A restricted nonlinear framework for studying wall turbulence

Dennice F. Gayme

Collaborators:
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Template courtesy of Rajat Mittal

Context: A word about models

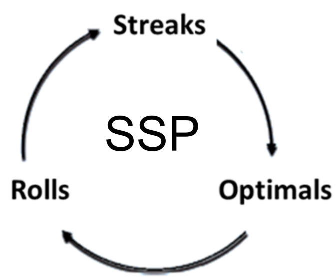
All models are wrong but

some models are useful!

Attributed to George Box

- Utility of a particular model is determined by the information one is seeking with its use

Bigger swirls hav
smaller swirls
That feed on the
velocity …



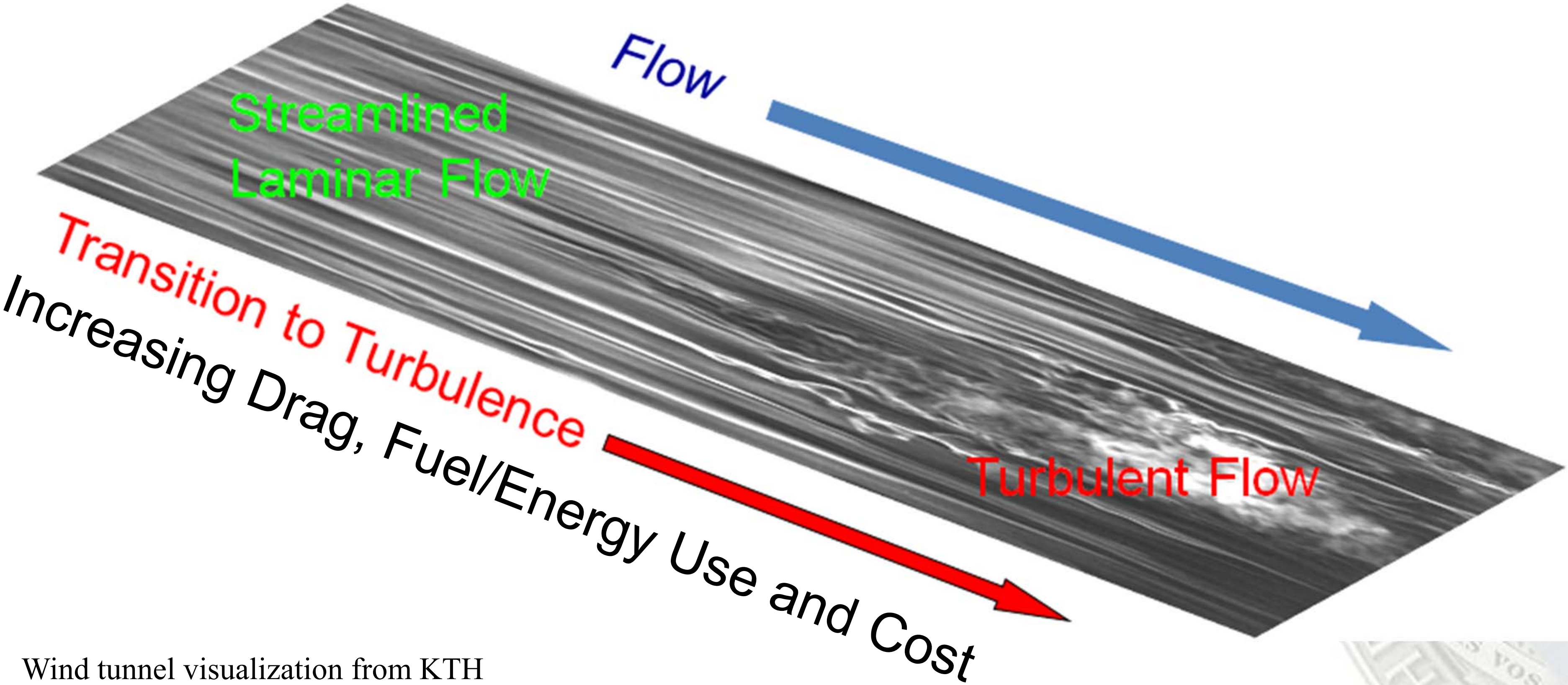
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p = \frac{1}{R}\Delta\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0$$

- Key: know what you want/expect from the model and understand its limitations

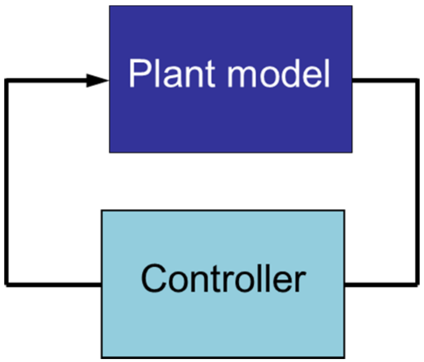
Motivation

- Models that can be used to inform flow control strategies for reducing skin friction drag in wall- turbulence applications



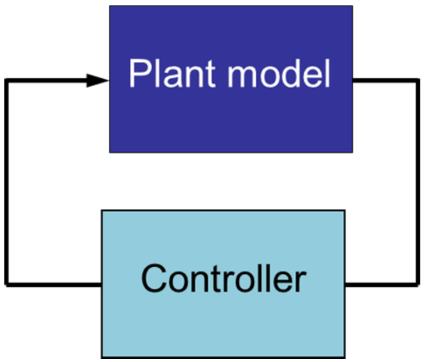
Wind tunnel visualization from KTH

The flow control problem



1. Select plant model that captures the important spatial and temporal interactions that lead to skin friction drag
2. Design a controller that can actuate at appropriate spatial and temporal scales

The flow control problem



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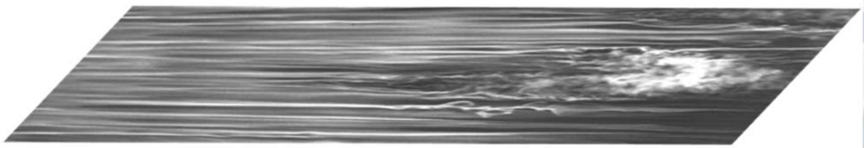
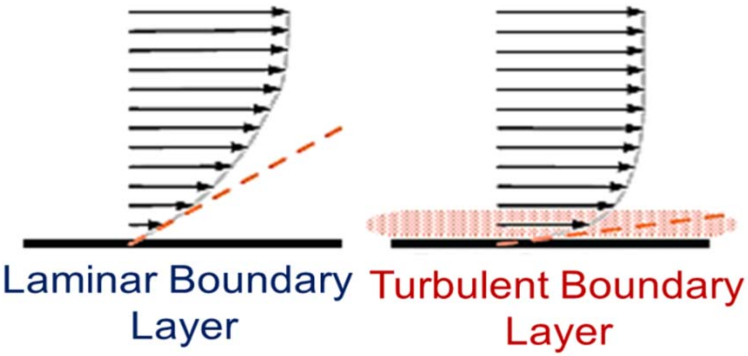
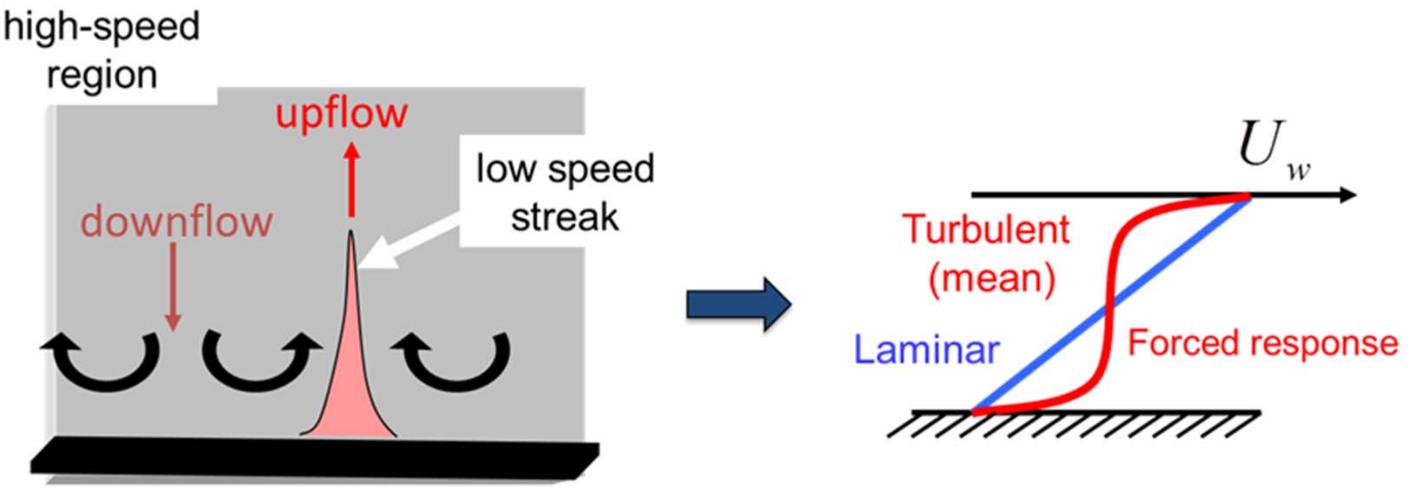


Need to capture the relevant dynamics accurately!



Approach: Isolate the key physics

1. Momentum transfer leading to turbulent mean velocity profile

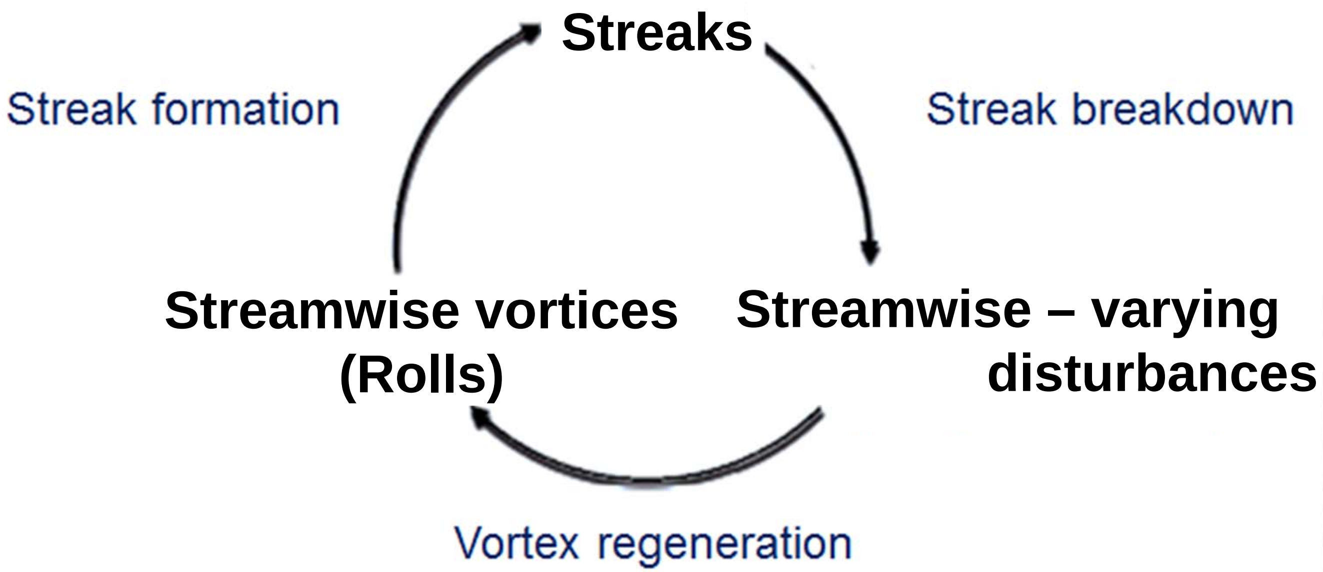


"Science begins by asking simple questions about complex phenomena, but advances by asking more penetrating questions about simpler systems, whose solution could be obtained with rigor and explained with clarity."

