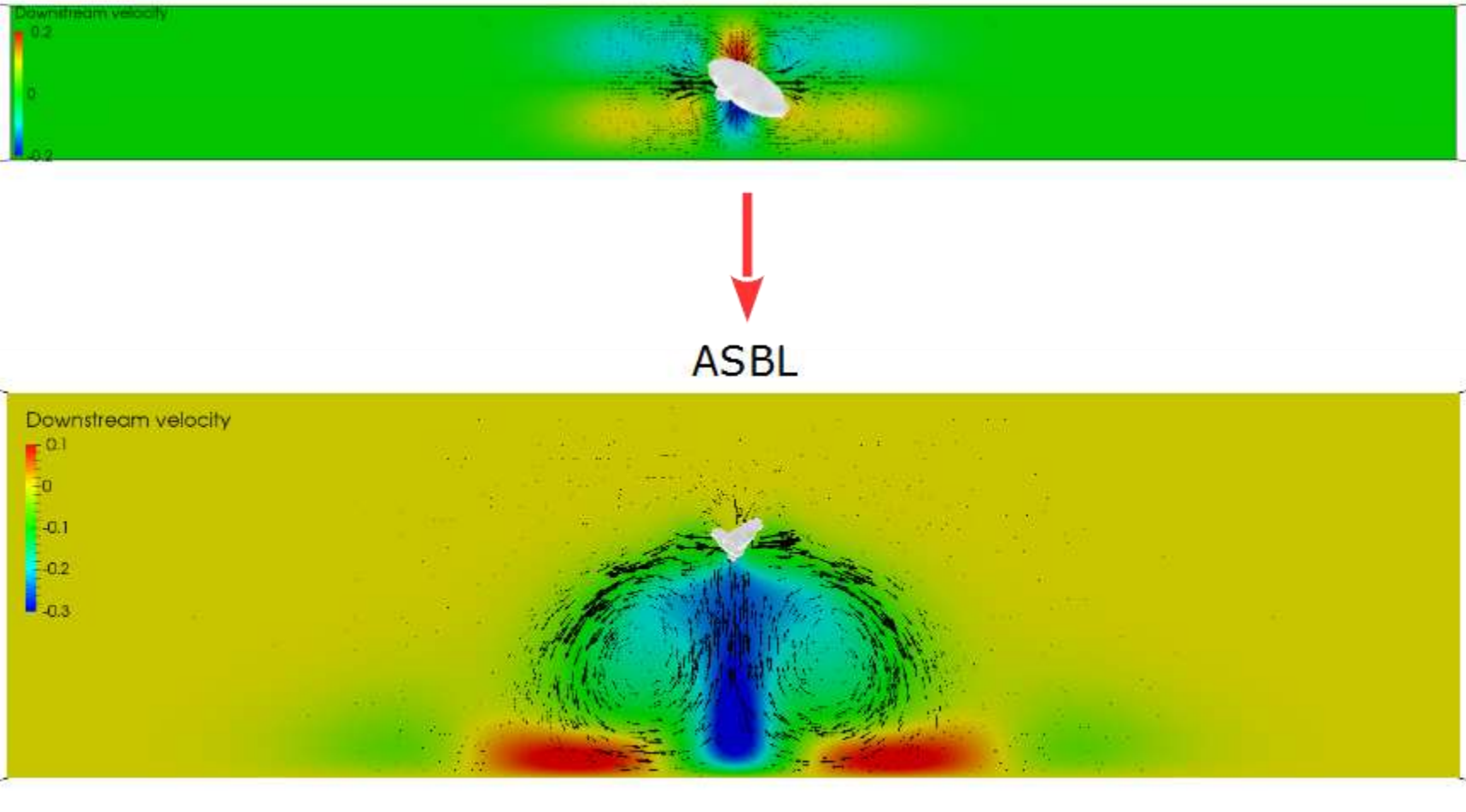
All structures close to the wall → “wall mode”

Kreilos, Gibson & Schneider, in preparation

EQ7-1

How are they connected to turbulence and transition?

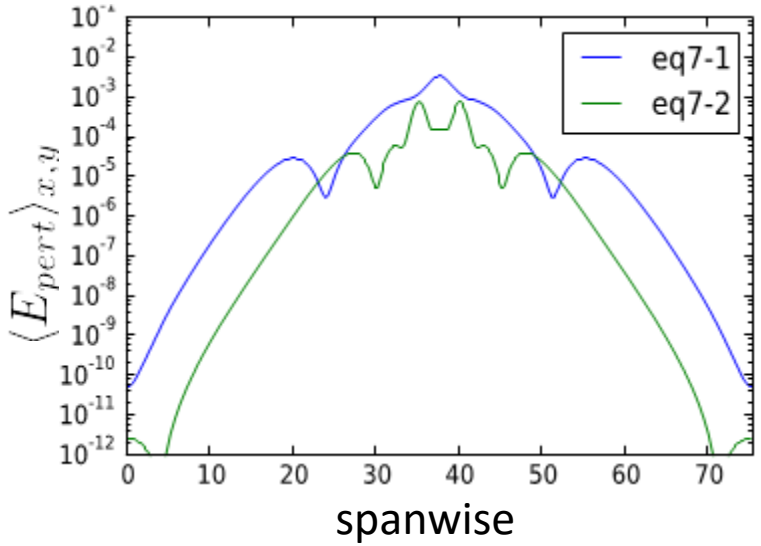
Which do we know?

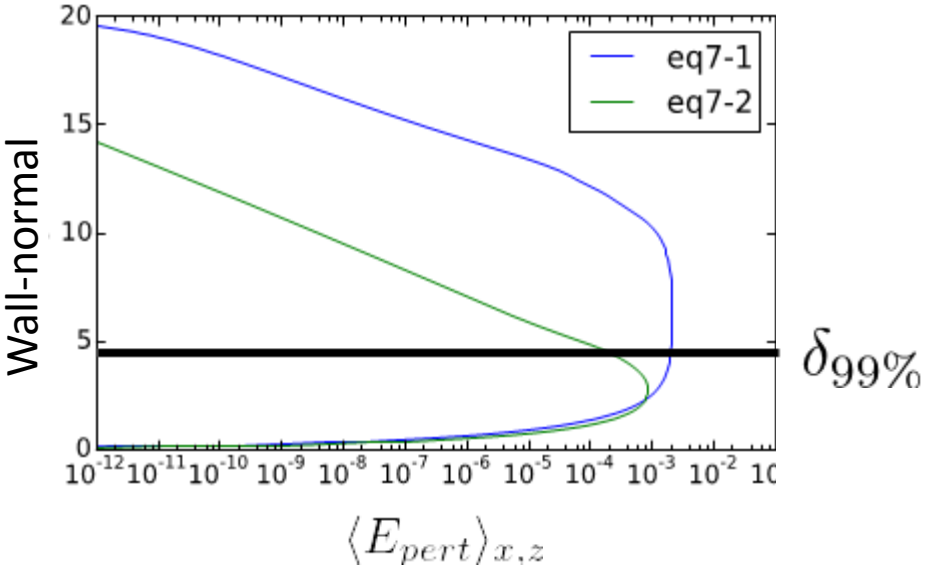


Main structures far away from the wall → free-stream mode

Kreilos, Gibson & Schneider, in preparation

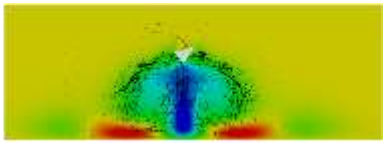
Localization properties





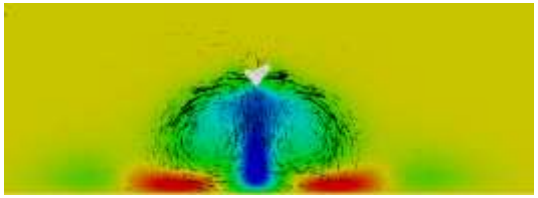
- Exponential localization in spanwise and wall-normal direction
- EQ7-1 extends far into the free-stream

