Why are CFD RANS models good and how can they be better?

Zhen-Su She

State Key Lab for Turbulence and Complex System,

College of Engineering, Peking University

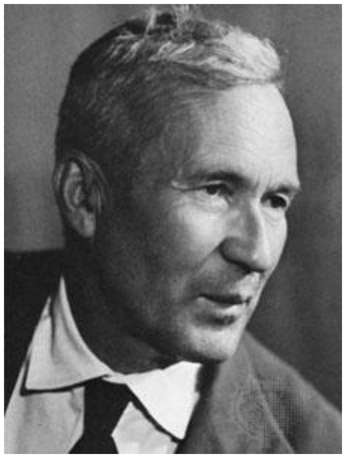
RANS model for CFD:

Are turbulent mean flow quantities predictable ?

We look for macroscopic flow equations for accurate predictions of mean quantities (velocity, kinetic energy, etc) of wall-bounded flows (aerodynamic applications when massive separations and shock waves are abundant).

1. **Results of accurate predictions with modified K-omega equations (from K-omega-Wilcox to K-omega-SED)**
2. **Interpretations of these results**

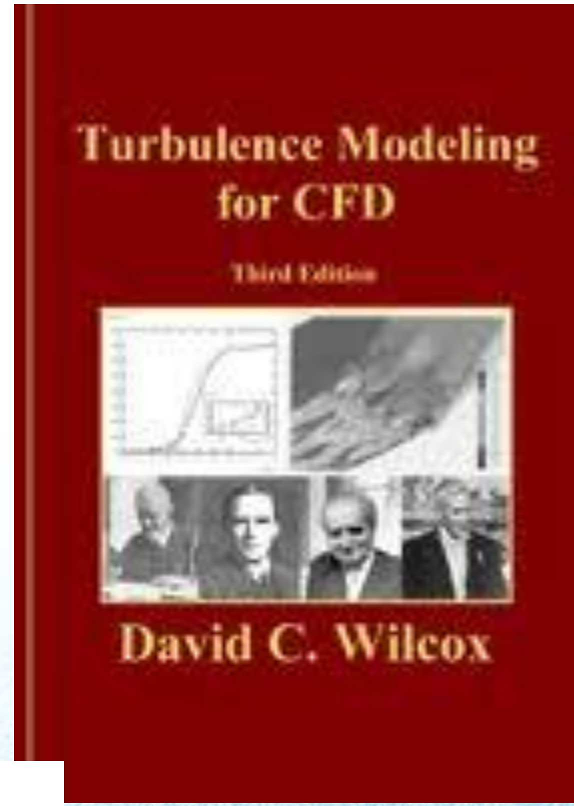
History for CFD:



Kolmogorov

Proposed k-omega model in 1942. Very nonlinear, multi-parameters

60 years later



$$S^{+}W^{+}-\beta^{*}k^{+}\omega^{+}+\frac{d^{2}k^{+}}{dy^{+2}}+\frac{d}{dy^{+}}\left(\sigma^{*}\alpha^{*}\frac{k^{+}}{\omega^{+}}\frac{dk^{+}}{dy^{+}}\right)=0$$

$$\alpha S^{+}W^{+}-\beta k^{+}\omega^{+}+\frac{k^{+}}{\omega^{+}}\frac{d^{2}\omega^{+}}{dy^{+2}}+\frac{k^{+}}{\omega^{+}}\frac{d}{dy^{+}}\left(\sigma\alpha^{*}\frac{k^{+}}{\omega^{+}}\frac{d\omega^{+}}{dy^{+}}\right)=0$$

Analyzing K Eq.

$$S^+W^+ - \beta^* k^+ \omega^+ + \frac{d^2k^+}{dy^{+2}} + \frac{d}{dy^+}\left(\sigma^*\alpha^* \frac{k^+}{\omega^+}\frac{dk^+}{dy^+}\right) = 0$$