Stanley Corrsin

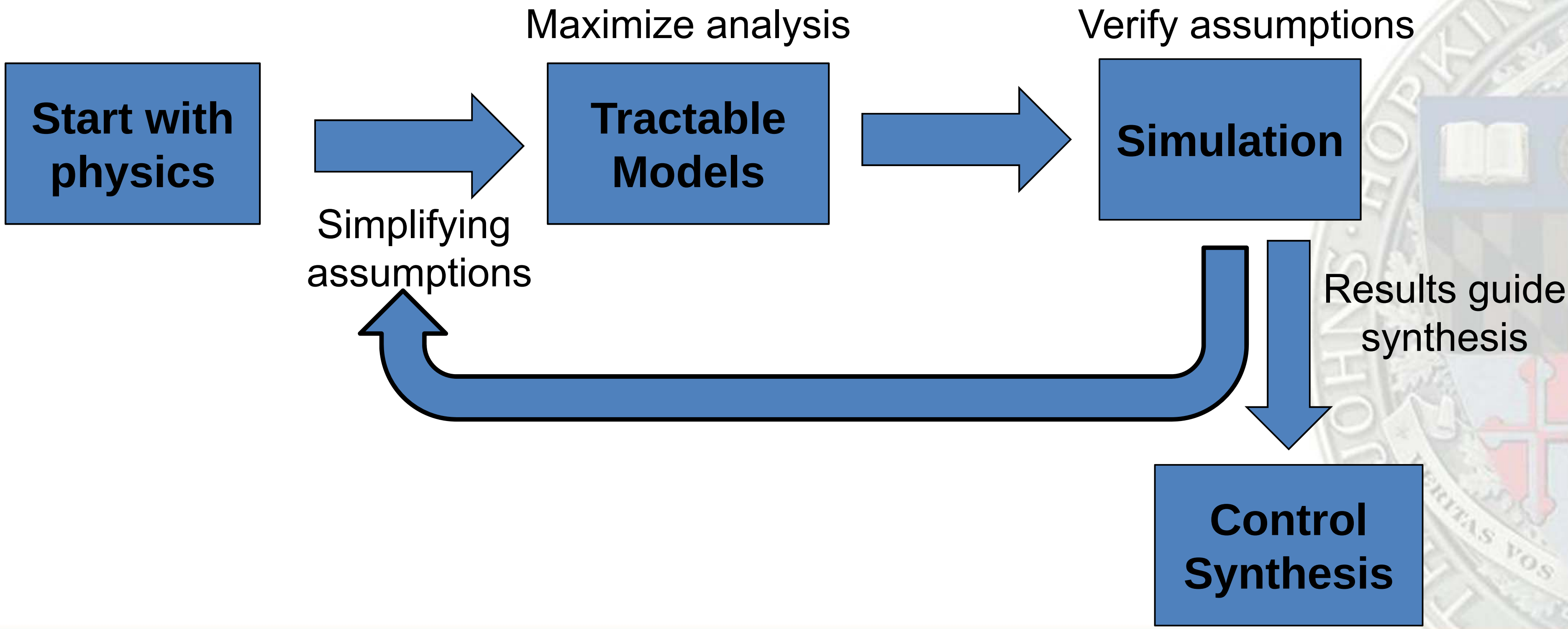
Approach: Isolate the key physics

2. The dynamics that maintain the turbulent state



Isolate the key physics

- Want an analytical model
- Most comprehensive one available is Navier Stokes
  - Not analytically tractable
  - Full DNS is Reynold number limited



Selecting a modeling framework

- Many previous analyses of linearized Navier Stokes

[e.g. Bamieh, Cossu, Farrell, Jiménez, Jovanović, Kim, Henningson, Ioannou, McKeon, Schmid, Sharma, Reddy, Trefethen…]

- Analysis of linear energy growth particularly successful

[e.g. Butler & Farrell 92, Farrell & Ioannou 93, Bamieh & Dahleh 01, Jovanović & Bamieh 04, 05]

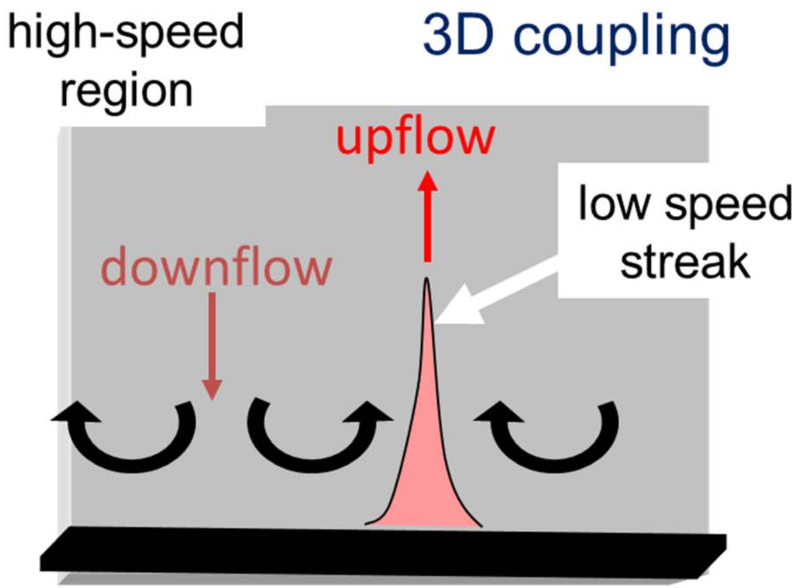
$$d(k_x, y, k_z, t) \longrightarrow \boxed{\text{LNS}} \longrightarrow \mathbf{u}(k_x, y, k_z, t)$$

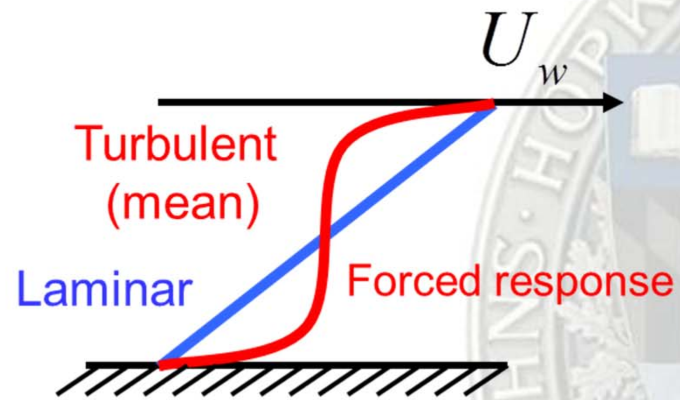
  - Identified initial conditions that exhibit maximum growth
  - Identified most energetic structures
- Linear mechanisms tied to the maintenance of turbulence

[Kim & Lim 00]

Why a nonlinear model

- LNS do not capture change in profile from laminar to turbulent
- Momentum redistribution comes from nonlinear interactions

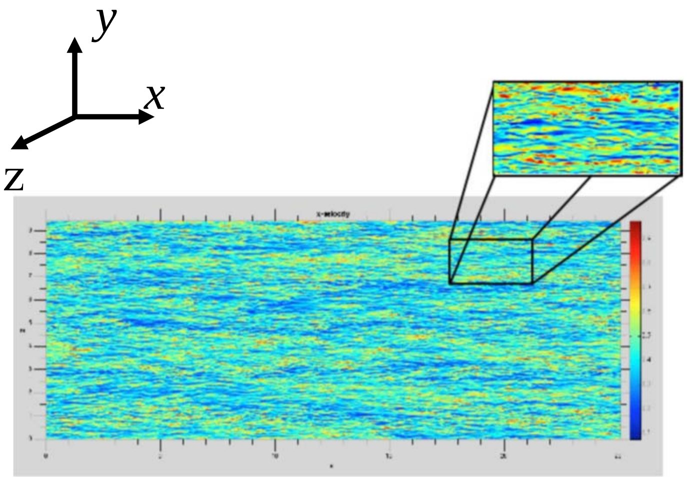




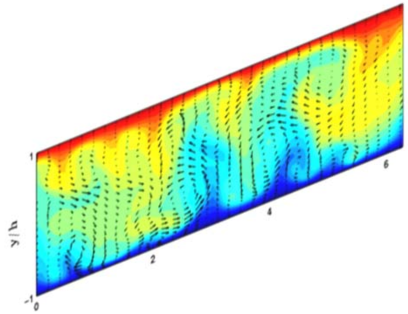
Question of which nonlinearity?

A coherent structure based model

Ubiquity of streamwise (flow direction) elongated flow structures



Picture: JHU Turbulence database (C. Meneveau)

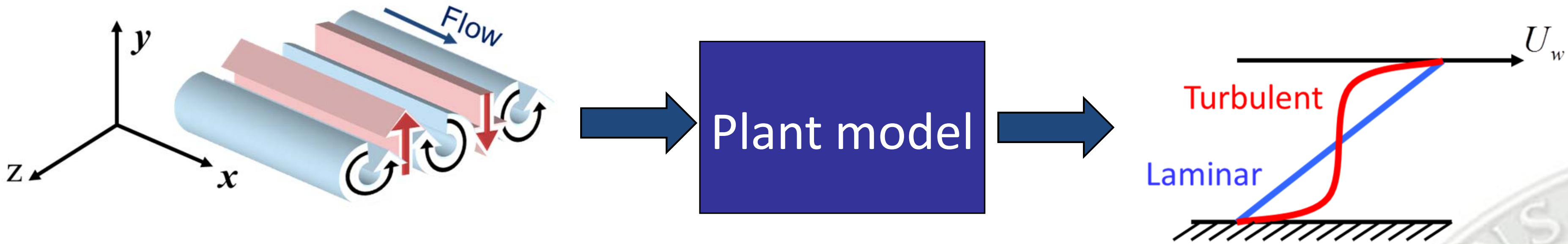


High and low speed fluid Caltech (McKeon)

- Streamwise constant disturbances show maximum growth in studies of the linearized Navier Stokes
- Near wall dominated by elongated streaks/vortices
  [eg. Jiménez & Moin 1991, Hamilton et al. 1995, Waleffe 1997, Schoppa & Hussain 2002]
- Longer structures throughout the height of the channel
  [eg. Kim & Adrian 99, Morrison et al. 02, Guala et al. 06, Hutchins & Marusic 07 …]