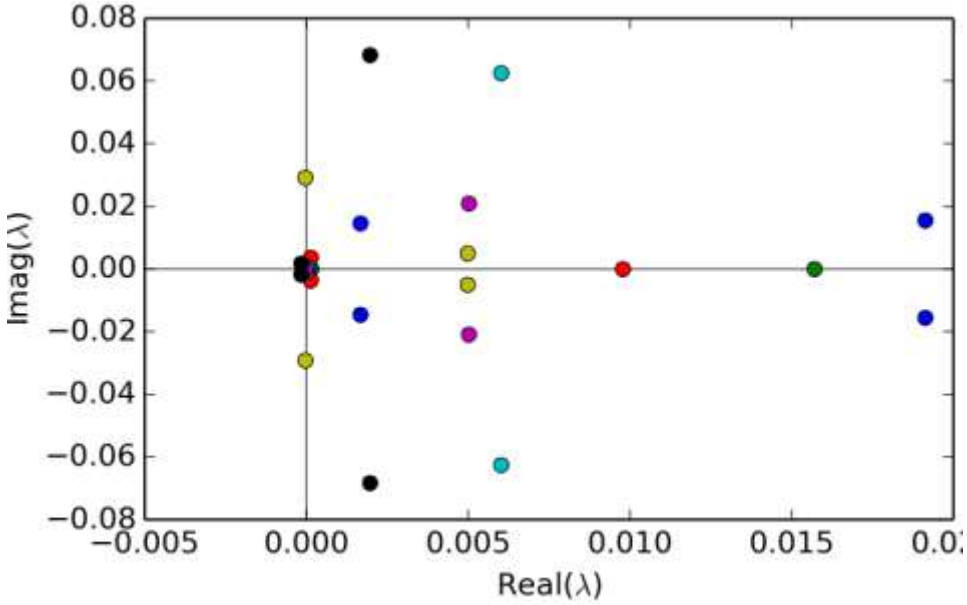
EQ7-1



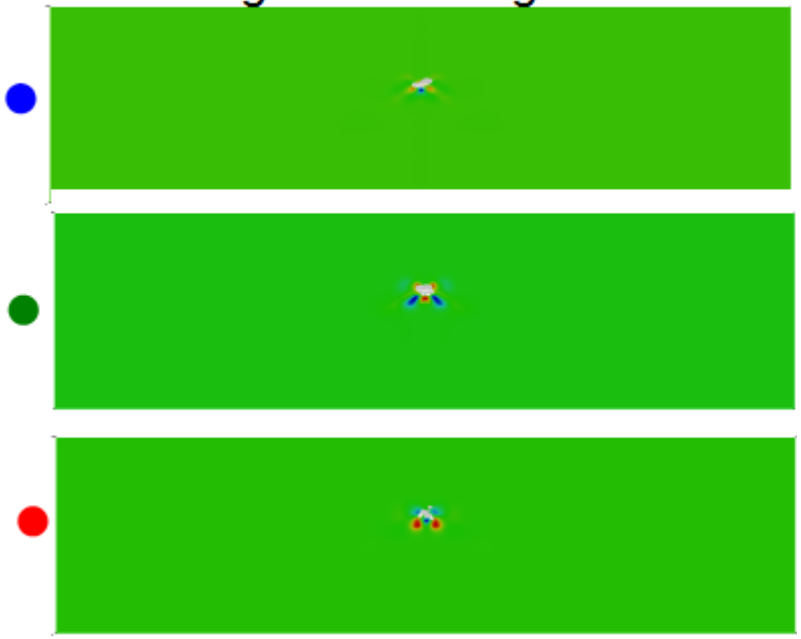
EQ7-2

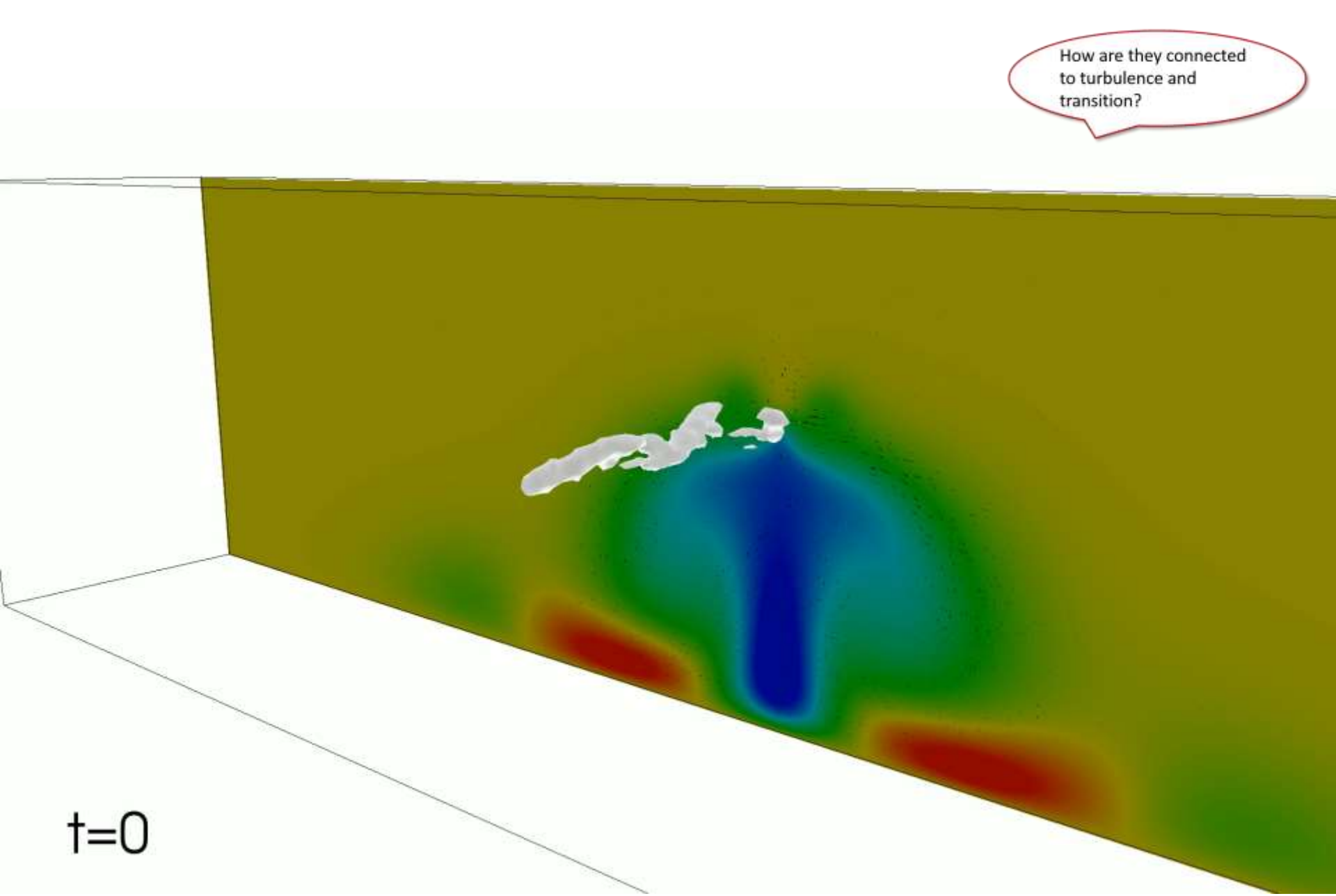
Spectrum of FSC



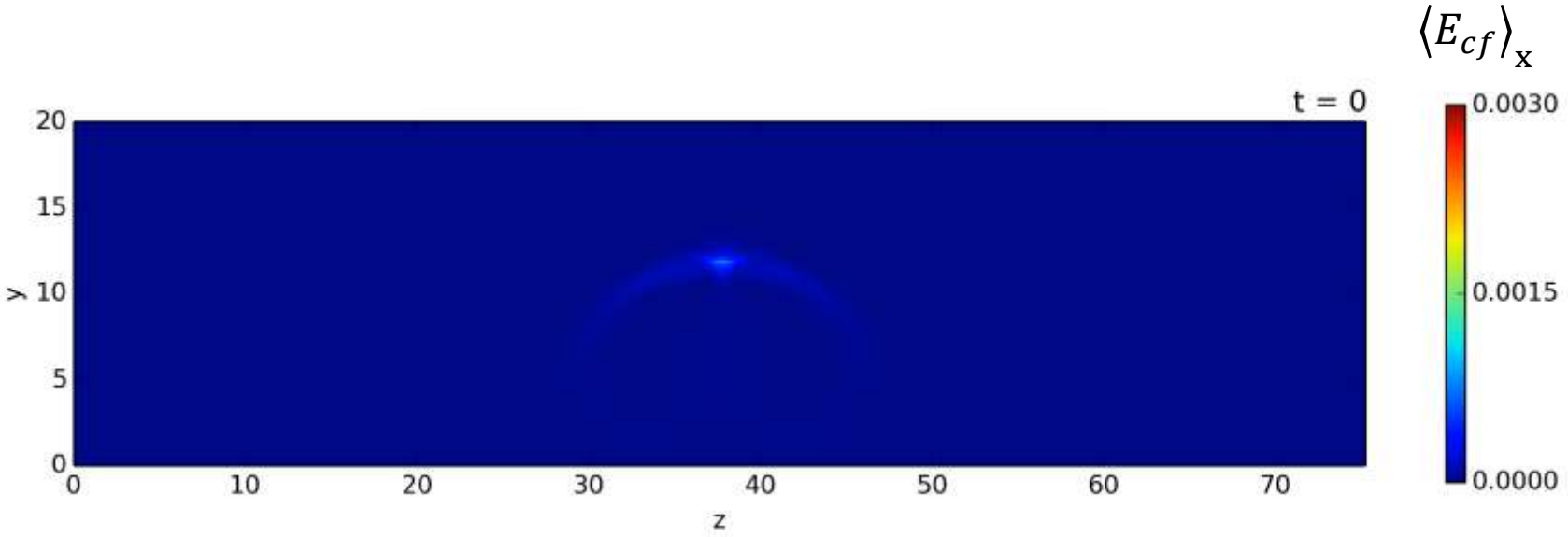


Leading unstable eigenmodes

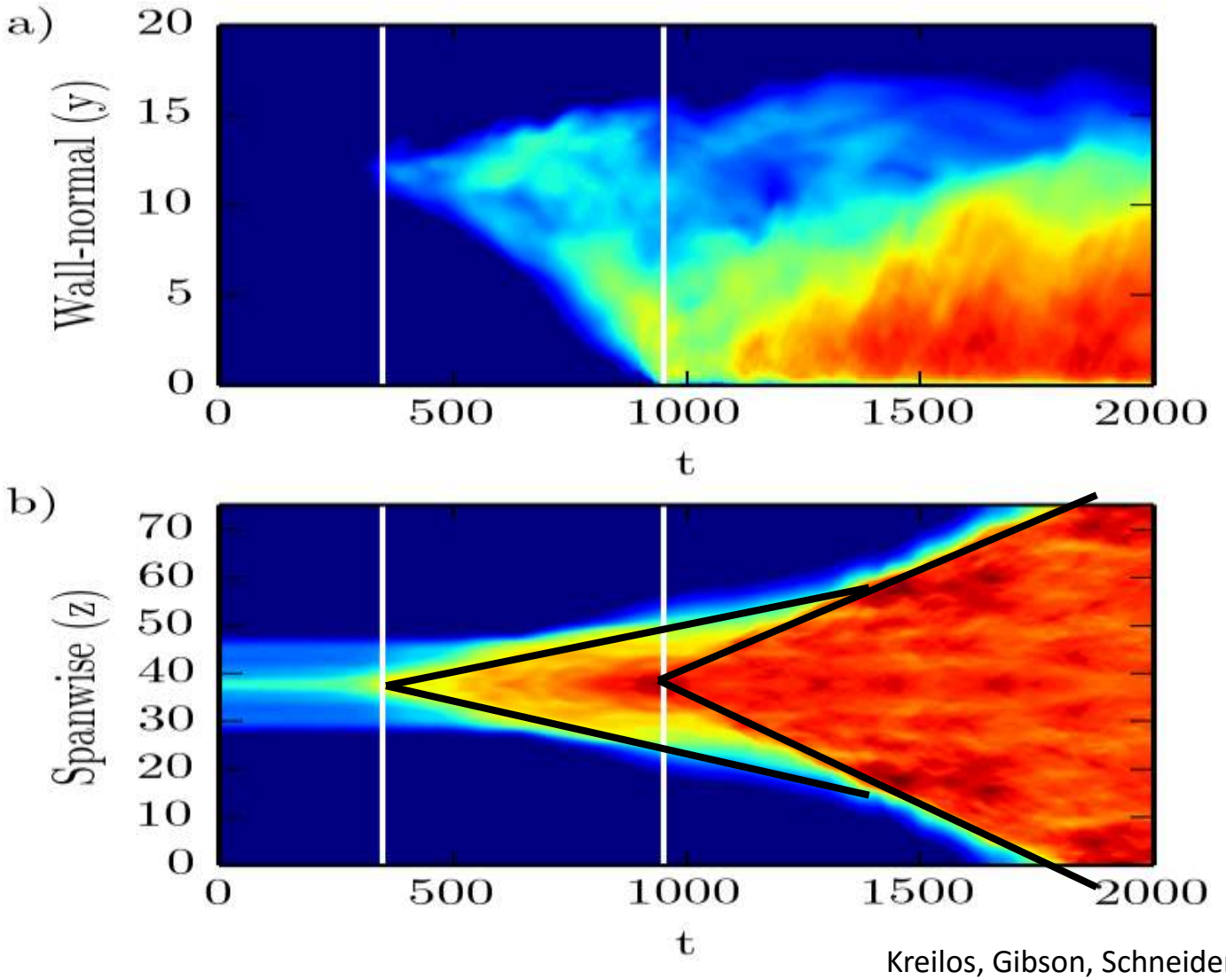



How are they connected to turbulence and transition?
t=0

Evolution in x-y plane



Evolution of FCS

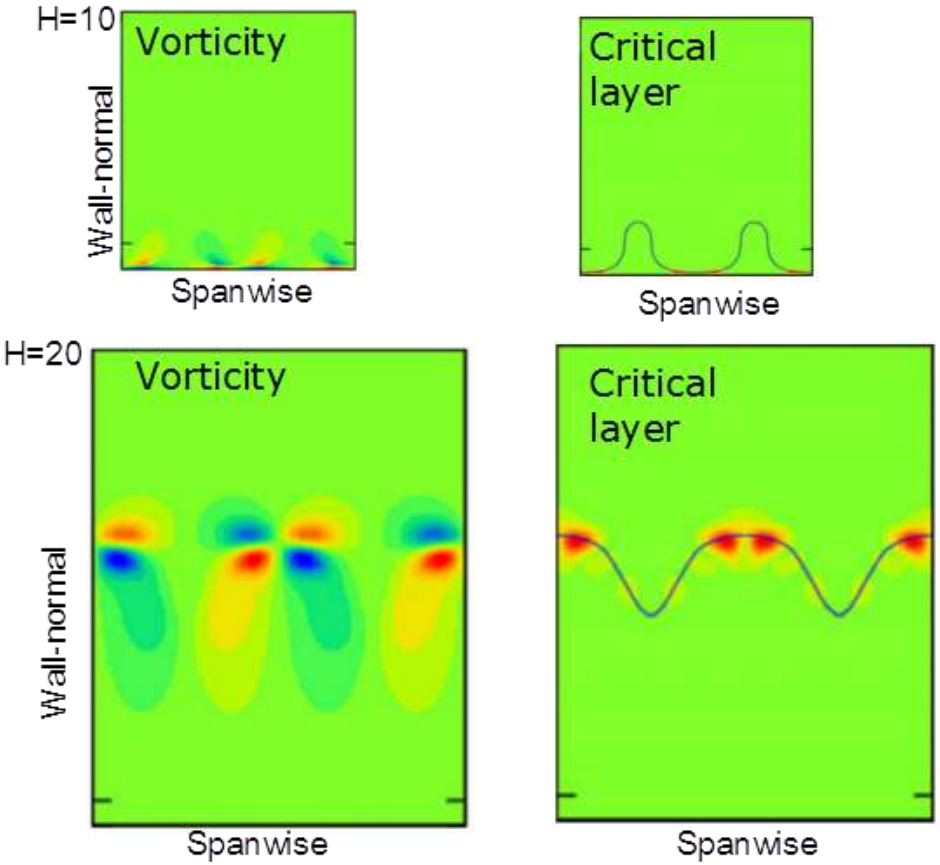


Kreilos, Gibson, Schneider, in preparation

Small periodic domain

Which do we know?

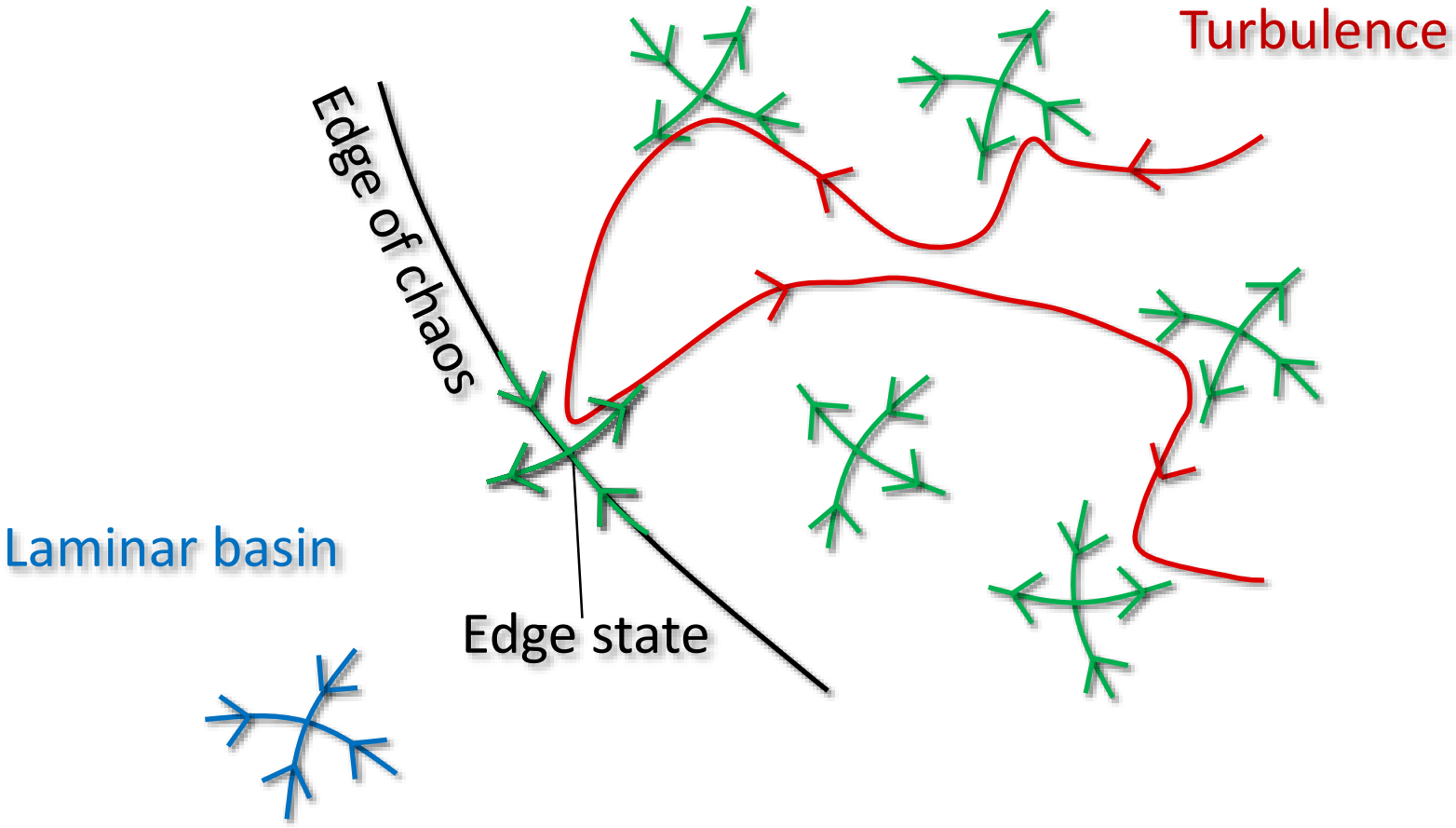
Wall mode



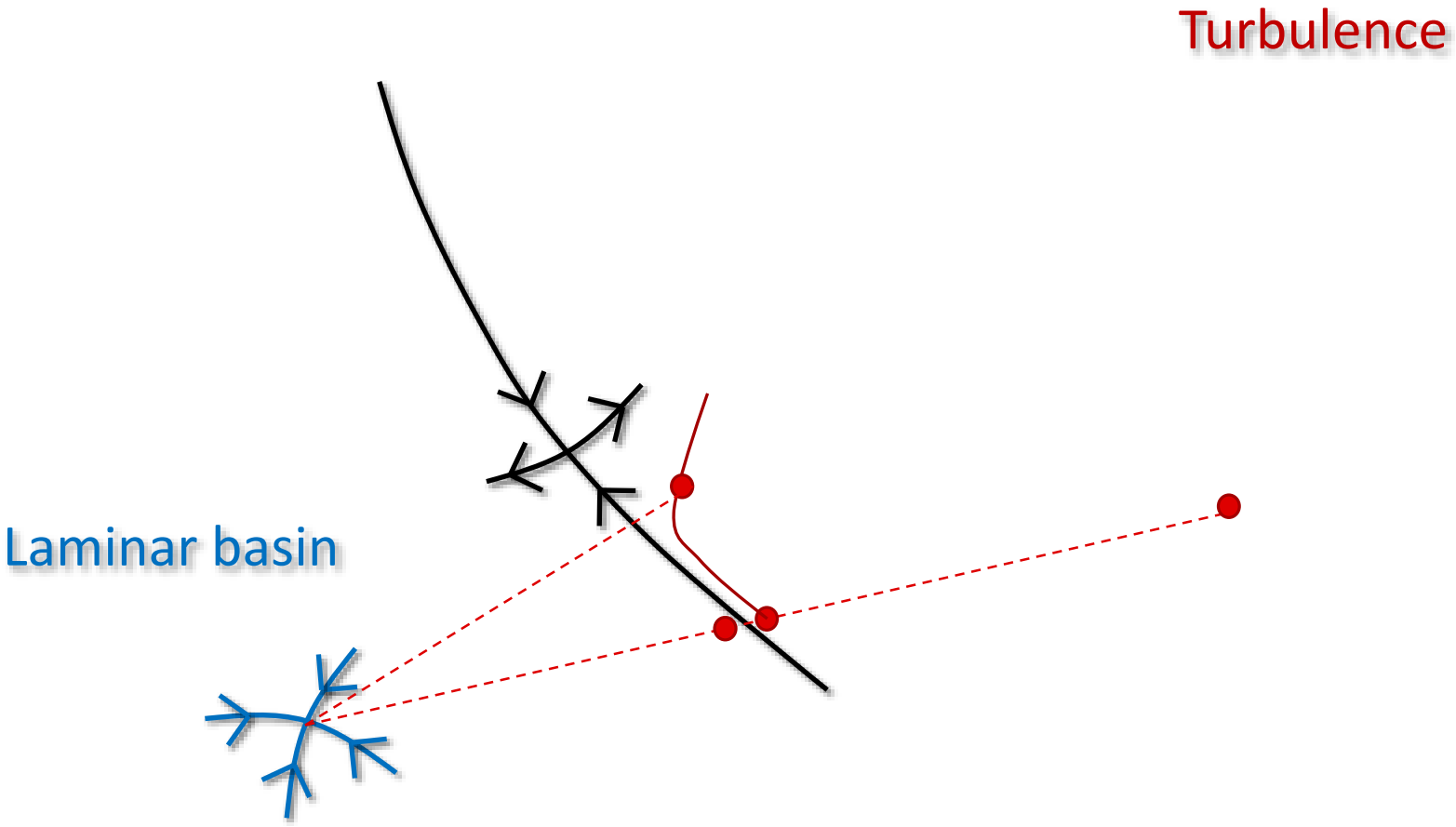
Free-stream coherent structure

Deguchi & Hall, JFM 2014

The edge state

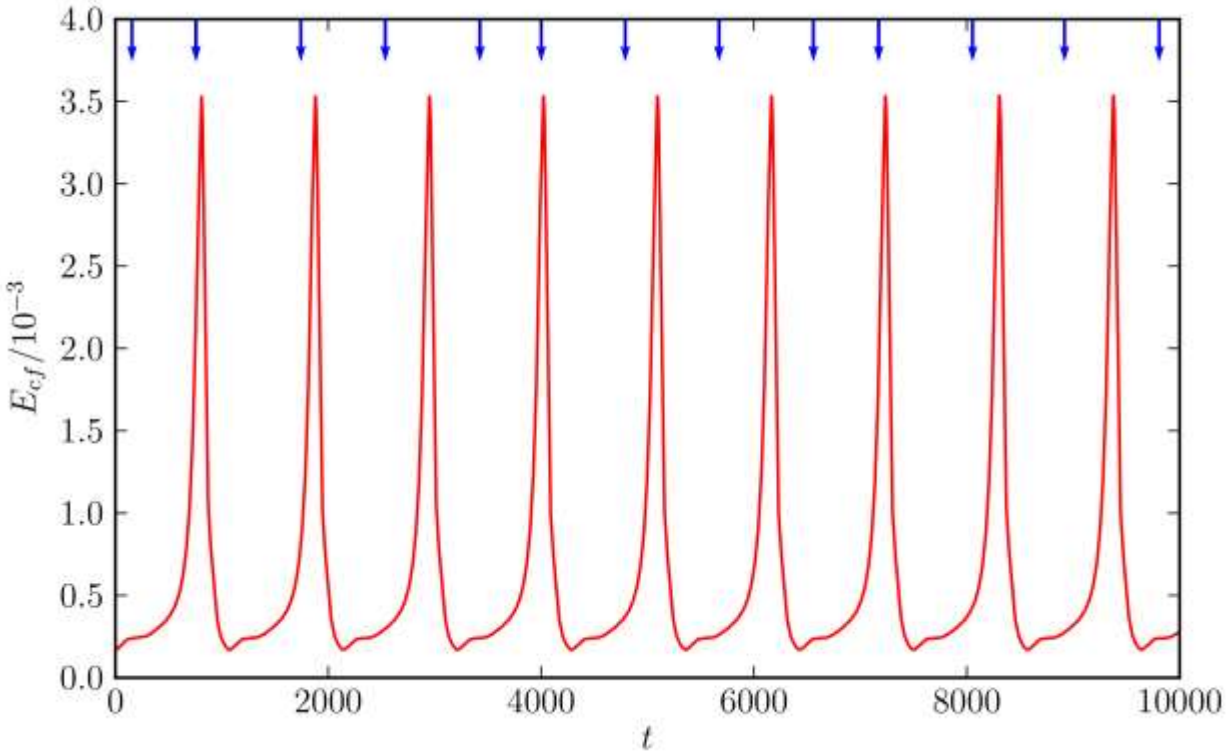


Finding the edge state



Toh & Itano, JFM 2003
Schneider, Eckhardt, Yorke, PRL 2007

The edge state in the ASBL



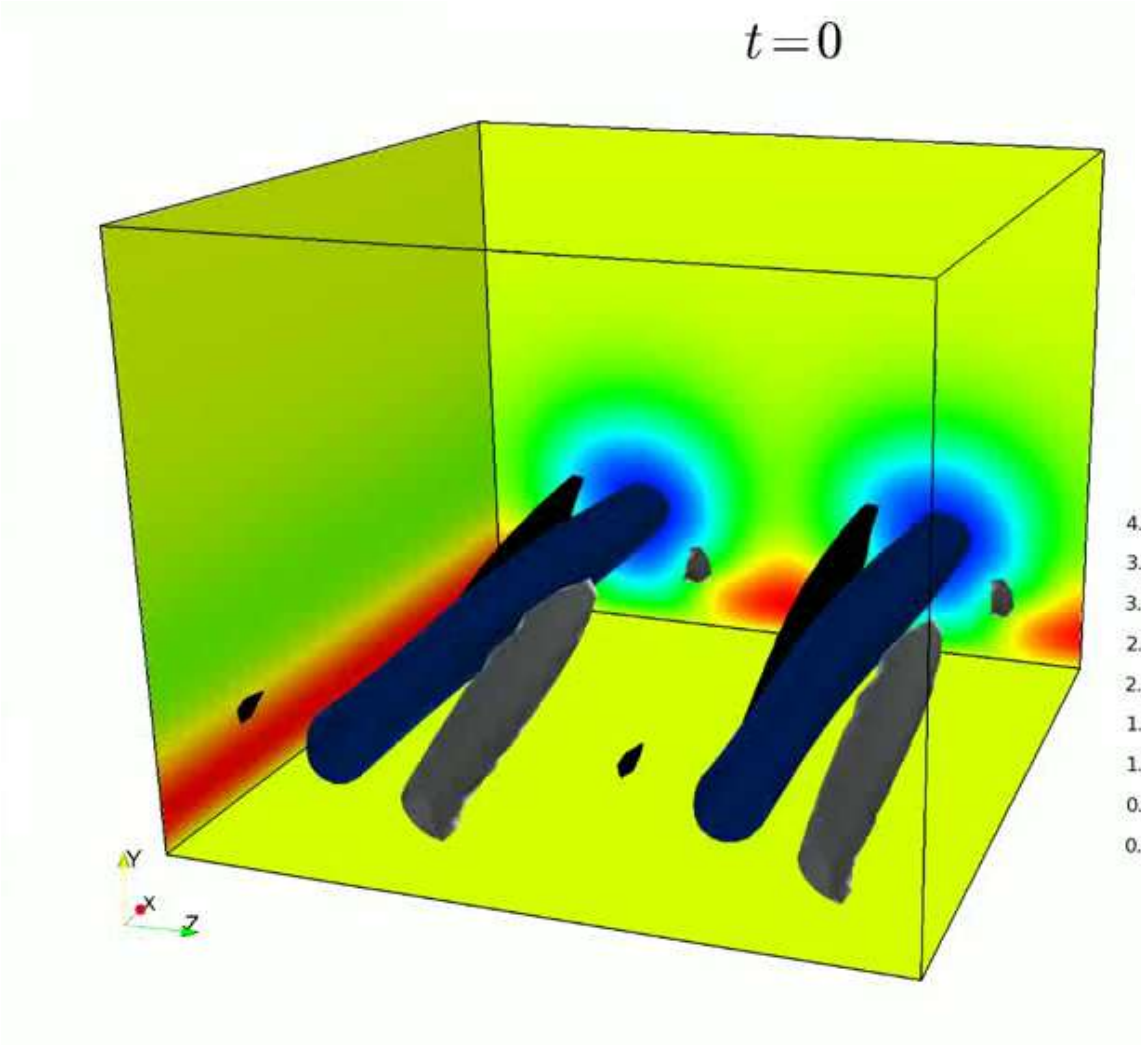
$$E_{cf} = \frac{1}{V}\int_V (v^2 + w^2) dV$$