Streamwise coherent modeling framework

Model that connects these structural features to the velocity shape



Streamwise coherent modeling framework

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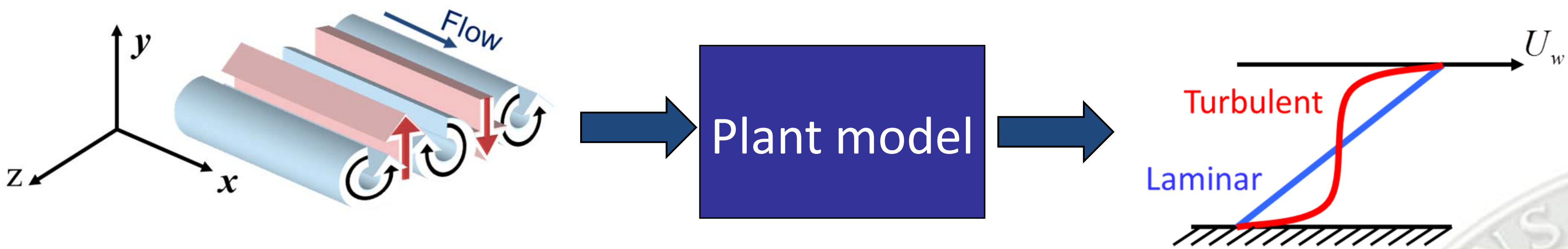


Decompose Navier Stokes:

Streamwise constant mean $\mathbf{U}$ + perturbation $\mathbf{u}$

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = -\langle \mathbf{u}\cdot\nabla\mathbf{u}\rangle$$

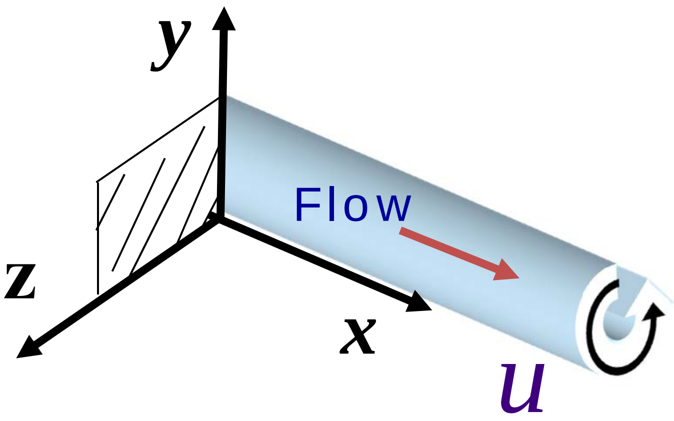
Streamwise constant (2D/3C) mean flow dynamics

$$\mathbf{u}_t + \mathbf{U}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{U} + \nabla p - \frac{\Delta\mathbf{u}}{R} = -\underbrace{\left(\mathbf{u}\cdot\nabla\mathbf{u} - \langle \mathbf{u}\cdot\nabla\mathbf{u}\rangle\right)}_{\text{Perturbation-Perturbation Nonlinearity}}$$

$\langle\cdot\rangle$ Denotes a streamwise average

Isolating the mean flow dynamics

IDEA: MEAN flow is 2D, Use all 3 velocity components (3C), to capture 3D nature of turbulence.



$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} + \nabla P = \frac{1}{R} \Delta \mathbf{U}$$

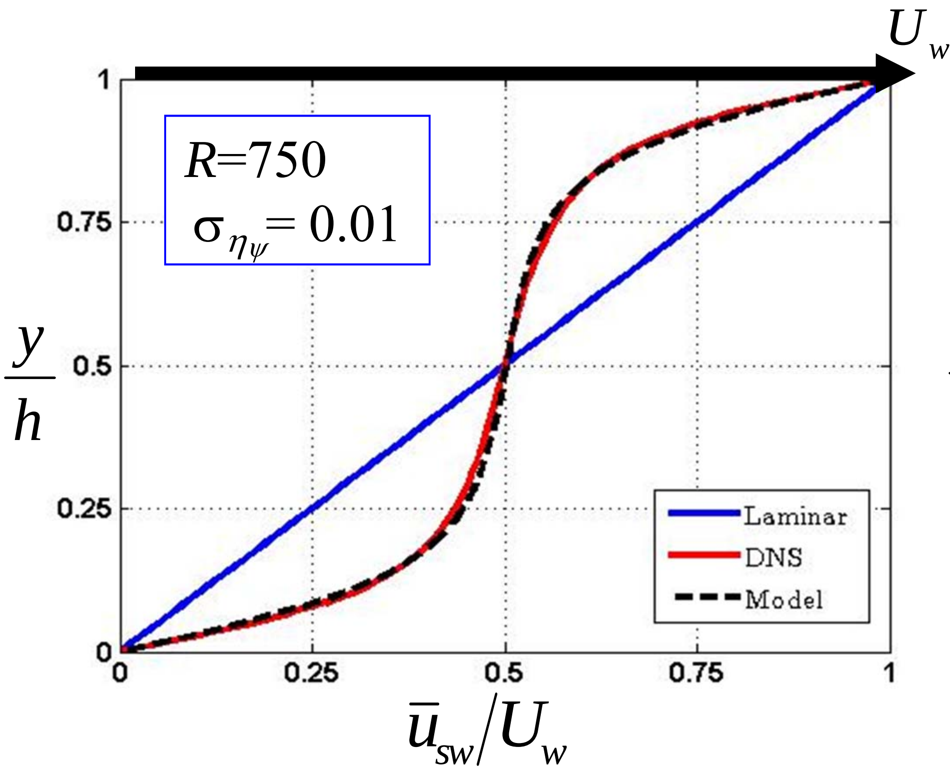
$$\nabla \cdot \mathbf{U} = 0 \qquad \text{at } k_x = 0$$

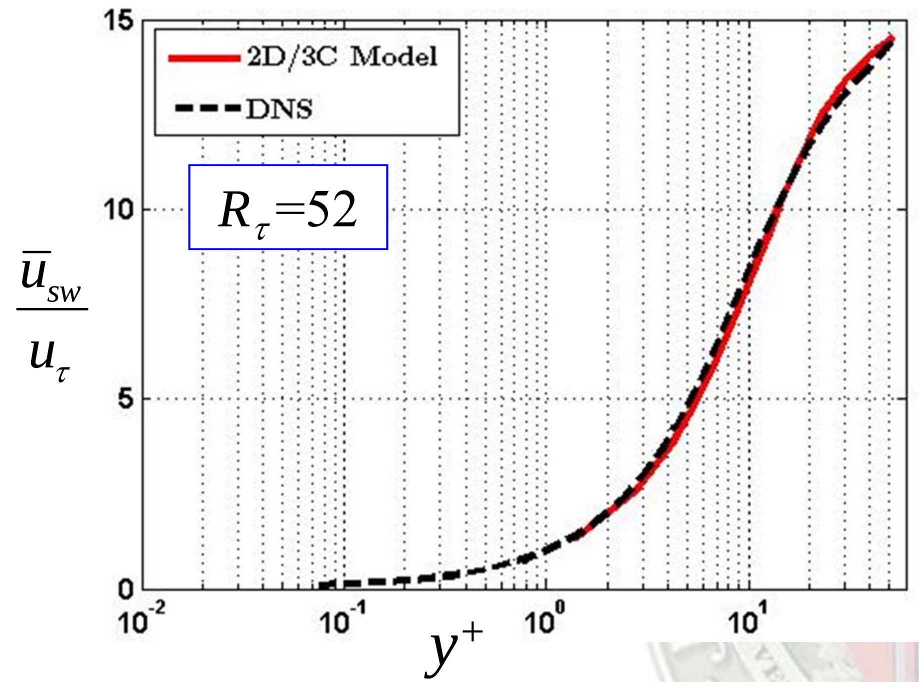
Hypothesis: streamwise constant nonlinearity is the key

Probing the mean flow dynamics

Simulating the mean flow dynamics

Small Amplitude White Noise → **2D/3C Model** → (U, V, W)





$$R = \frac{U_w \delta}{\nu}$$

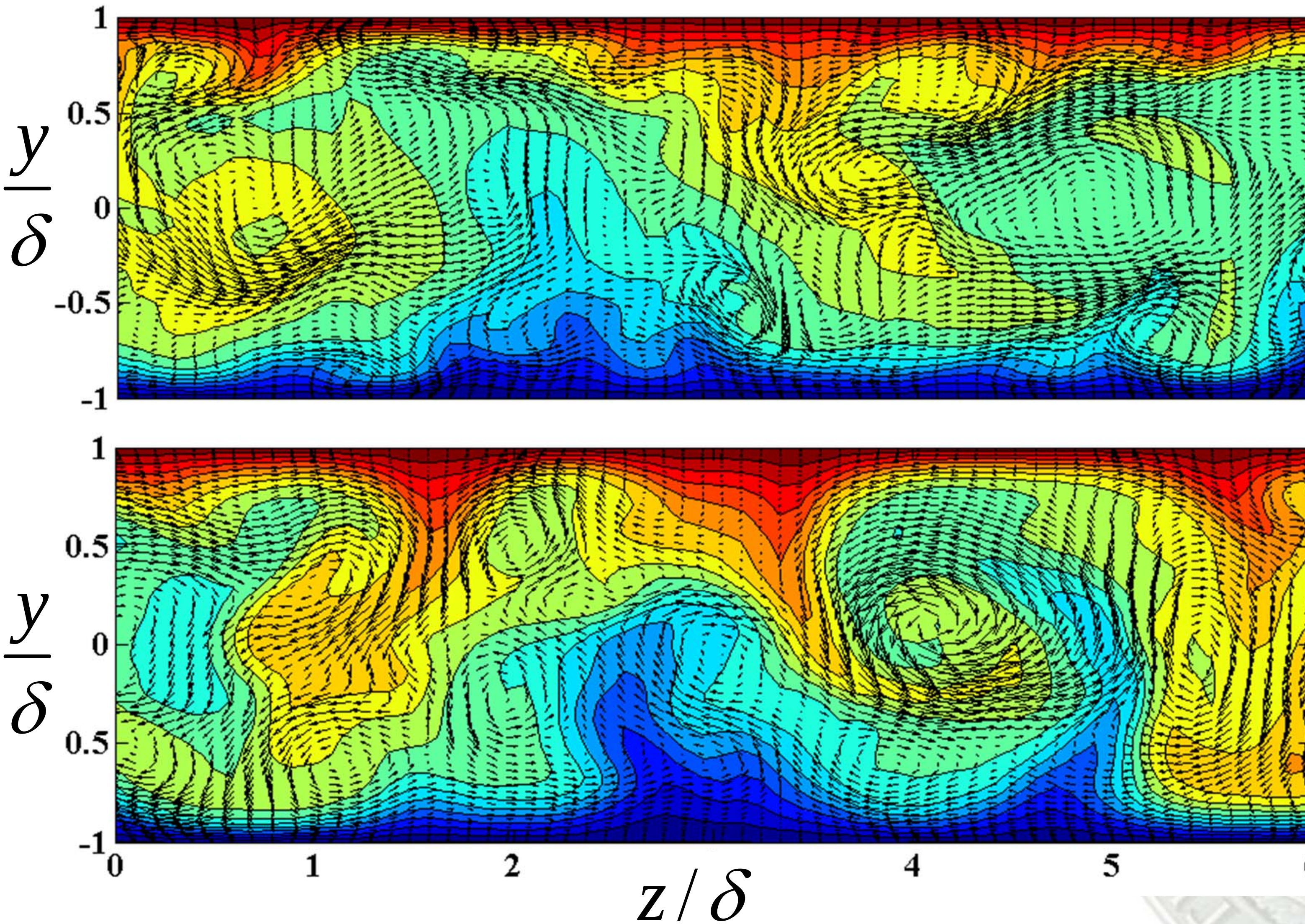
δ:= channel half height,
U_w:= plate velocity