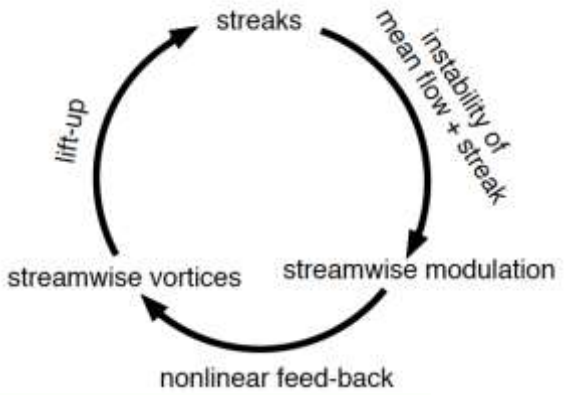
Looks like we found a periodic orbit

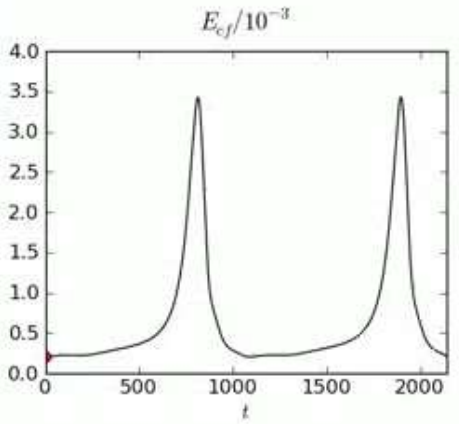
Kreilos et al., JFM 2013

The edge state in the ASBL

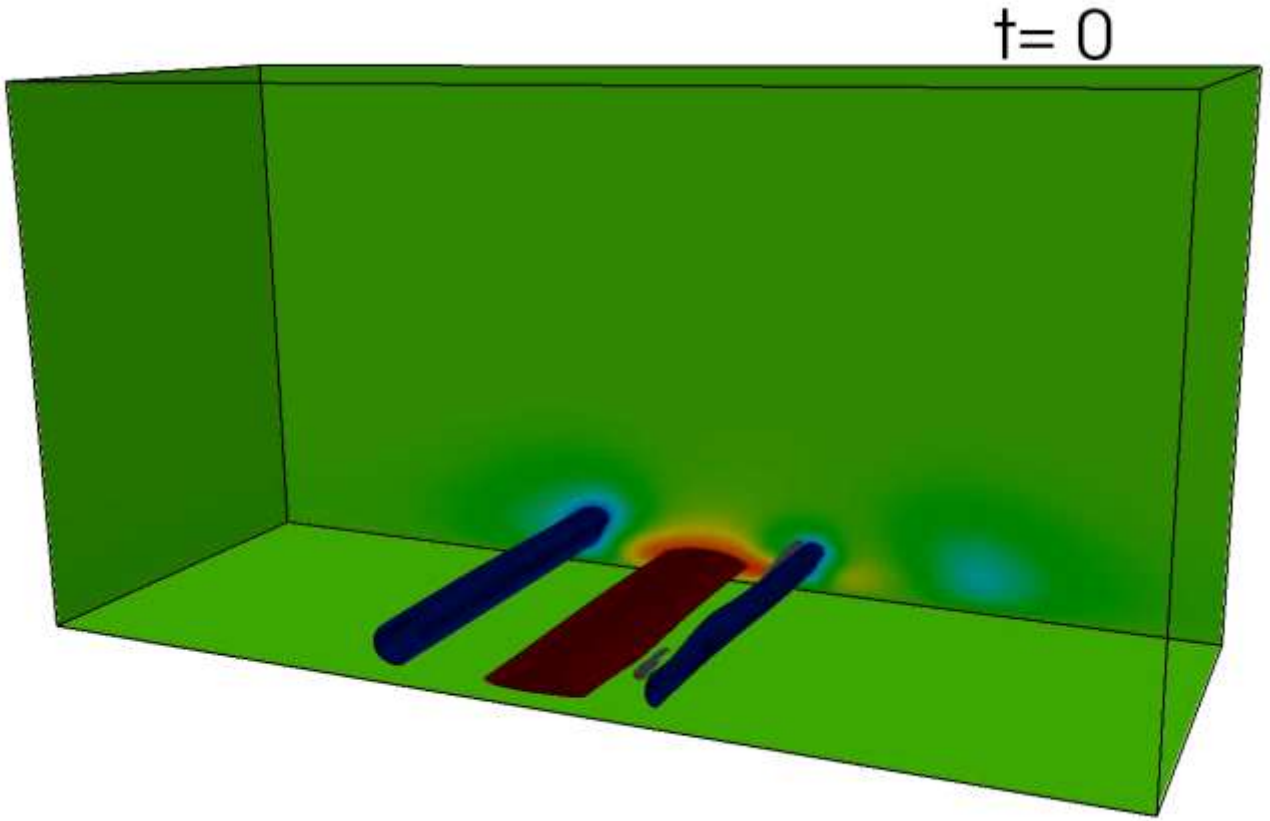
Which do we know?







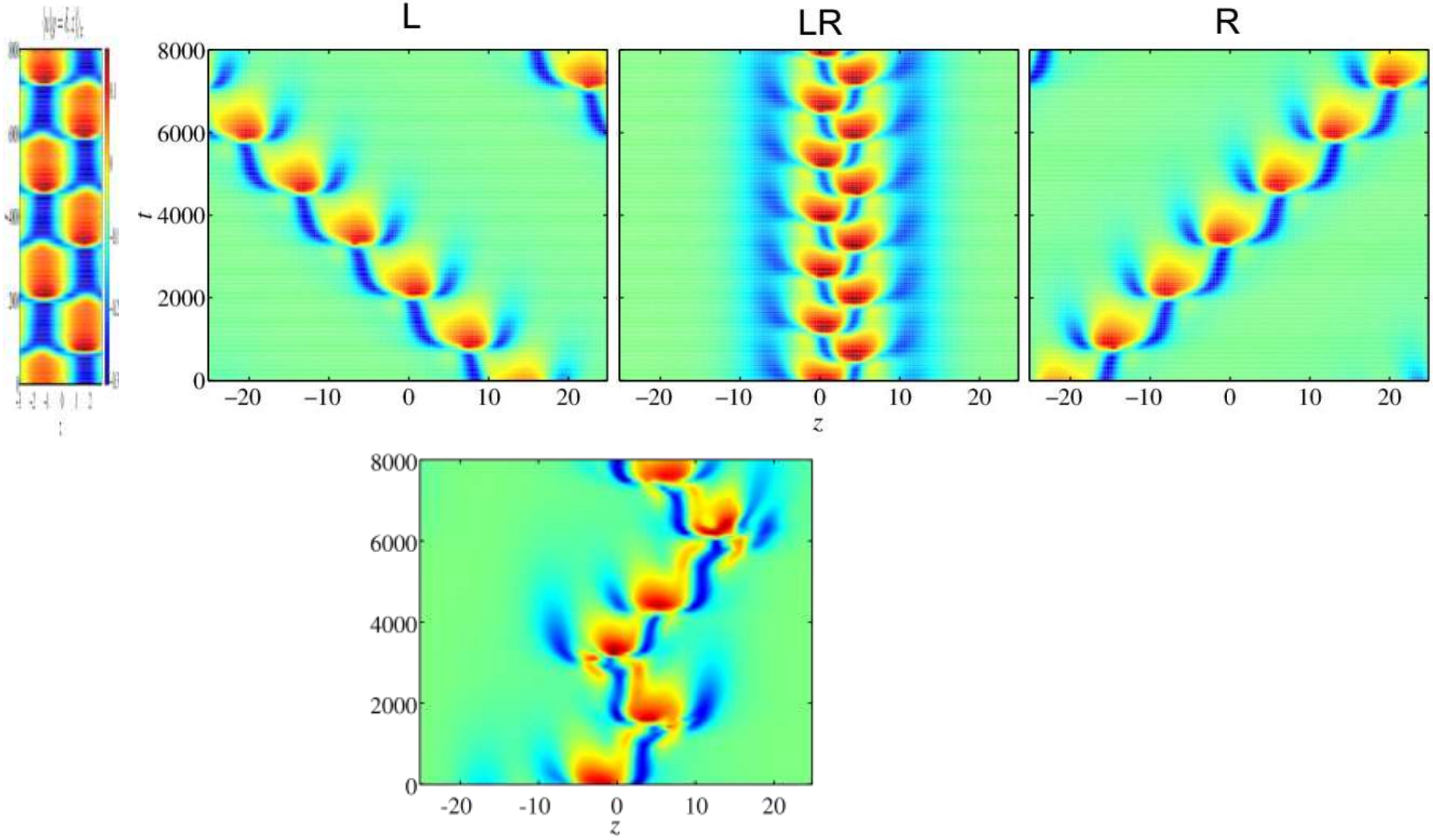
Spanwise extended domains - localization



Khapko, Kreilos, et al., JFM 2013
Khapko, Kreilos, et al. EPJE 2014

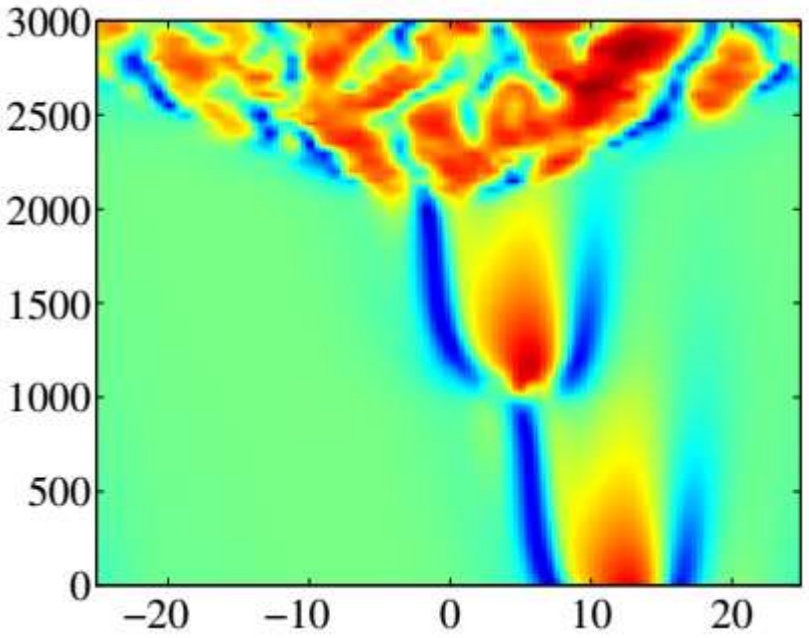
Spanwise extended domains

Which do we know?

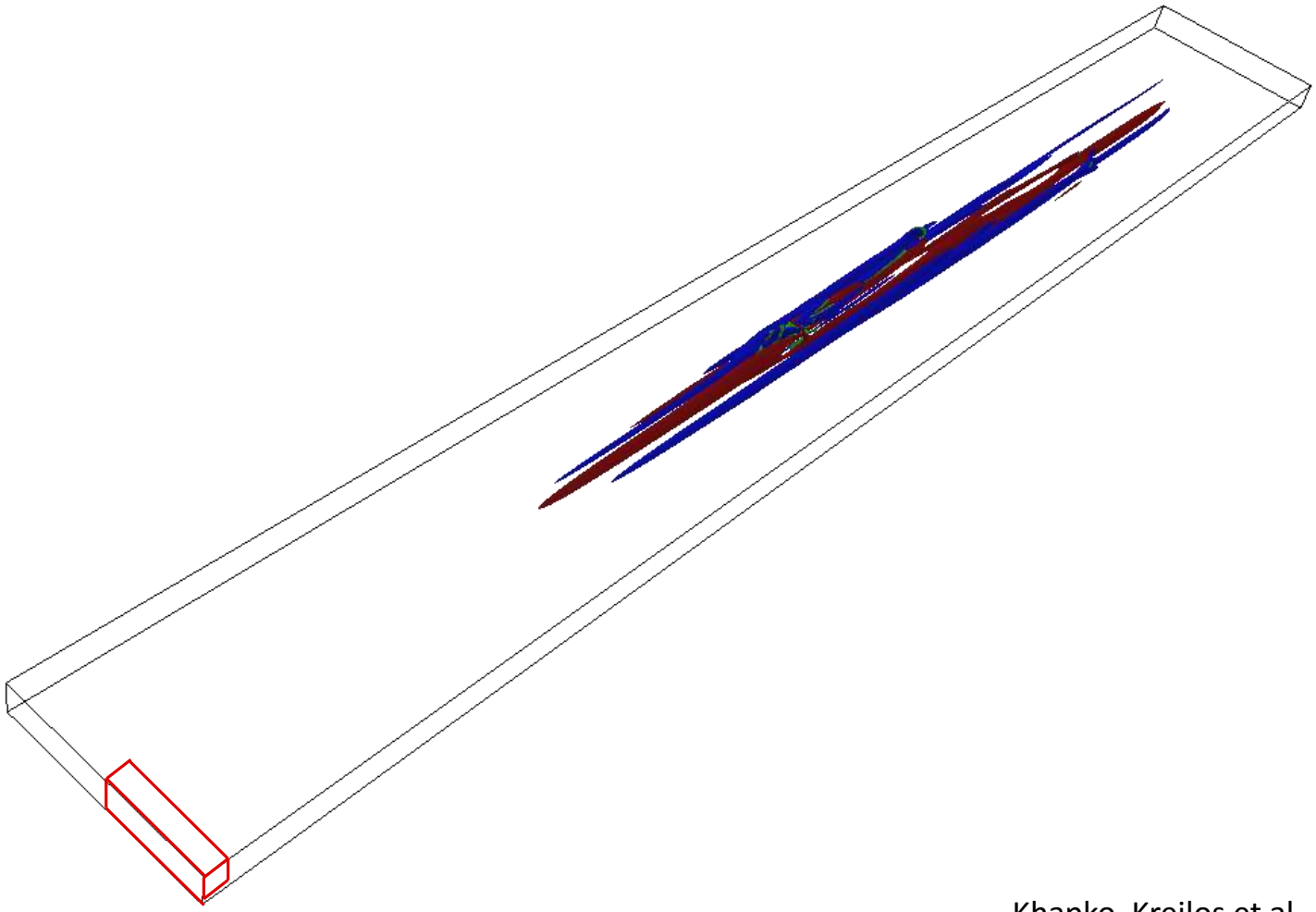


Transition to turbulence

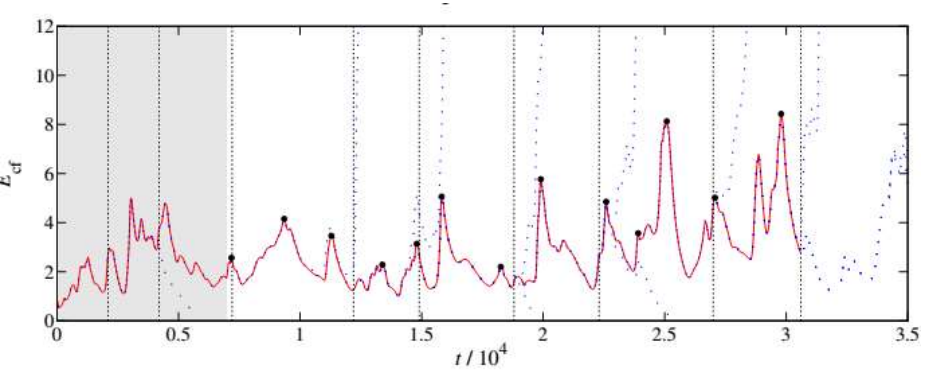
How are they connected to turbulence and transition?



Increasing the domain size

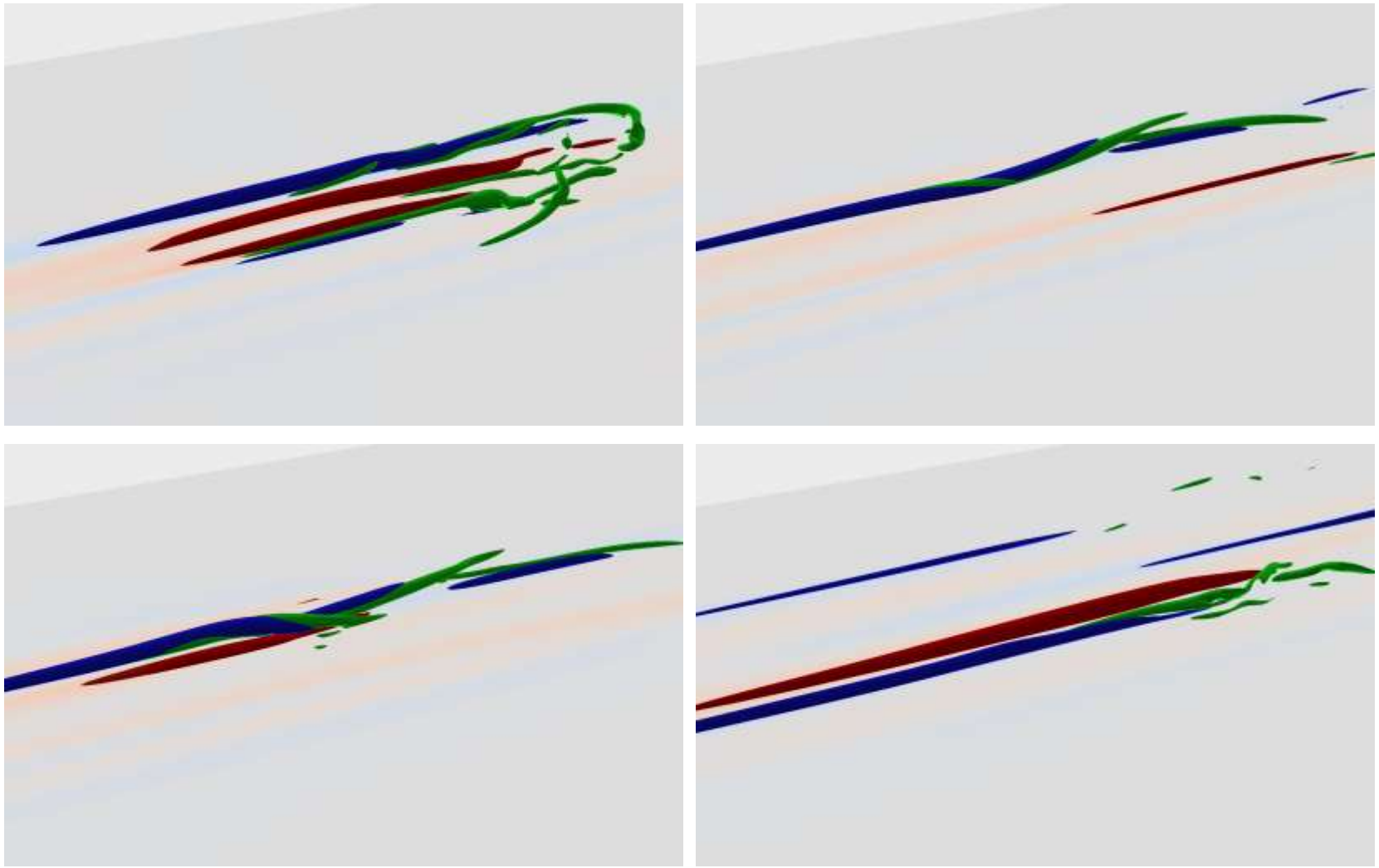


Khapko, Kreilos et al., in preparation



Time: 0

Zoom into a burst

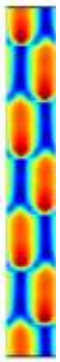


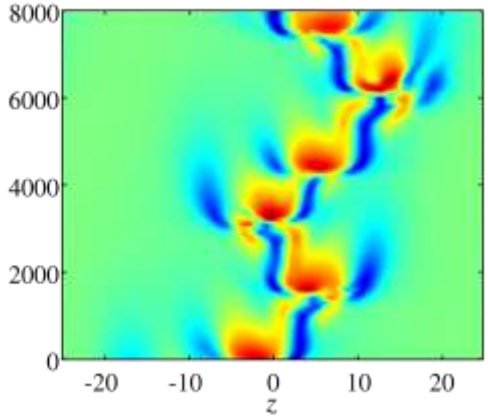
Edge states in the ASBL

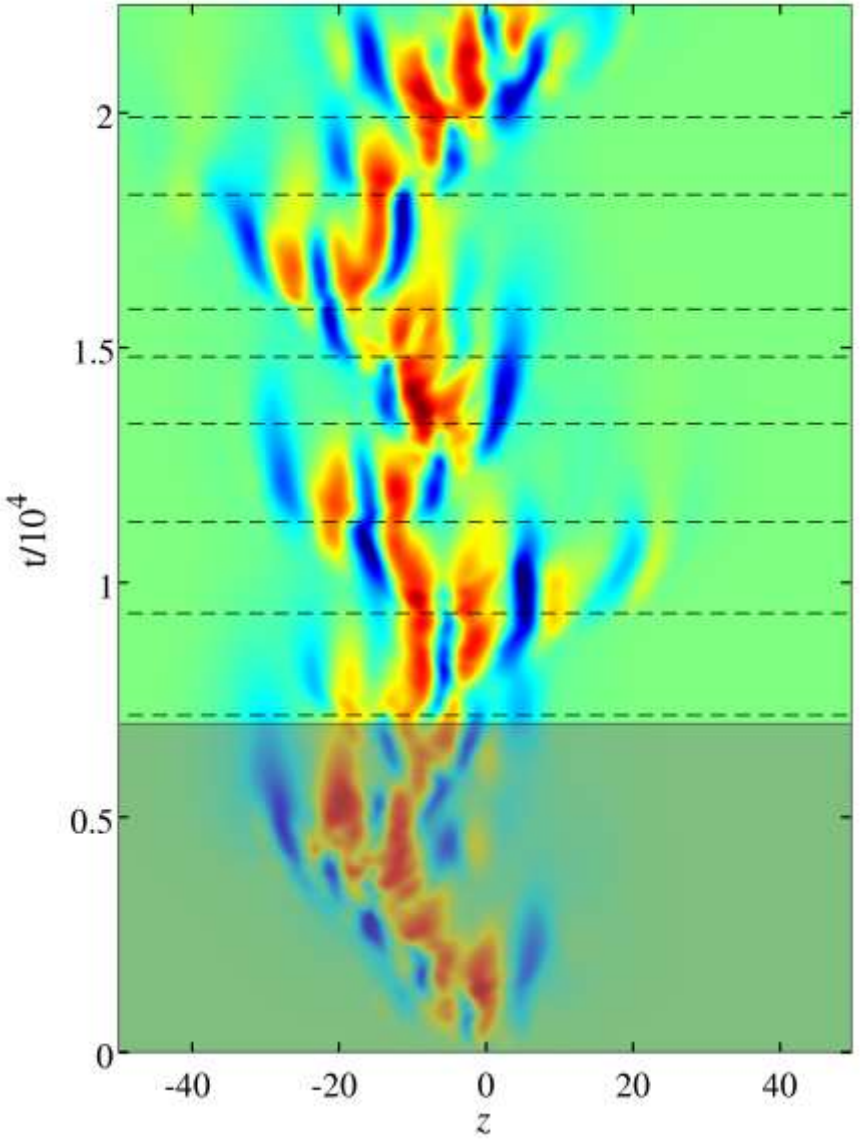
Which do we know?

All show same qualitative behavior:

- Streaks flanked by vortices
- Vortices cross over the streaks
- Streaks break up
- Structures reform at shifted location

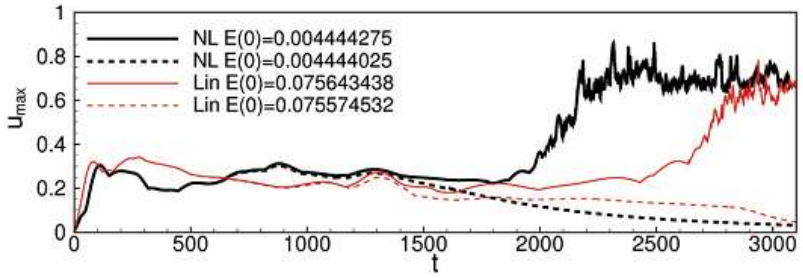




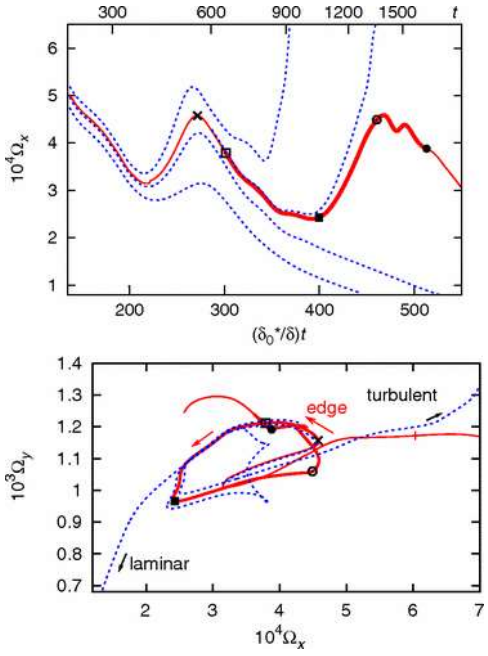


Edge states in the Blasius BL?

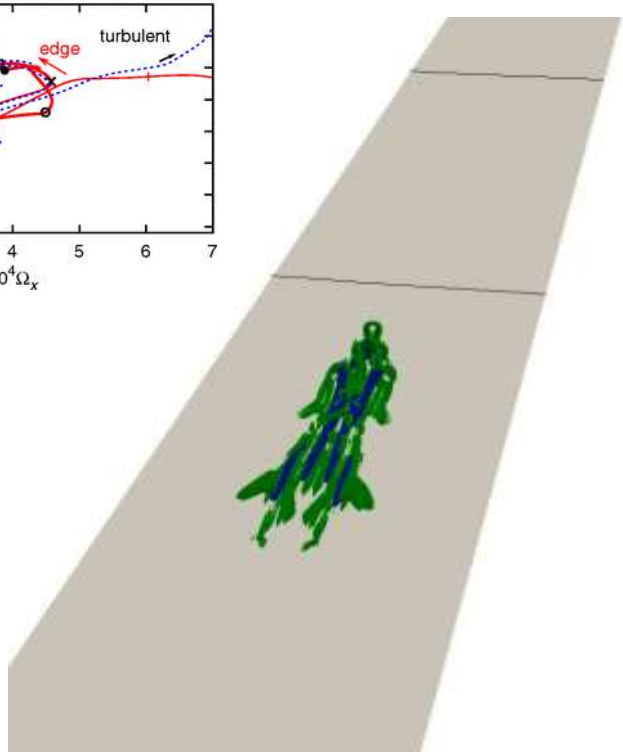
Which do we know?