DNS, Tsukahara et al., 2006

[Gayme et. al 2010]

Flow field visualizations

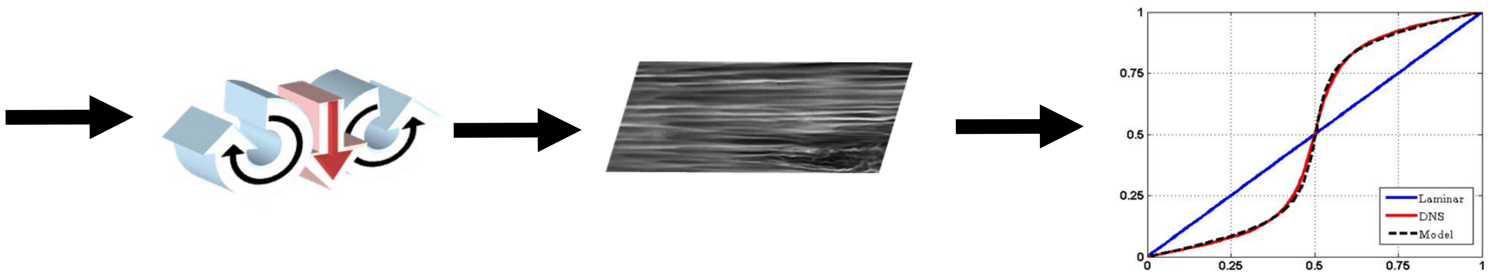
DNS



2D/3C

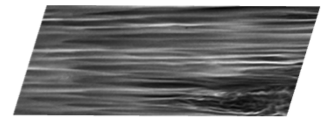
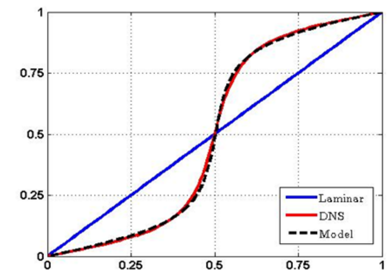
Mean flow dynamics: Implications

Stochastic forcing



- Supports the idea that streamwise constant model forms a reasonable mean flow

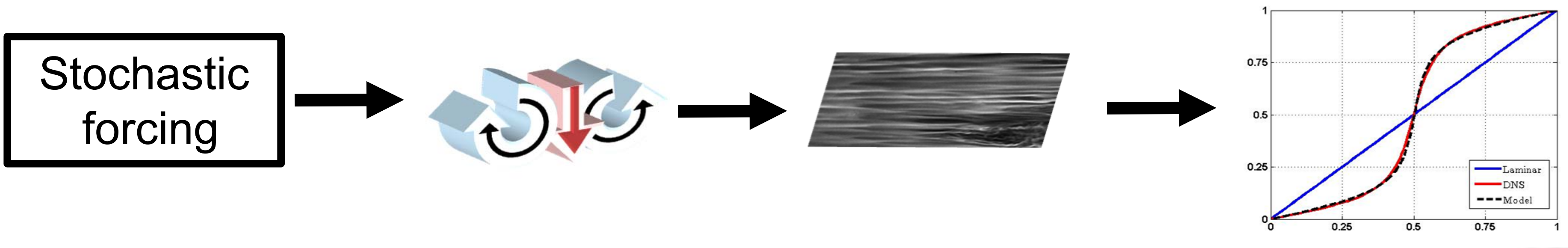
Mean flow dynamics: Implications

Stochastic forcing →  →  → 

What does this mean about the model?

1. The momentum transfer mechanism is reproduced
2. The turbulence is not self-sustaining

Mean flow dynamics: Implications



What does this mean about the model?

1. The momentum transfer mechanism is reproduced
2. The turbulence is not self-sustaining

Quantitative reproduction of the mean velocity serves as a proof of concept but not the key point

Streamwise average decomposition of NS

Start with a streamwise constant mean $\mathbf{U}$ + perturbation $\mathbf{u}$ decomposition of Navier Stokes

$$\mathbf{u}(t) = (u, v, w) \text{ perturbation,}$$

$$\mathbf{U}(t) = (U, V, W) \text{ streamwise constant mean flow}$$

$$\mathbf{u}_t + \mathbf{U}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{U} + \nabla p - \frac{\Delta\mathbf{u}}{R} = -\underbrace{\left(\mathbf{u}\cdot\nabla\mathbf{u} - \langle\mathbf{u}\cdot\nabla\mathbf{u}\rangle\right)} + \boldsymbol{\varepsilon}$$

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = -\langle\mathbf{u}\cdot\nabla\mathbf{u}\rangle$$

External noise forcing

Perturbation nonlinearity

Mean/perturbation coupling

$\langle\cdot\rangle$ Denotes a streamwise average

[Farrell & Ioannou JFM 2012]

RNL∞ model: A second order closure

Ergodic assumption along with stochastic perturbations of the form $f\,F\xi(t)$ where $\xi(t)$ is delta correlated

The perturbation covariance evolves according to

$$C_t = A(\mathbf{U})C + CA^\dagger(\mathbf{U}) + f^2 Q$$

Linear perturbation dynamics

$$\mathbf{U}_t = G(\mathbf{U}) + \mathcal{L}C$$

Nonlinear streamwise constant (2D/3C) mean flow dynamics

$C = \langle \mathbf{u}\mathbf{u}^\dagger \rangle$, $Q = FF^\dagger$ and $\mathcal{L}C$ are streamwise Reynolds stresses

A second order closure of the dynamics of the statistical state

Restricted nonlinear (RNL) model

- Computationally tractable approximation of covariance
- Represents the statistical state dynamics of one member of the infinite ensemble
- Analogous to DNS being a single realization of Navier Stokes

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = -\langle \mathbf{u}\cdot\nabla\mathbf{u} \rangle$$

$$\mathbf{u}_t + \mathbf{U}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{U} + \nabla p - \frac{\Delta\mathbf{u}}{R} = e$$

Depends on the instantaneous $U(y, z, t)$

$\mathbf{U}(t) = (U, V, W)$ Streamwise constant mean flow

$\mathbf{u}(t) = (u, v, w)$ Streamwise varying perturbations about the mean flow

$\langle\cdot\rangle$ Denotes a streamwise average

Restricted nonlinear (RNL) model

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta \mathbf{U}}{R} = -\langle \mathbf{u}\cdot\nabla\mathbf{u}\rangle$$

Nonlinear 2D/3C

Linearized

$$\mathbf{u}_t + \mathbf{U}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{U} + \nabla p - \frac{\Delta \mathbf{u}}{R} = e$$

Zero mean Gaussian

Perturbation Dynamics

Mean flow $\mathbf{U}(\mathrm{t})$ regulates $\mathbf{u}(\mathrm{t})$

Perturbations $\mathbf{u}(\mathrm{t})$ drive mean flow $\mathbf{U}(\mathrm{t})$

Mean flow Dynamics

Restricted nonlinear (RNL) model

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = -\langle \mathbf{u}\cdot\nabla\mathbf{u} \rangle$$

Nonlinear 2D/3C

Linearized

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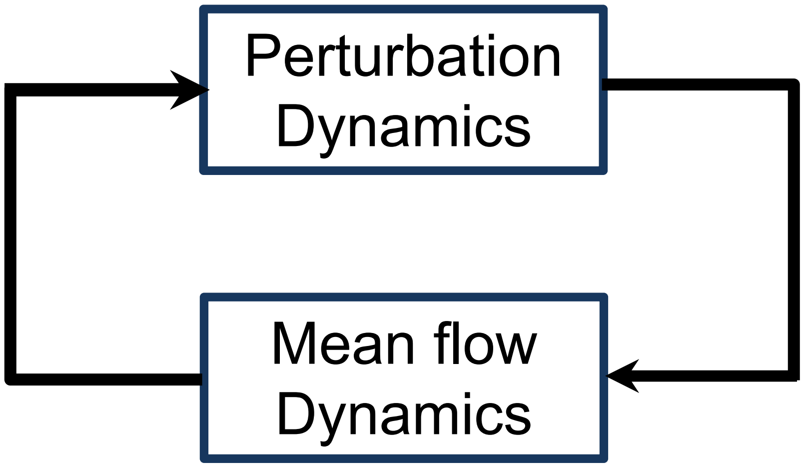
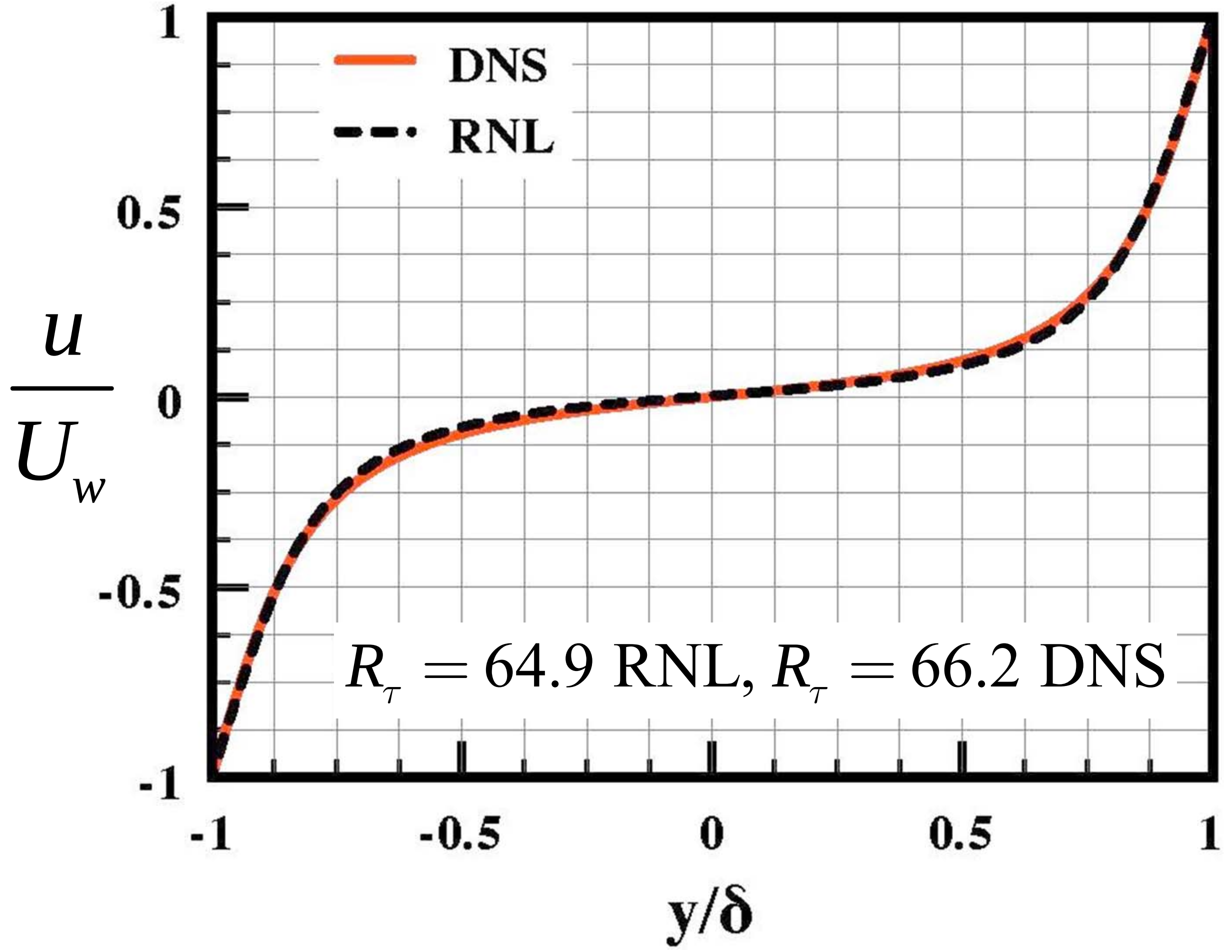
Zero mean Gaussian