Cherubini et al., PoF 2011



Duguet et al., PRL 2011

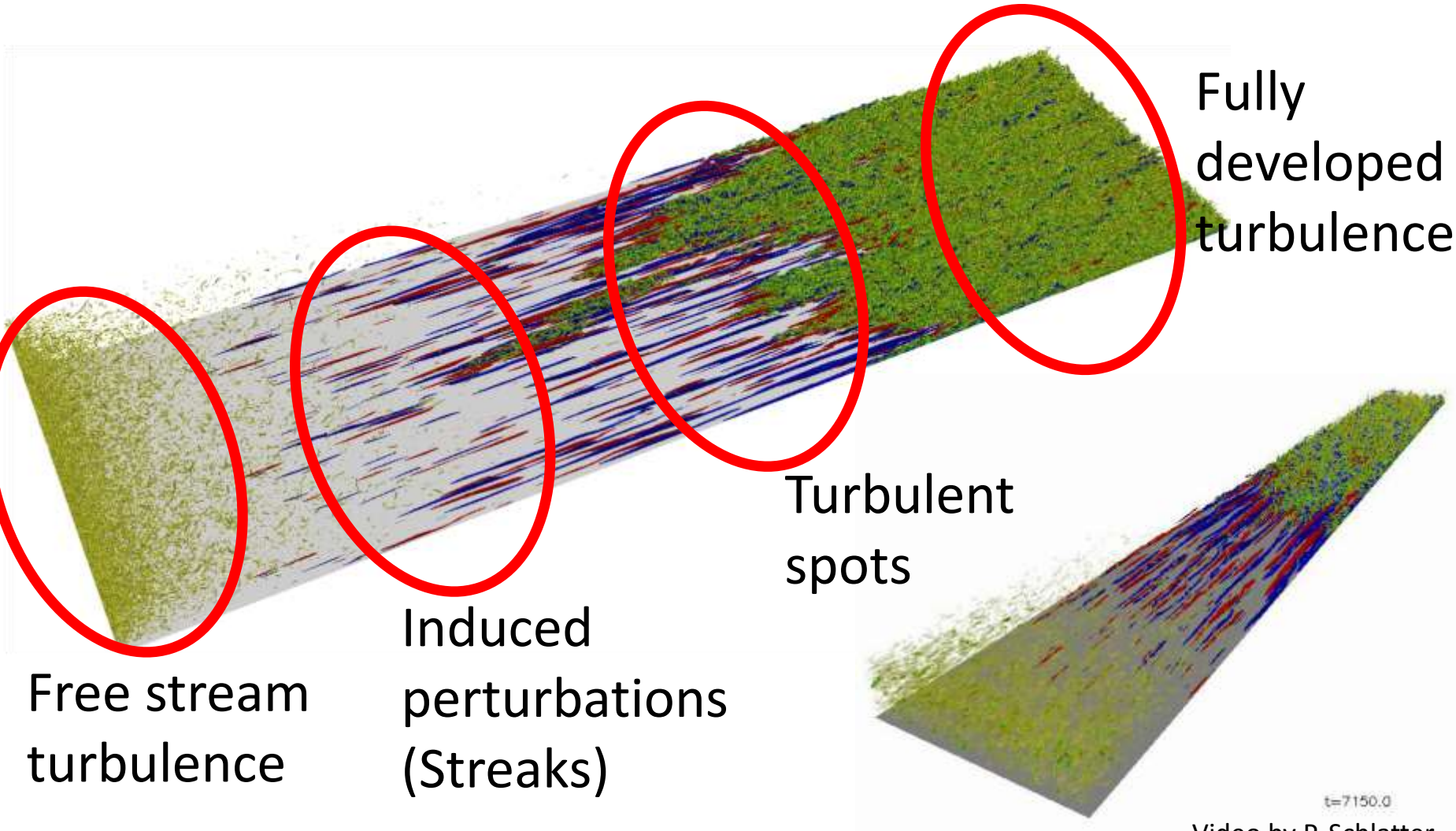


Conclusions:

- There is an edge!
- But: no access to asymptotic dynamics

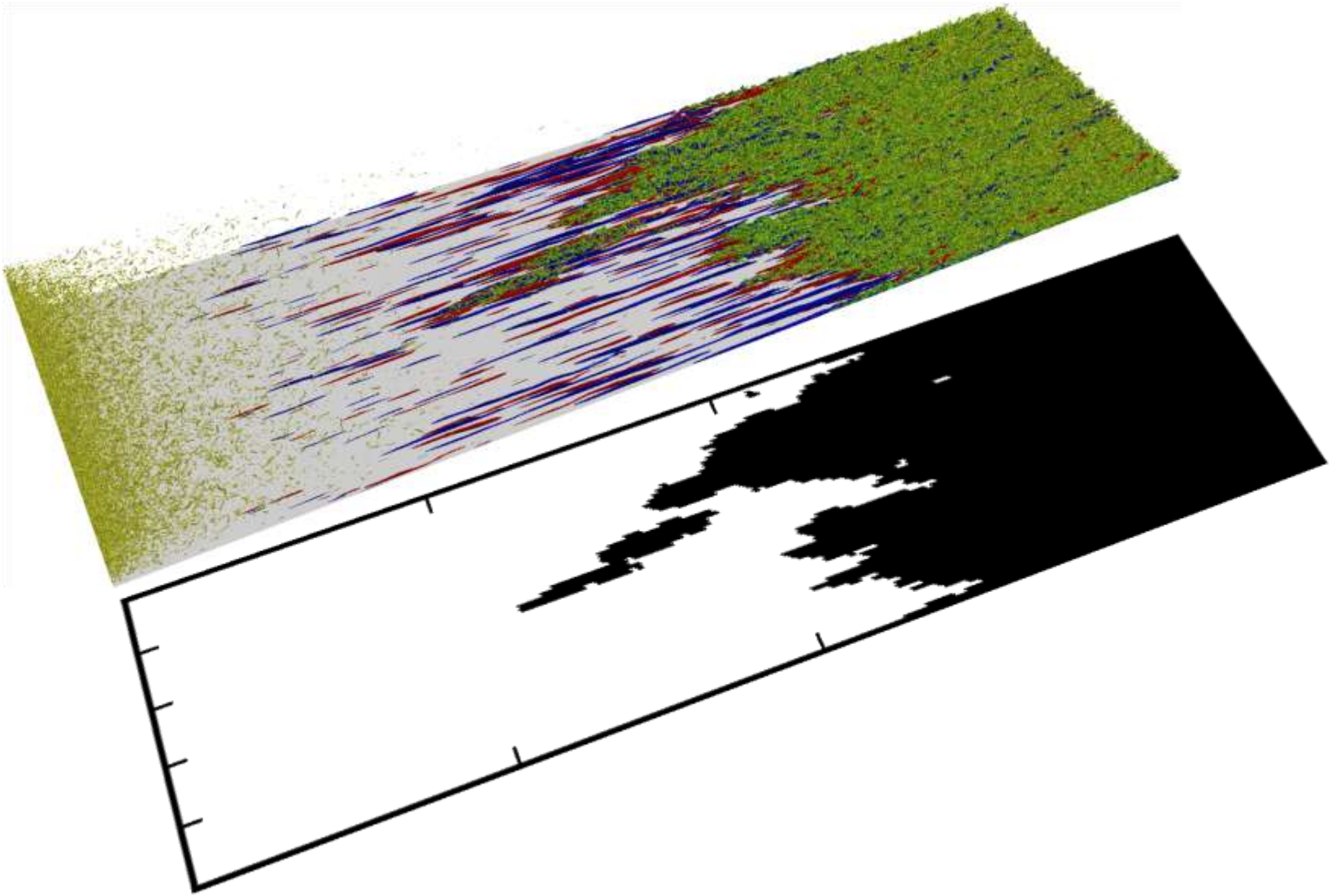
Application: turbulent spots

Where can we go from there?

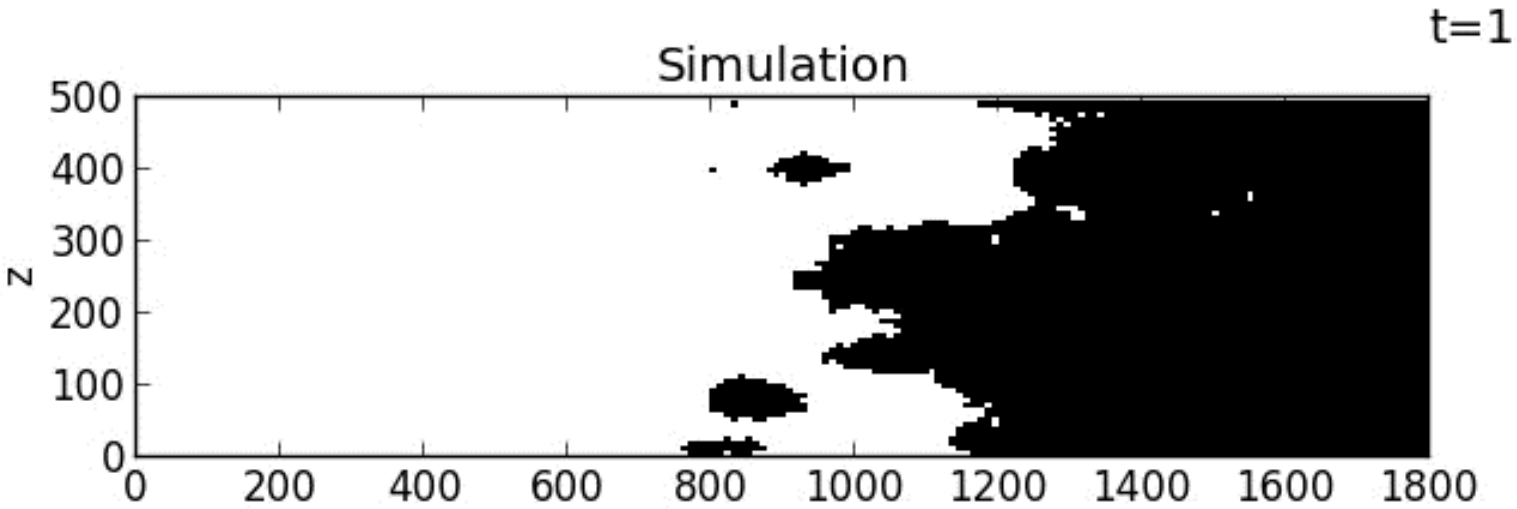


Video by P. Schlatter

Binary representation



Binary representation

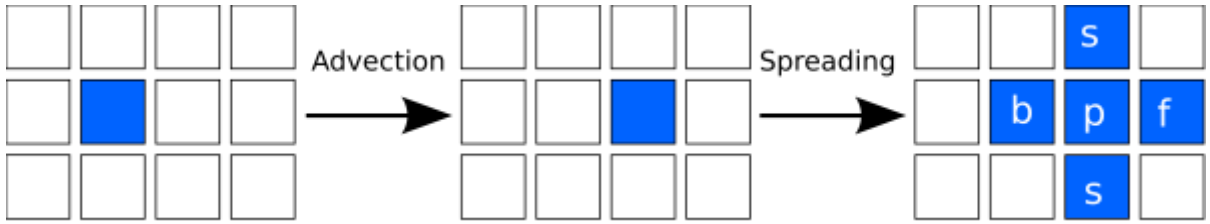


2-step model:

- Evolution of spots
- Nucleation of spots

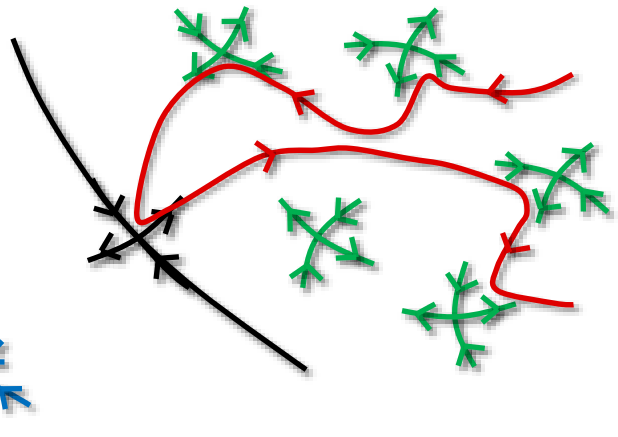
Kreilos et al, under review

Spot evolution: cellular automaton

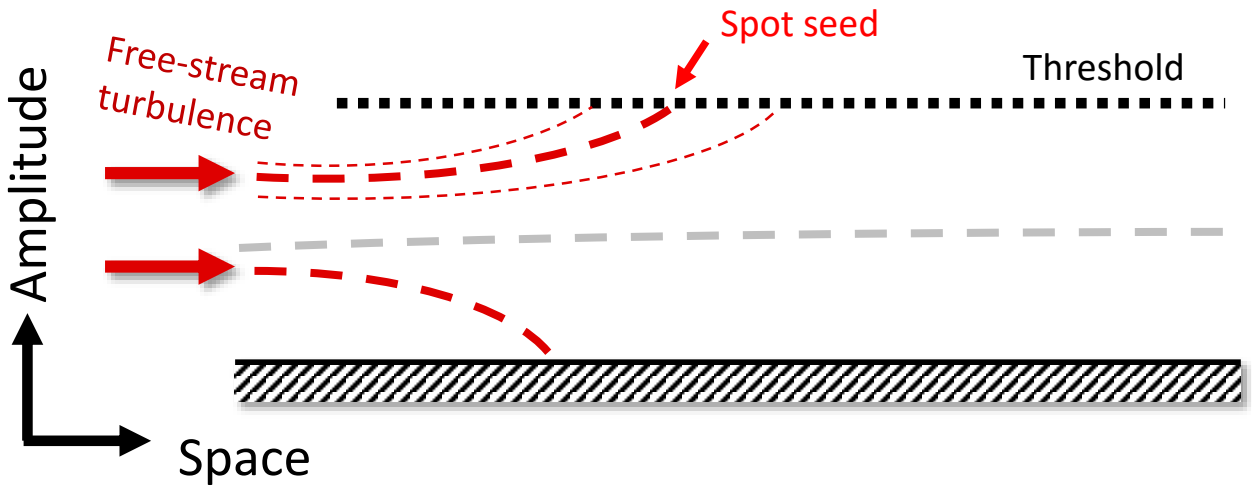


- Proabilities fitted directly from LES simulations
- All probabilities are almost constant
- Values independent of intensity of free-stream turbulence
- **Spot evolution is an activated process**