Mean flow $\mathbf{U}(\mathrm{t})$ regulates $\mathbf{u}(\mathrm{t})$



Perturbations $\mathbf{u}(\mathrm{t})$ drive mean flow $\mathbf{U}(\mathrm{t})$

Mean velocity profile – low Reynolds number



$$R = \frac{U_w \delta}{\nu} = 1000$$

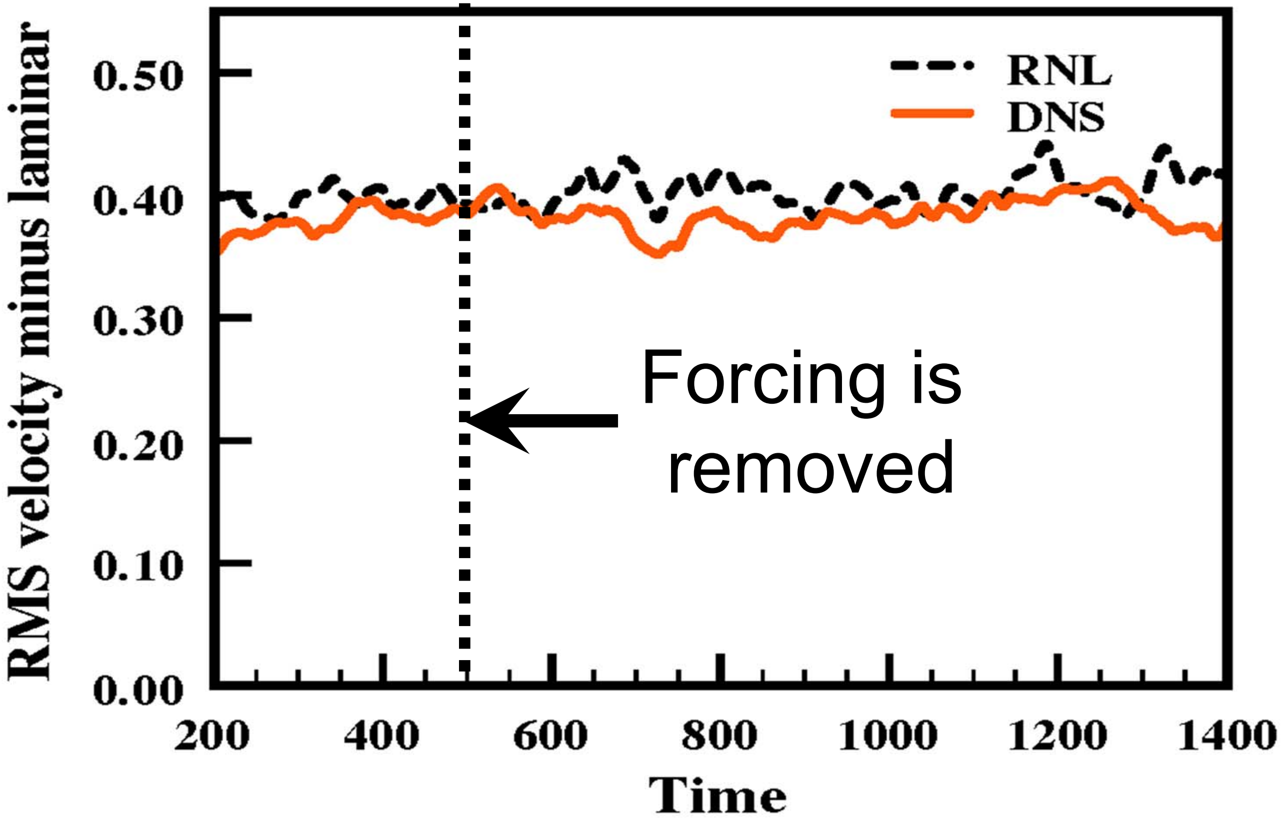
δ:= channel half height

U_w:= plate velocity

[Thomas et. al 2014]

DNS Gibson 2012

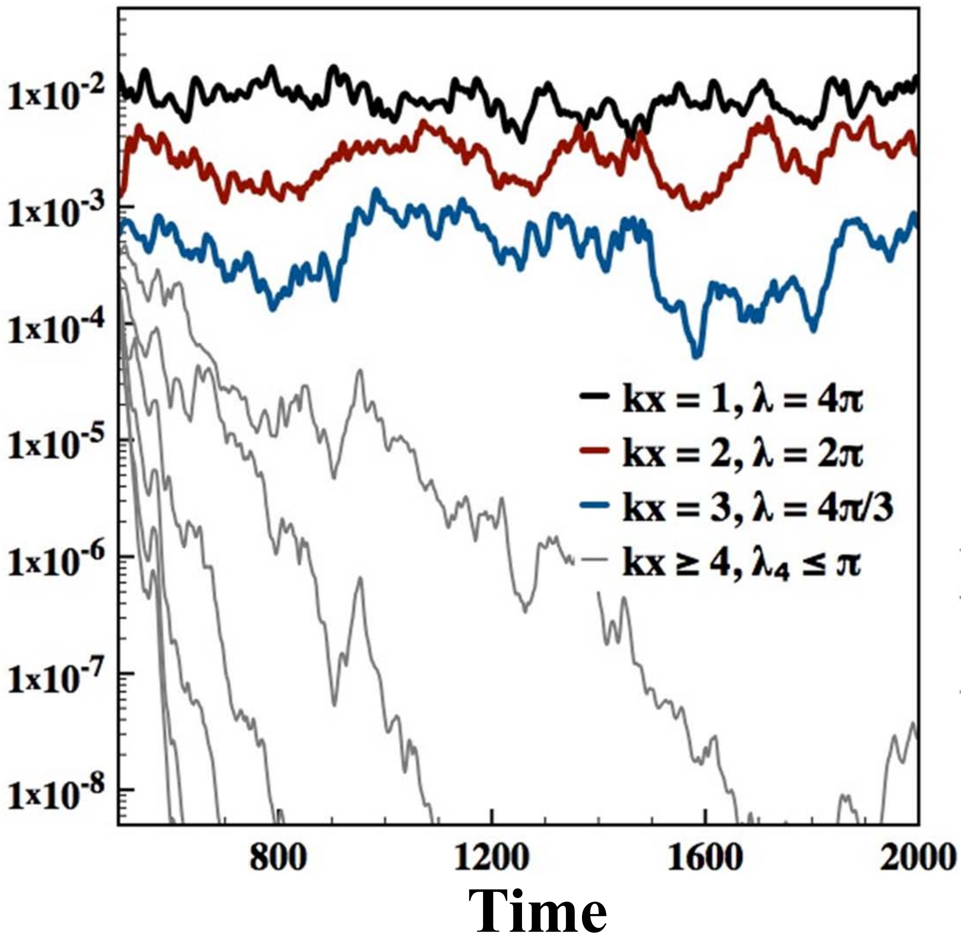
Self sustaining turbulence



	x	y	z	$M_x \times M_y \times M_z$
DNS	$[0, 4\pi]$	$[-1, 1]$	$[0, 4\pi]$	$83 \times 65 \times 41$
RNL	$[0, 4\pi]$	$[-1, 1]$	$[0, 4\pi]$	$9 \times 65 \times 41$

RNL as a minimal model

$$E_{\lambda_n}(t) = \int_o^{L_z} \int_{-\delta}^{\delta} \int_0^{L_x} u_{\lambda_n}(x,y,z,t)^2 \, dx \, dy \, dz,$$



- RNL in a 4π channel naturally collapses to 3 streamwise Fourier components (modes)
- Maintains all spanwise modes

	x	y	z	$M_x \times M_y \times M_z$
DNS	$[0, 4\pi]$	$[-1, 1]$	$[0, 4\pi]$	$83 \times 65 \times 41$
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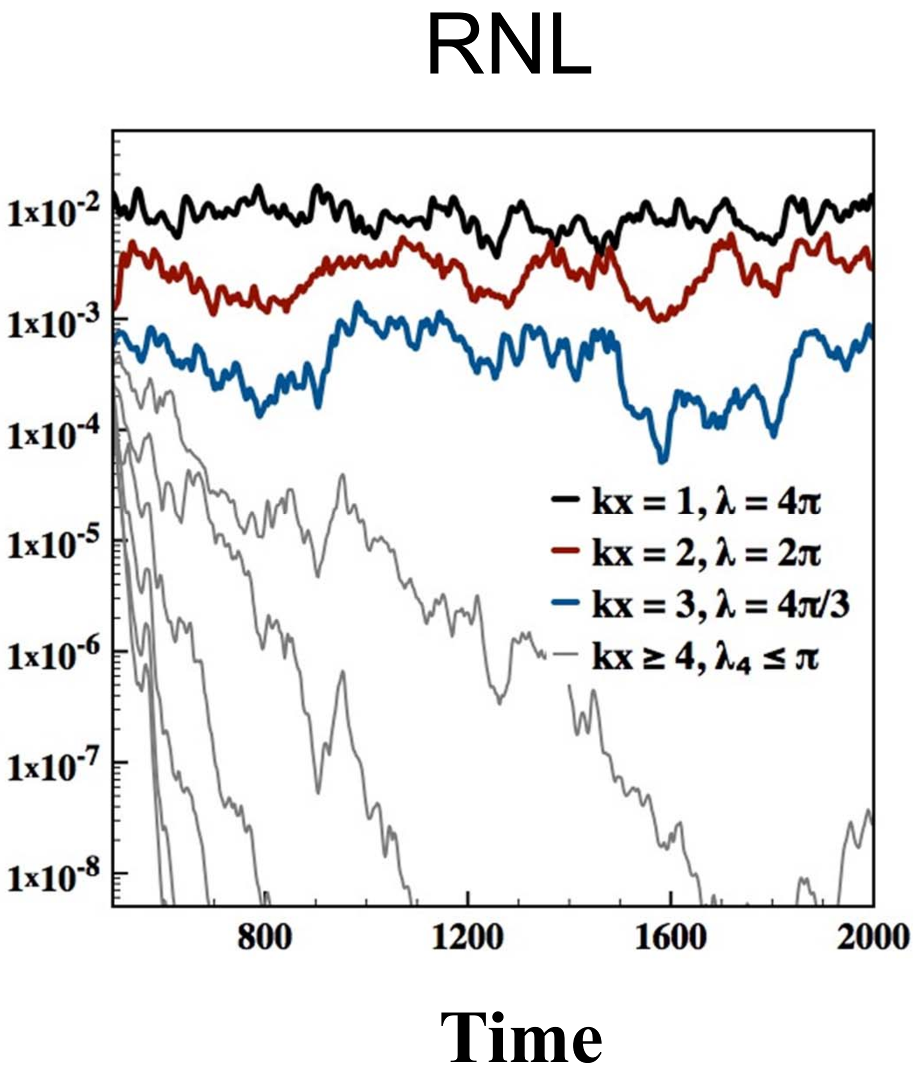
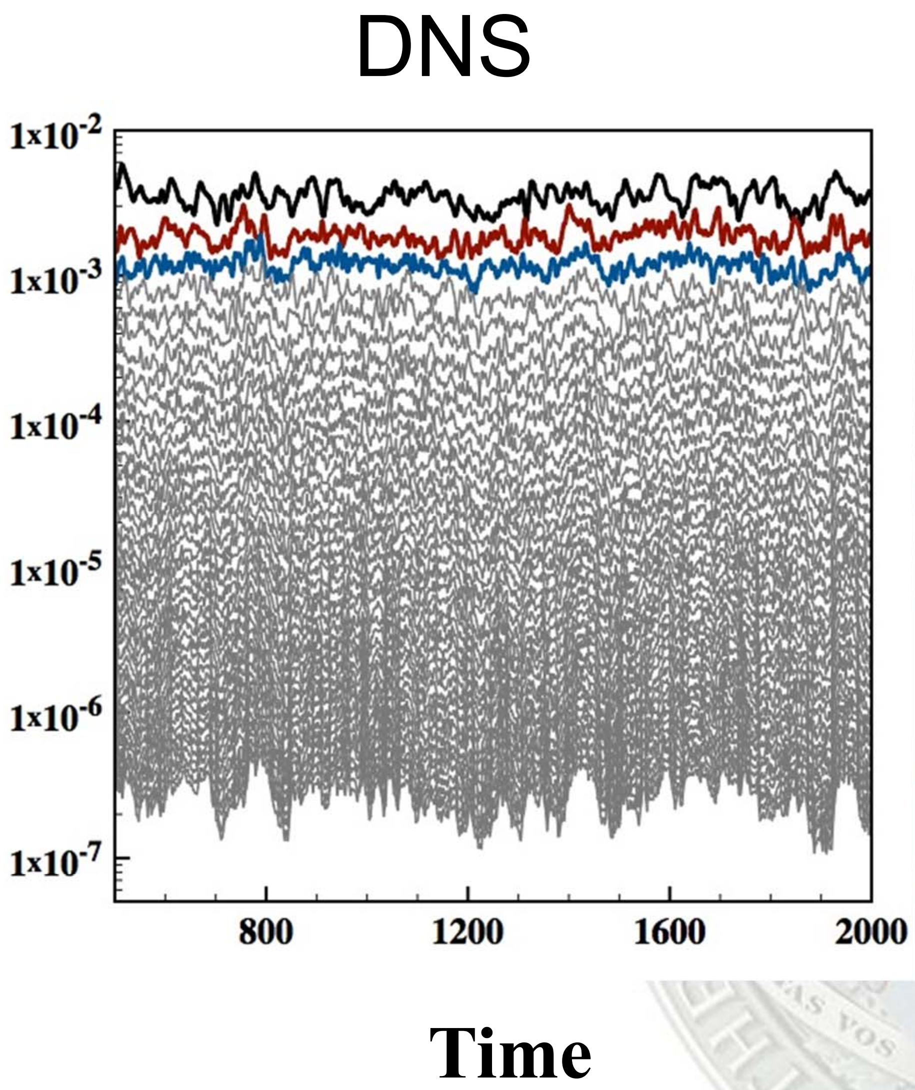
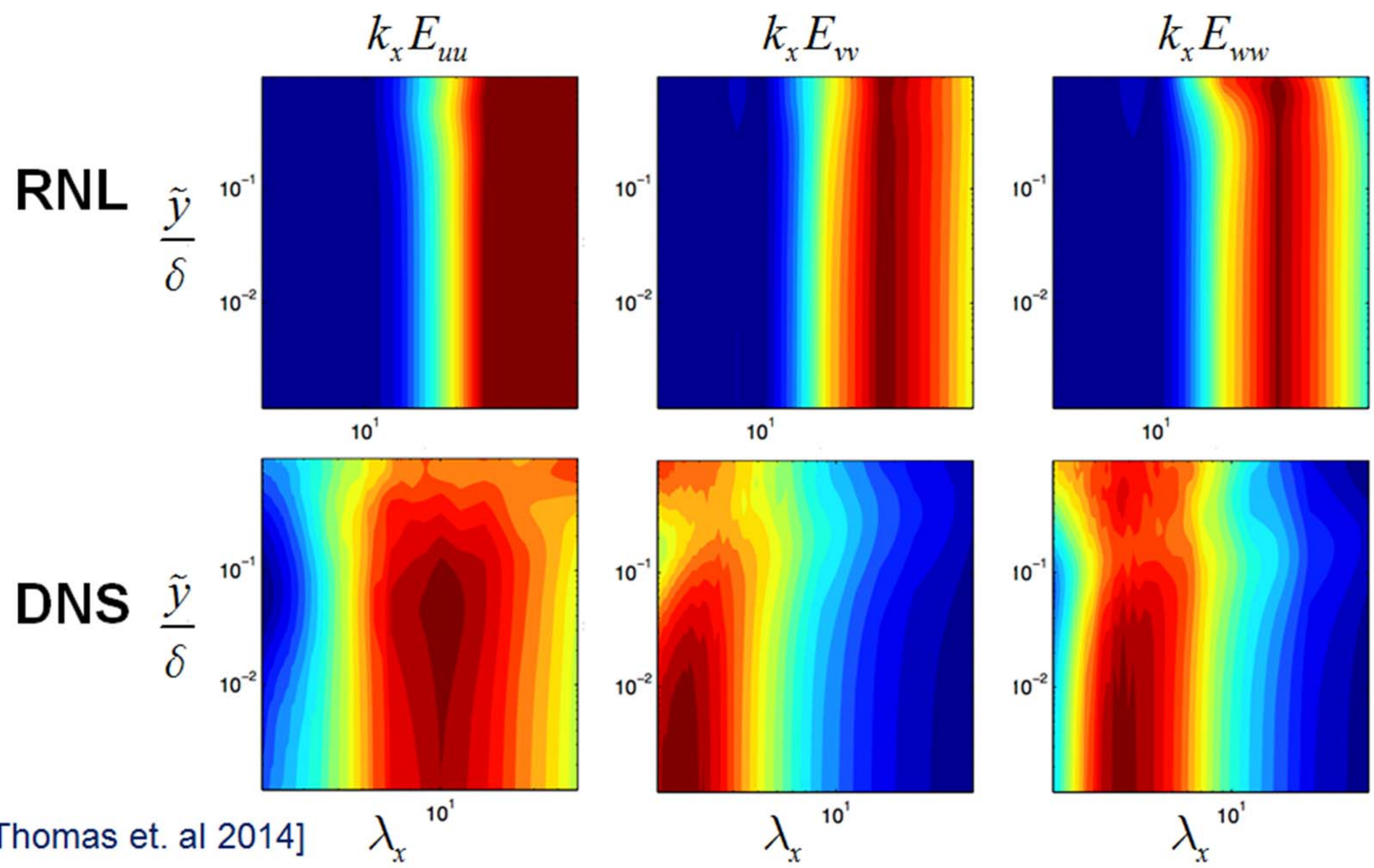
[Thomas et. al 2015]

RNL as a minimal model: $E_{\lambda_n}(t)$ comparison





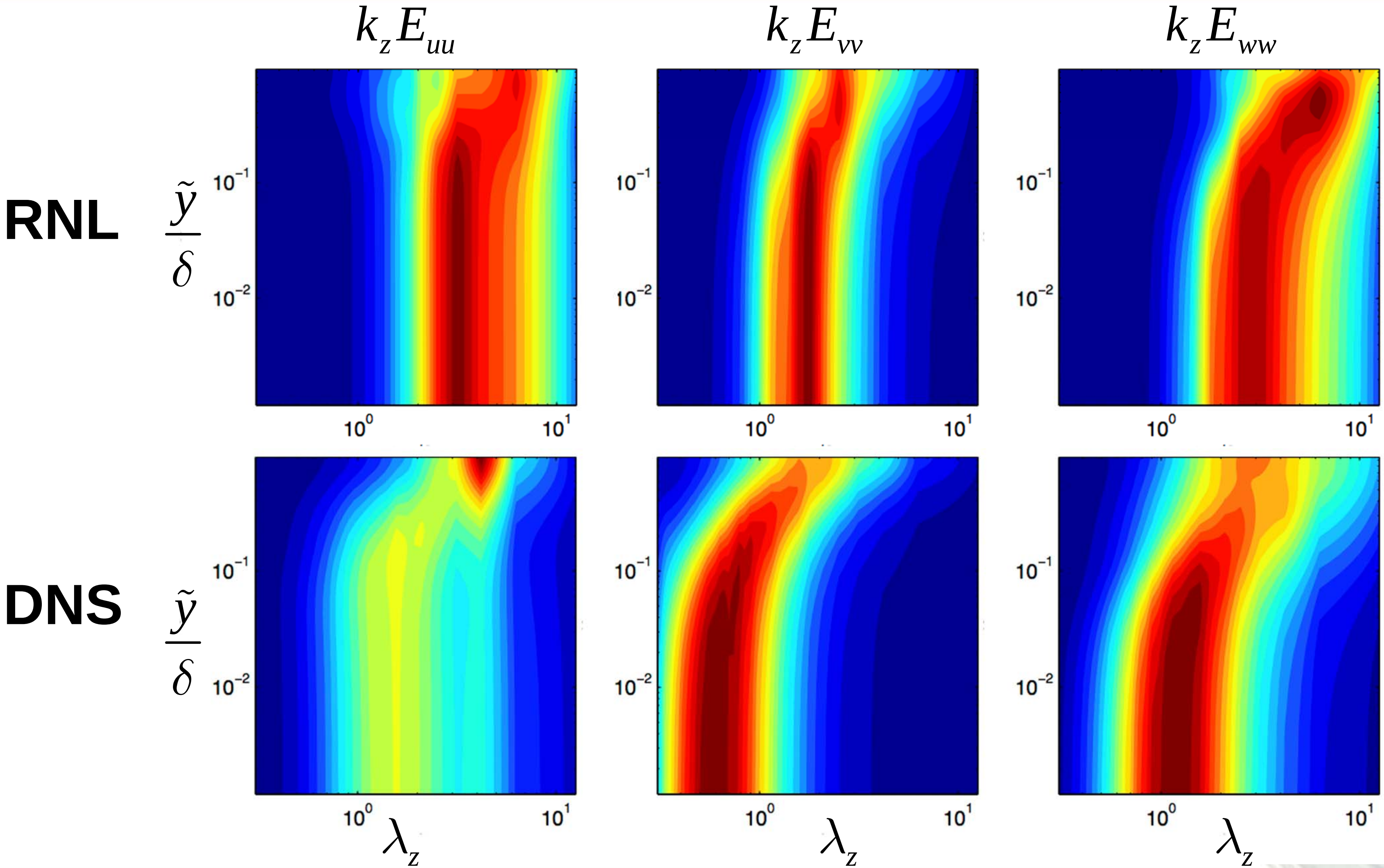
k_x premultiplied spectra(16π channel)