Spot nucleation rate

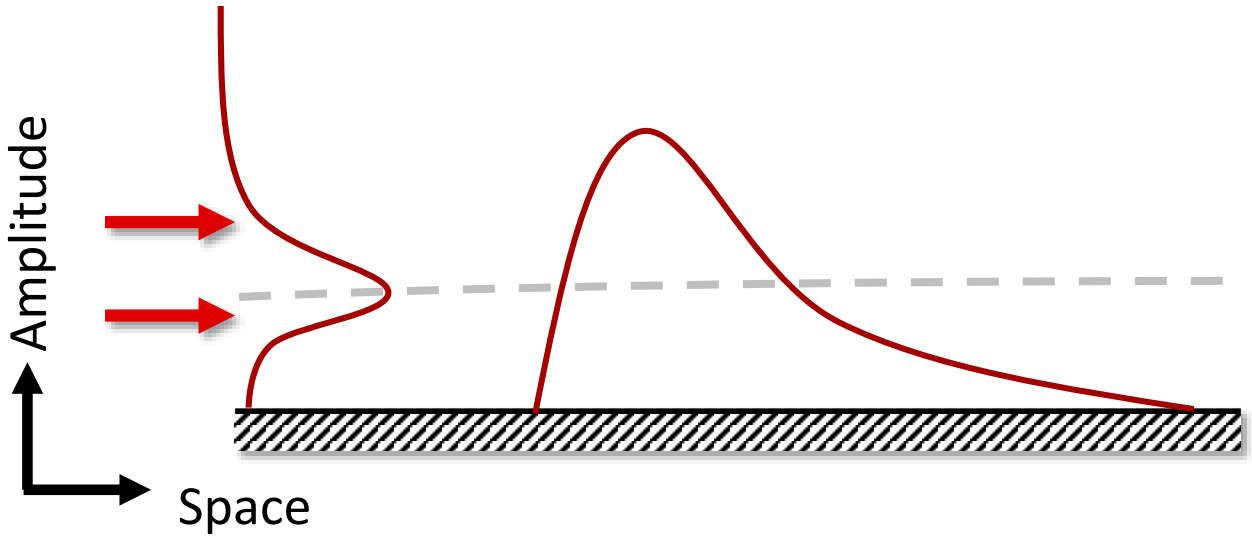


- Question: when and where are spots created?
- Model inspired by state space structure

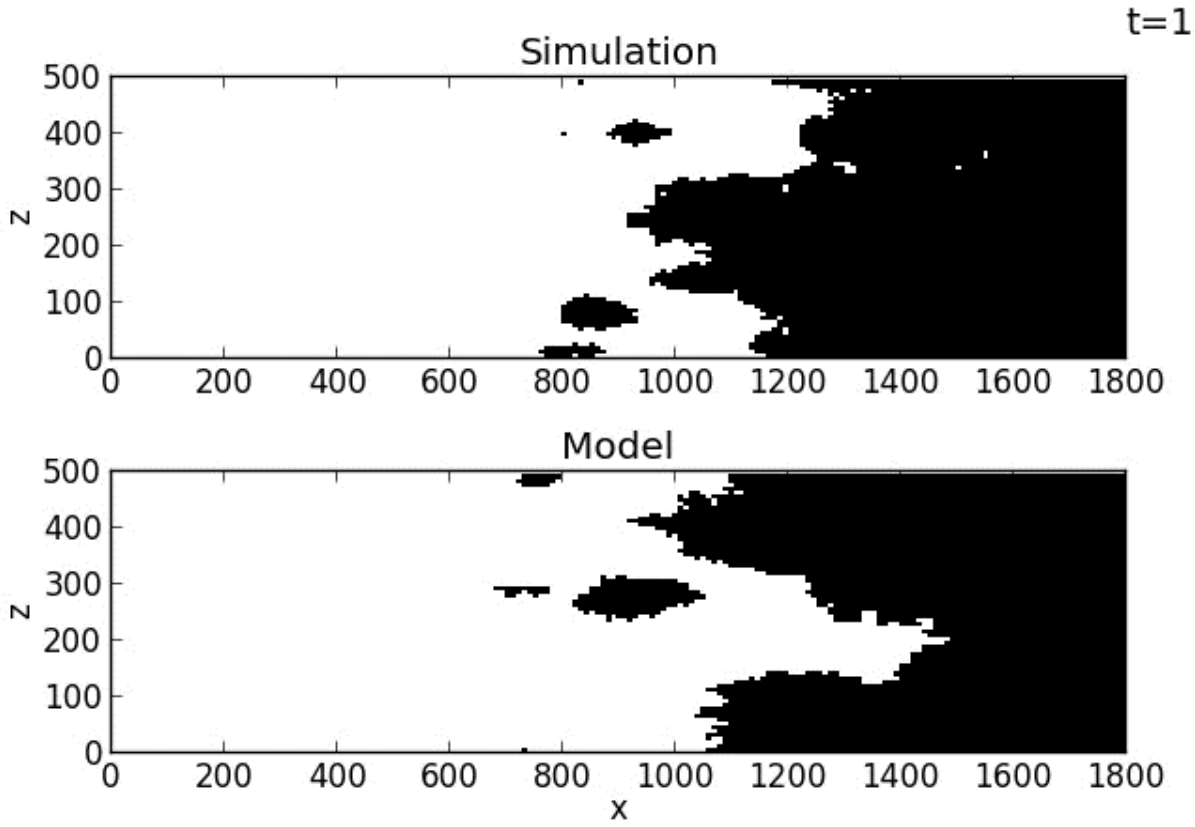


Spot nucleation rate

- Distribution of initial amplitudes → distribution of spot nucleations in space

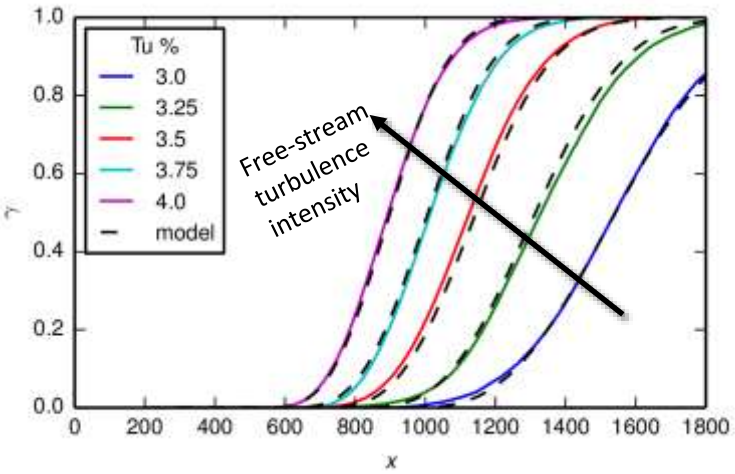


Combining nucleation and spreading

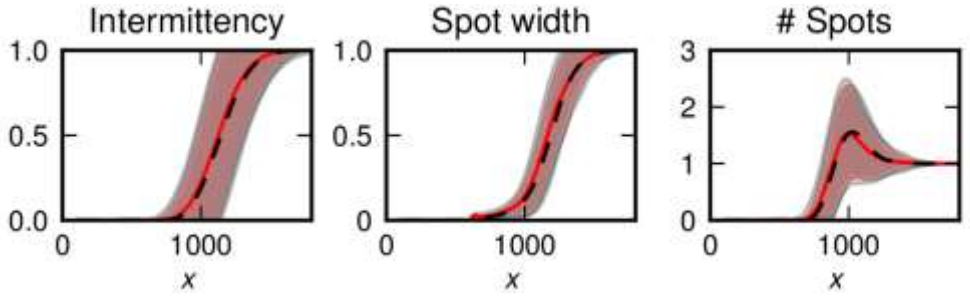


Quantitative comparison

- Intermittency factor: fraction of space covered by turbulence

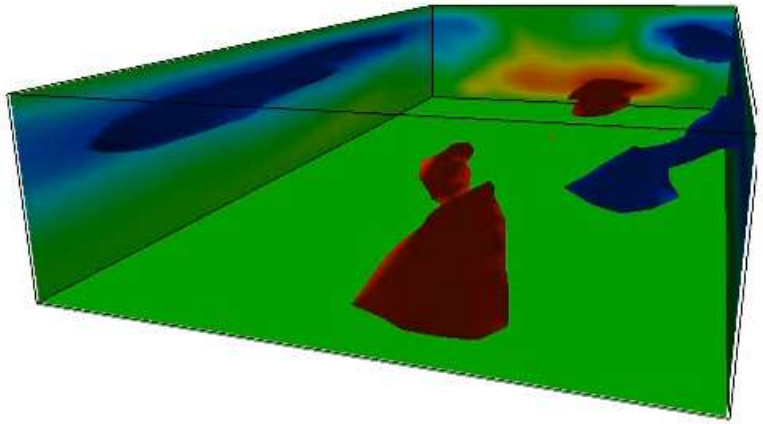


- Spot statistics

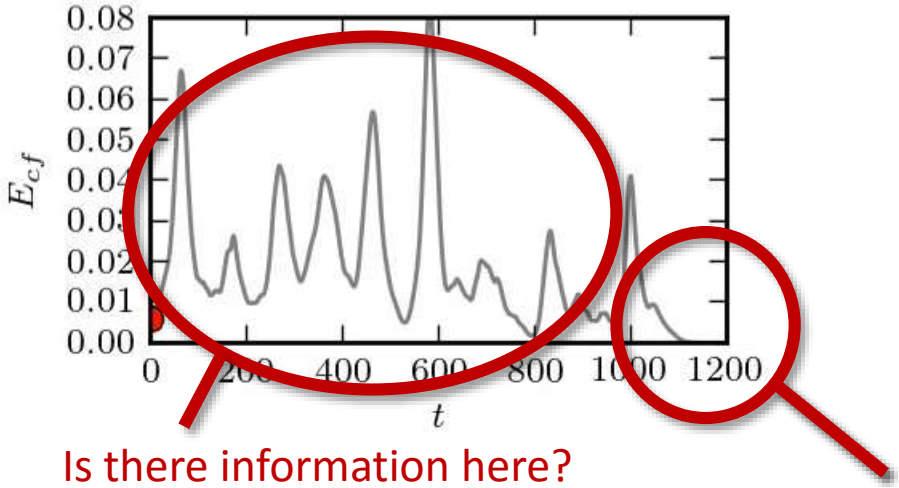


Application: relaminarization

Where can we go from there?

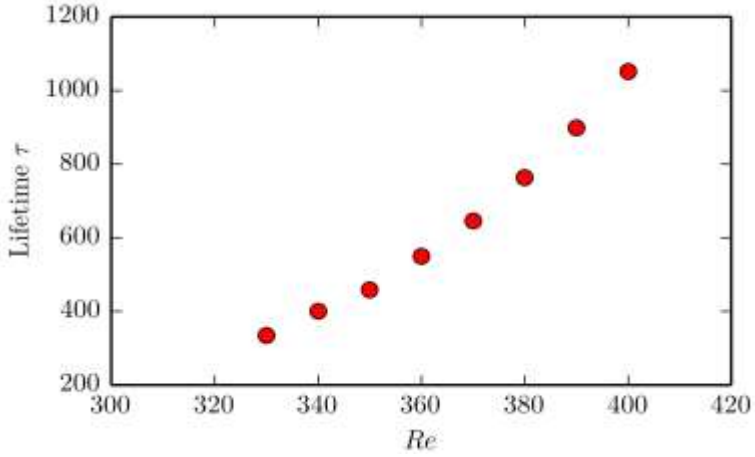


Can we predict lifetimes by measurements in the turbulent part?



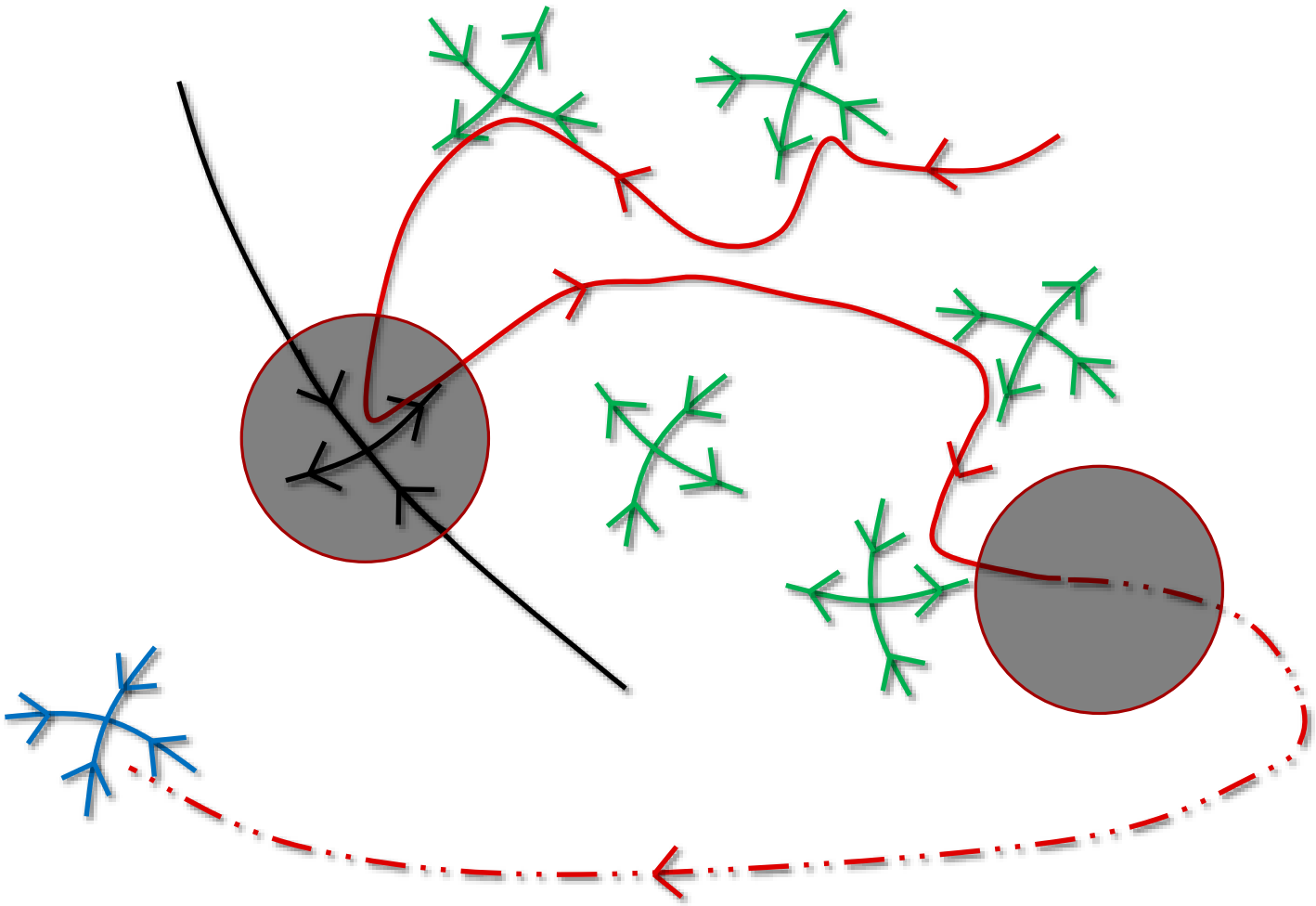
Is there information here?