[Thomas et. al 2014]

- Collapse to the longest wavelength structures

k_z premultiplied spectra(16π channel)



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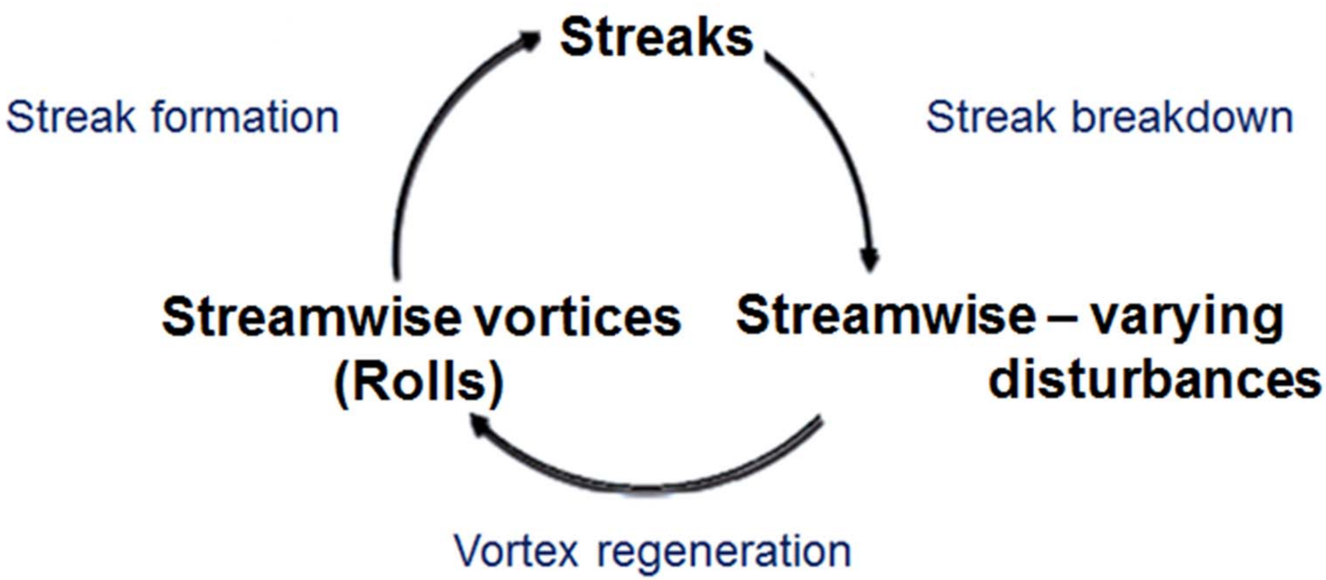
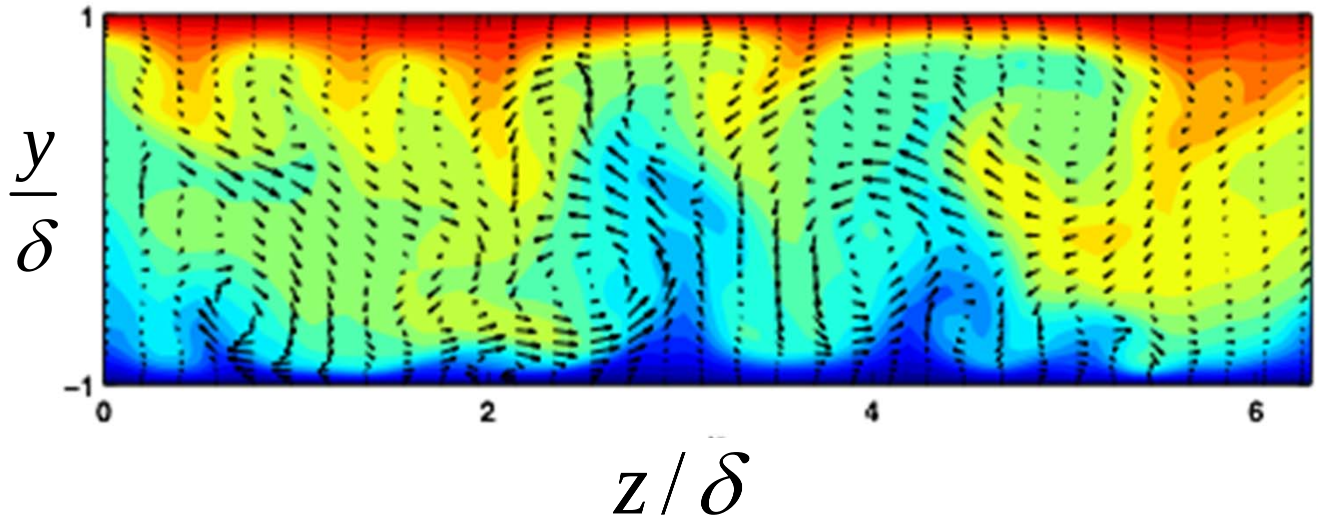


- Rolls/streaks are critical to transition and turbulence

[e.g., Kim et al. 1971, Jiménez & Moin 1991, Hamilton et al. 1995, Waleffe 1997, Jiménez & Pinelli 1999, Schoppa & Hussain 2002, Hall & Sherwin 2010 …]

Isolating the critical flow structures



$$Streak_{RMS} = \sqrt{U_s^{\,2}} = \sqrt{\int_0^{L_z}\int_{-\delta}^{\delta}\left(U-[U]\right)^2 dydz}$$

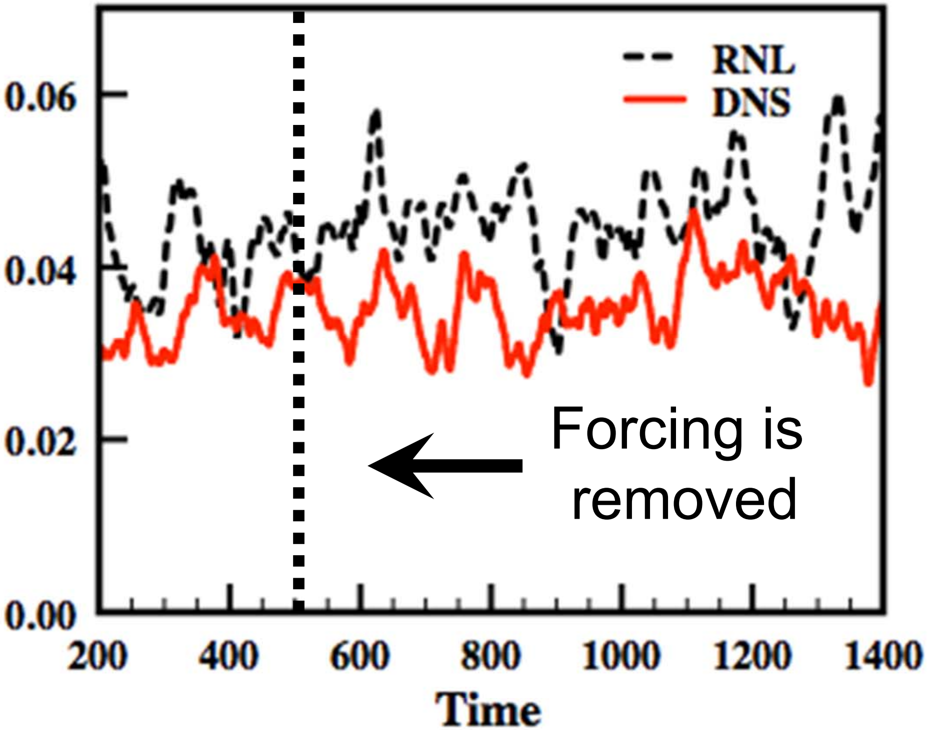
$$Roll_{RMS} = \sqrt{\int_0^{L_z}\int_{-\delta}^{\delta} V^2 + W^2 dydz}$$

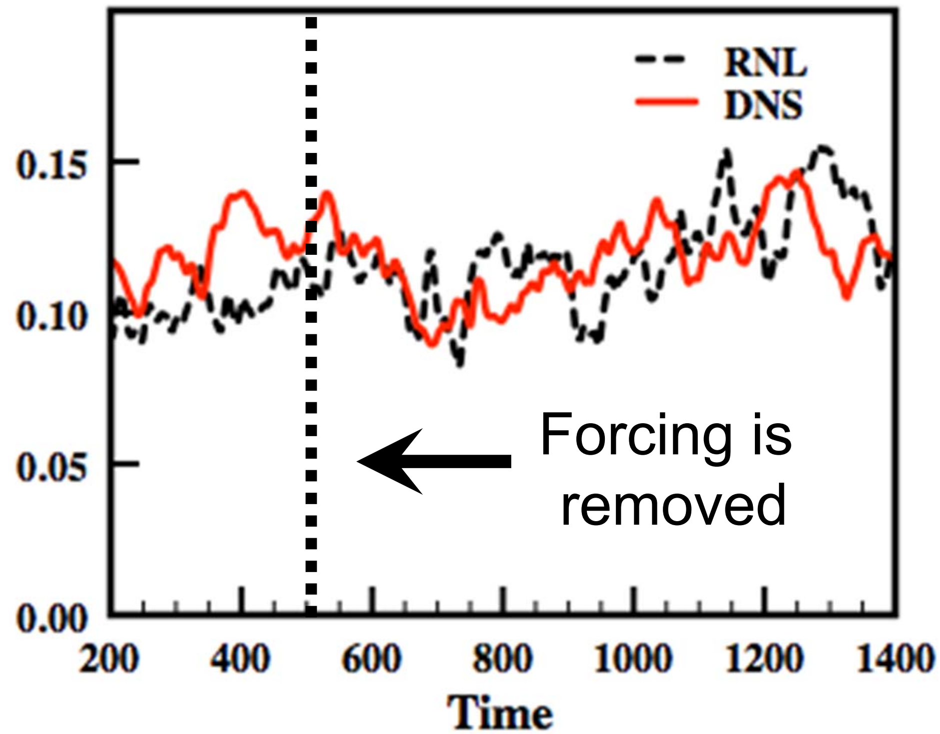
$[U]$: spanwise average

Structural features of the flow

$$Roll_{RMS} = \sqrt{\int_0^{L_z}\int_{-\delta}^{\delta} V^2 + W^2 dydz}$$

$$Streak_{RMS} = \sqrt{\int_0^{L_z}\int_{-\delta}^{\delta} \left(U - [U]\right)^2 dydz}$$