Usually used information



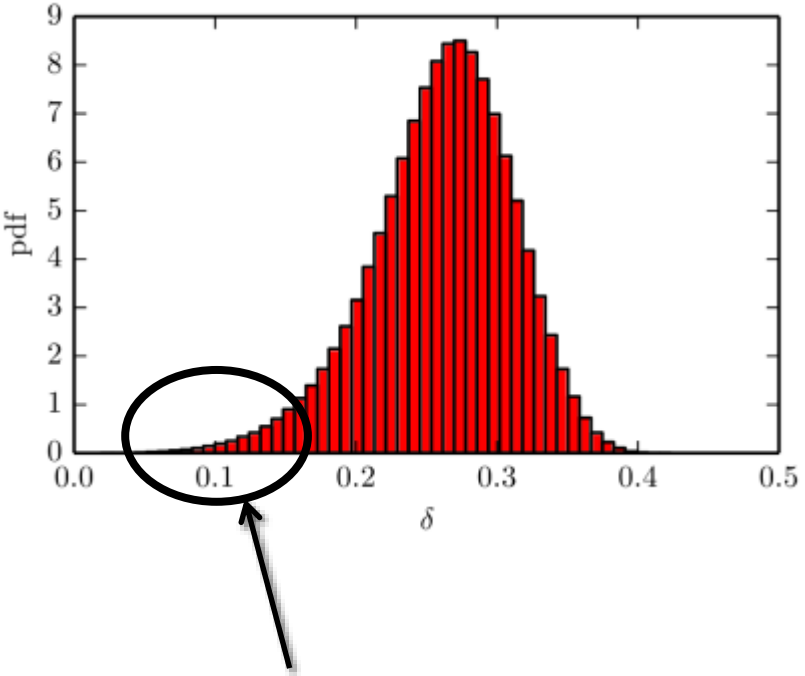
Kreilos et al, in preparation
cf. Goldenfeld, Guttenberg, Gioia, PRE 2010

The state space



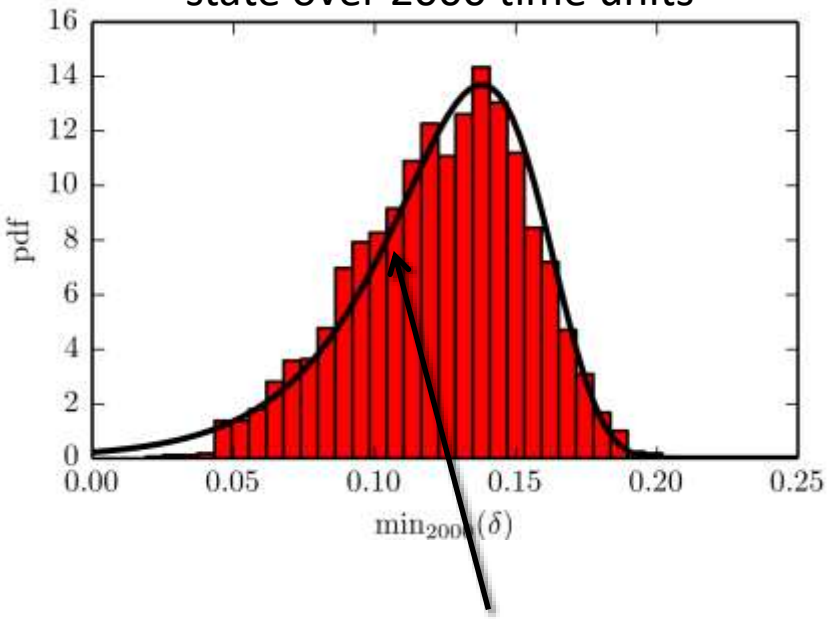
Sampling turbulent trajectories

Distance to the edge state



Close approaches to edge state

Minimum distance to the edge state over 2000 time units

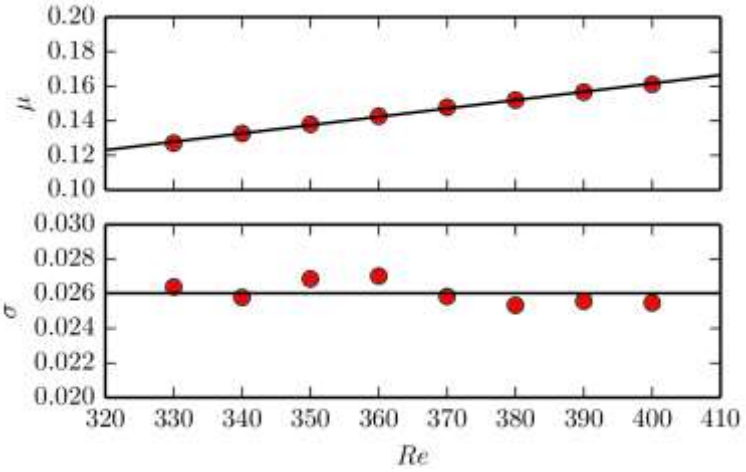


Gumbel distribution

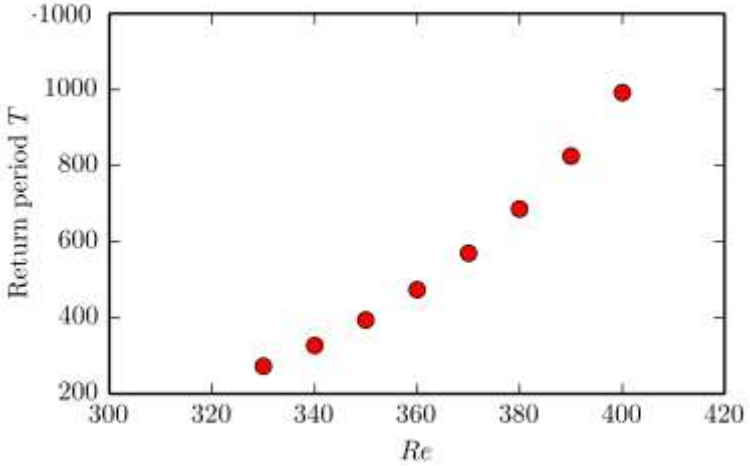
CDF: $F(\delta) = 1 - e^{-e^{\frac{\delta-\mu}{\sigma}}}$

Fitting parameters

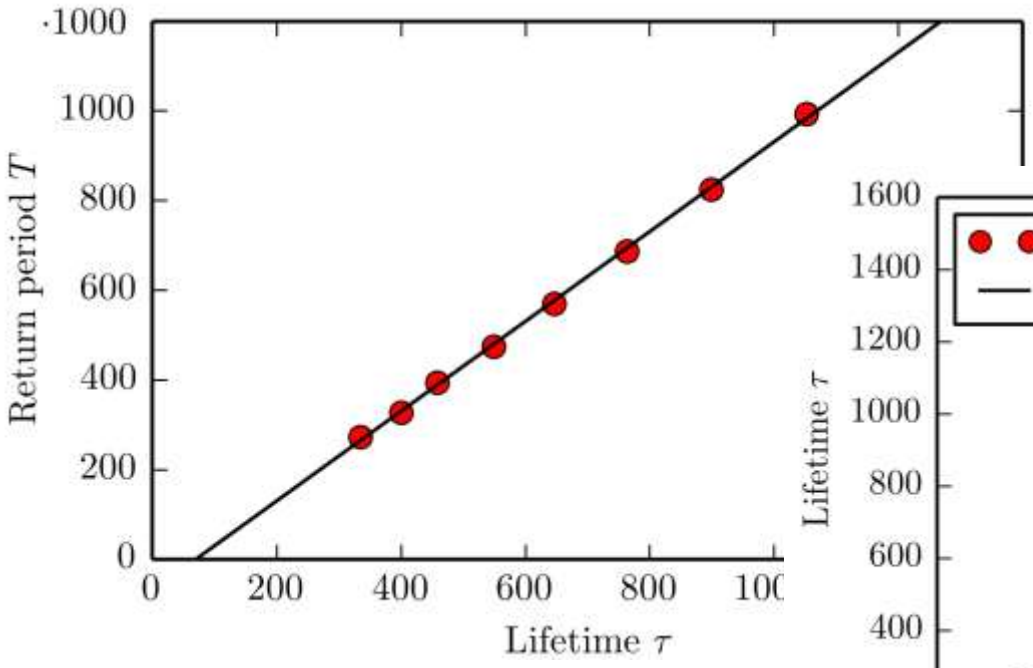
Gumbel CDF: $F(\delta) = 1 - e^{-e^{\frac{\delta-\mu}{\sigma}}}$

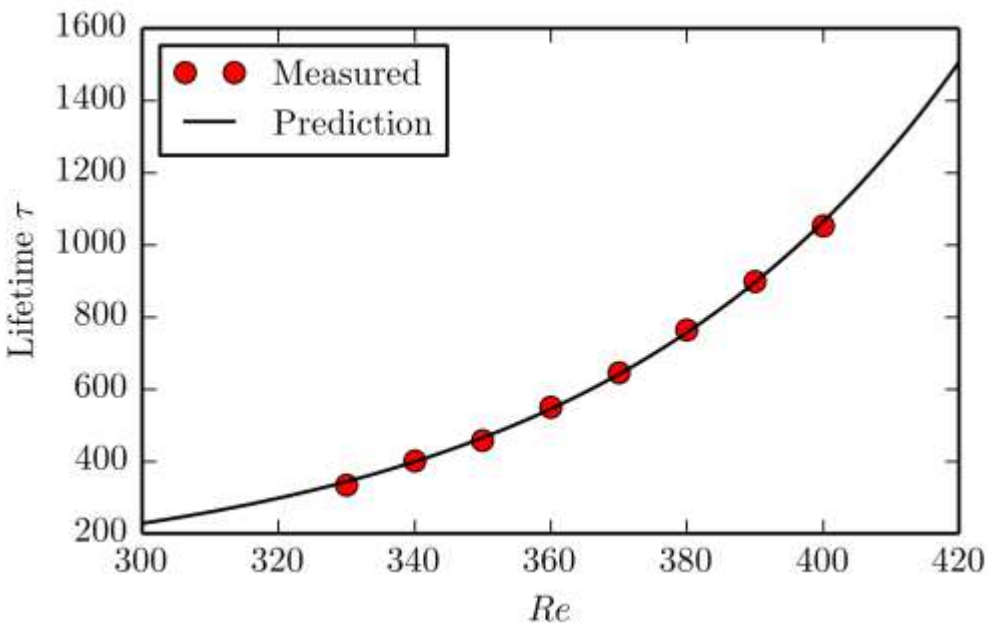


Return period: $T(\delta) = 1/\mathrm{F}(\delta)$



From return periods to lifetimes





Linear relation between approaches to the edge state and relaminarization!

Summary: coherent structures in BL

- **Which do we know? (ASBL)**

	Periodic domain	Spanwise localized	Fully localized
Wall modes	Periodic orbit (edge) Traveling wave (Hall&Deguchi)	Periodic orbit (edge) Traveling wave	Chaotic edge state Traveling wave?
Free-stream modes	Traveling wave (Hall&Deguchi)	Traveling wave	?

- **How are they connected to turbulence and transition?**
  - Edge states: marginally stable, define laminar-turbulent boundary
  - Free-stream modes:
    - Interaction between turbulence in free-stream and wall
    - Transition by spotlike evolution + fast spreading along wall
- **Where can we go from there?**
  - Lifetimes are linearly correlated to return periods to edge
  - Edge inspired nucleation model for turbulent spots

Thank you to

- Tobias M. Schneider, Bruno Eckhardt, John F. Gibson, Taras Khapko, Philipp Schlatter, Dan S. Henningson, Yohann Duguet, Stefan Zammert, Hecke Schrobsdorff, …
- John for providing and maintaining channelflow
- Greg, John and Joe
- **You** for your attention

The state space full slide