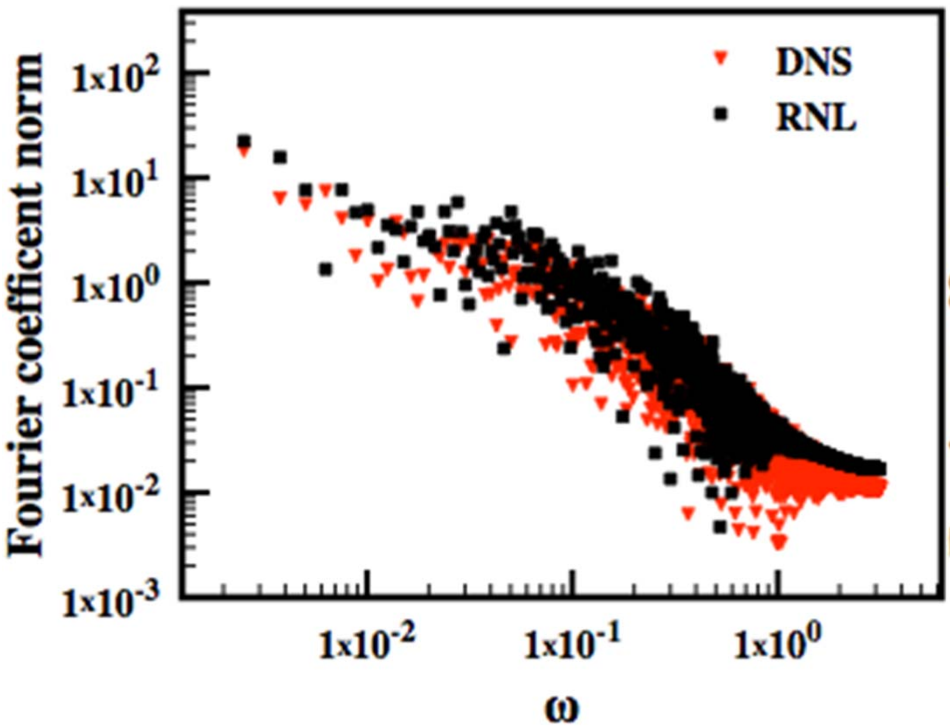


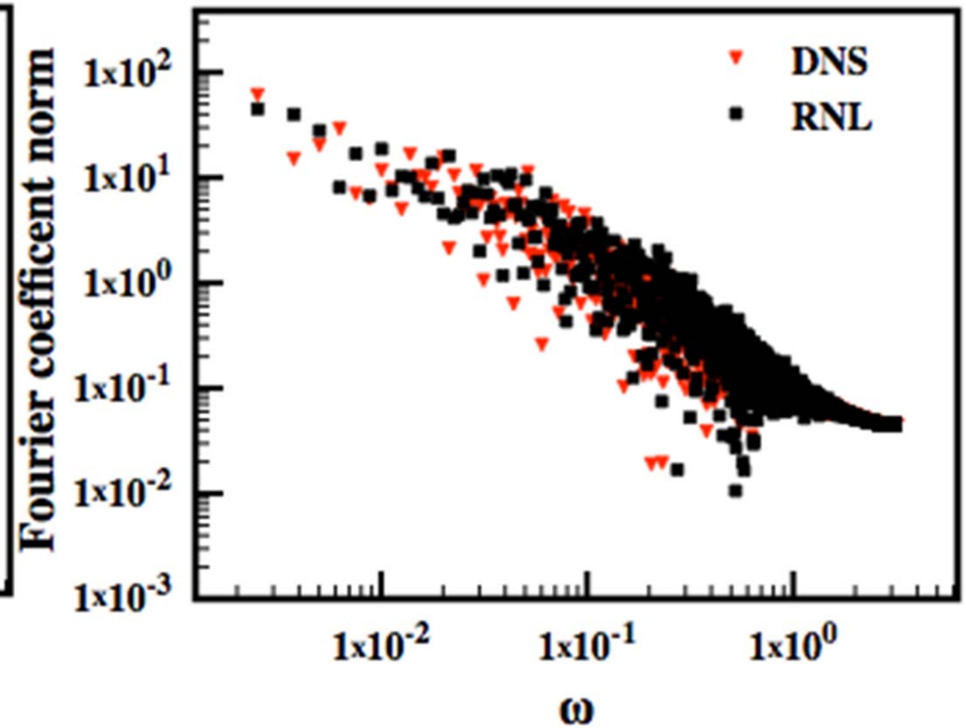


[Thomas et. al 2014]

Temporal spectra of the rolls and streaks

$$Roll_{RMS} = \sqrt{\int_0^{L_z} \int_{-\delta}^{\delta} V^2 + W^2 dydz} \qquad Streak_{RMS} = \sqrt{\int_0^{L_z} \int_{-\delta}^{\delta} \left(U - [U]\right)^2 dydz}$$

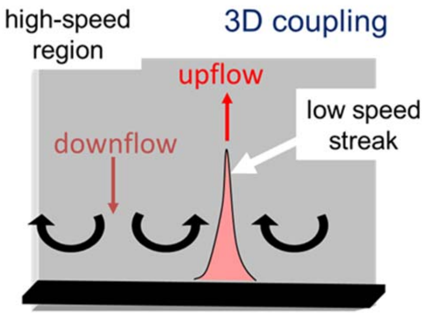


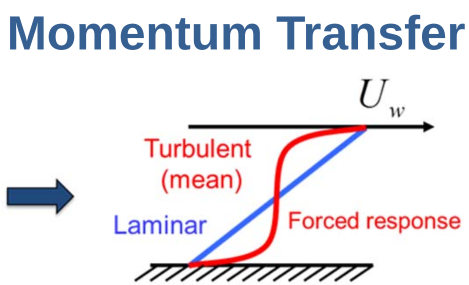


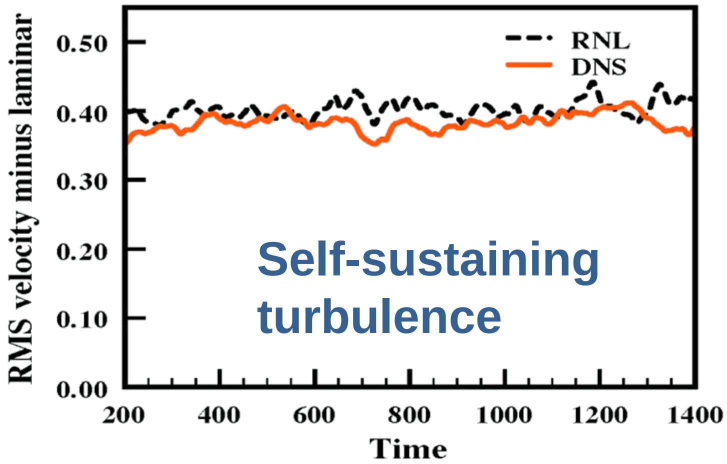
[Thomas et. al 2014]

Summary

- At low Reynolds numbers we have reproduced both phenomena of interest







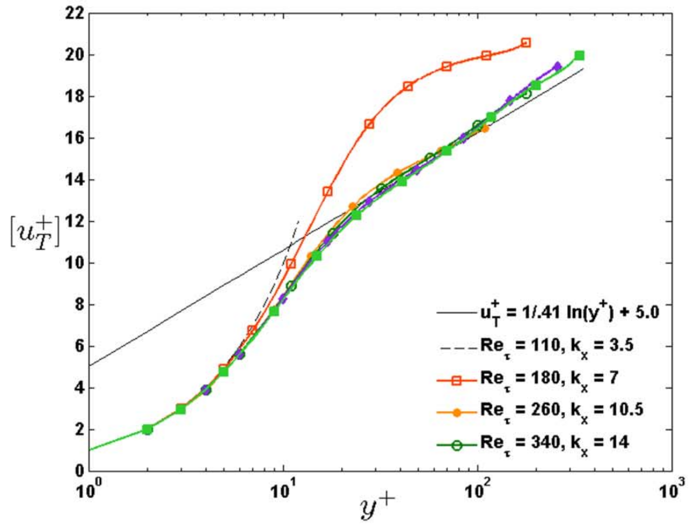
- Demonstrated spatial and temporal properties of key structures in the SSP (rolls and streaks) are consistent with DNS

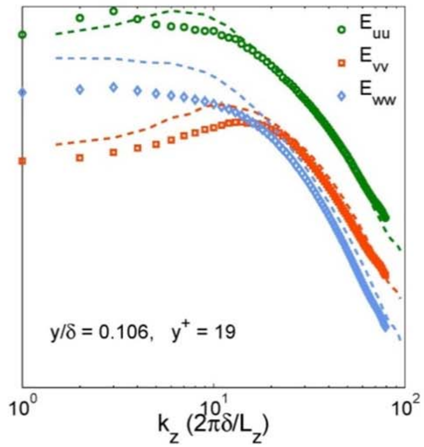
Conclusions

- RNL∞ provides a new theoretical framework for direct analysis of the dynamics of the statistical state of wall turbulence
- RNL naturally collapses to a 'minimal' model of turbulence supported by small number of streamwise modes
- RNL turbulence can be maintained by a single streamwise varying mode interacting with the mean flow
  - truncated system can be used to isolate important flow mechanisms
- Key: know what you want/expect from the model and understand its limitations
  - Ongoing work to test conclusions of RNL based analysis in DNS and experiments
  - Quantifying the unmodeled effects, e.g. Large deviation theory

Moving toward higher Reynolds numbers

- Engineered systems operate at high Reynolds numbers
- Need to verify that the model continues to perform as intended as the Reynolds number increases
- Results at moderate Reynolds numbers are promising





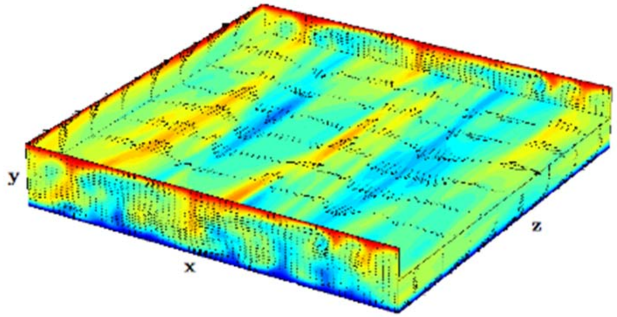
$$k_x = \frac{\delta 2\pi}{L_x}(n) = \frac{n}{2}\delta$$

DNS: Moser, Kim, Mansour 1999

[Bretheim, Meneveau & Gayme 2015]

- Currently looking at LES models to push Re further

Questions!



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