$$S^+ = dU^+ / dy^+$$

$$W^+ = -\langle u'v' \rangle^+$$

Production

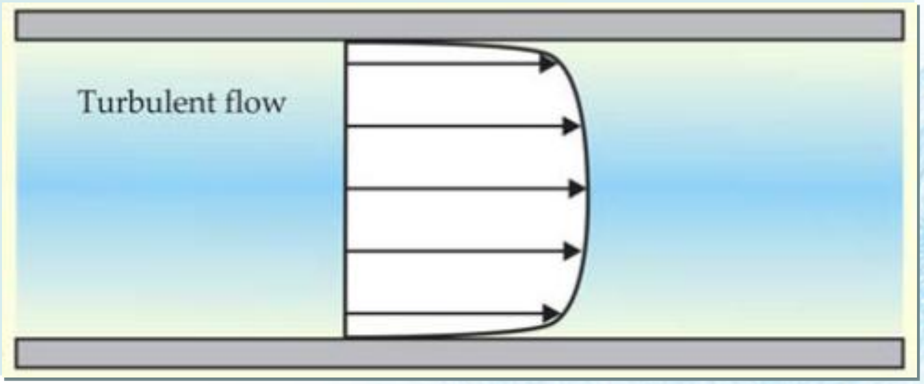
Dissipation

Diffusion

Turbulent transport

Constitutive relation:

$$\alpha^* k^+ / \omega^+ = W^+ / S^+ = \nu_T^+$$



Analyzing omega Eq.

$$\alpha S^+W^+ - \beta k^+\omega^+ + \frac{k^+}{\omega^+}\frac{d^2\omega^+}{dy^{+2}} + \frac{k^+}{\omega^+}\frac{d}{dy^+}(\sigma\alpha^*\frac{k^+}{\omega^+}\frac{d\omega^+}{dy^+}) = 0$$

Production

Dissipation

Diffusion

Turbulent transport

Complicated parameter setting:

Constitutive relation:

$$\alpha^* k^+ / \omega^+ = W^+ / S^+ = \nu_T^+$$

$$\alpha = \frac{\alpha_\infty}{\alpha^*}\frac{\alpha_0 + k^+/(R_\omega\omega^+)}{1 + k^+/(R_\omega\omega^+)}$$

$$\alpha^* = \frac{\alpha_0^* + k^+/(R_k\omega^+)}{1 + k^+/(R_k\omega^+)}$$

Amazingly, they are very close to data! Why?

SED theory define "Order functions" (symmetry) …

Ratio order function 1: $\nu_t = \frac{W}{S}$

$S^+ = dU^+ / dy^+$

Ratio order function 2: $\Theta_\nu = \frac{\varepsilon}{SW}$

$W^+ = -\langle u'v' \rangle^+$

$S^+ + W^+ = r$

These ratios capture changes in the balance mechanism of energy dynamics.

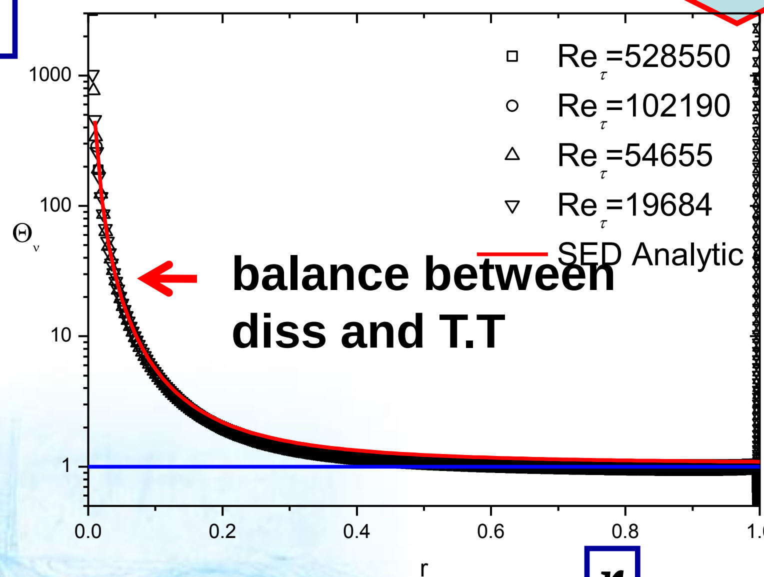
$S^+W^+ - \varepsilon + \frac{d^2k^+}{dy^{+2}} + T.T = 0$

SED multi-layer expression for Θ_ν

SED theory define "Order functions" (symmetry) …

Ratio order function 1: $\Theta_\nu = \dfrac{\varepsilon}{SW}$

Θ_ν



r

$$\Theta_\nu = \frac{\varepsilon}{SW} \approx \frac{1 + r_c^2 / r^2}{1 + r_c^2}$$

$$\Theta_\nu = \frac{\varepsilon}{SW}\begin{cases} = 1 & \text{for} \quad r \to 1 \\ \propto r^{-2} & \text{for} \quad r \to 0 \end{cases}$$

Quasi-balance
prod and diss

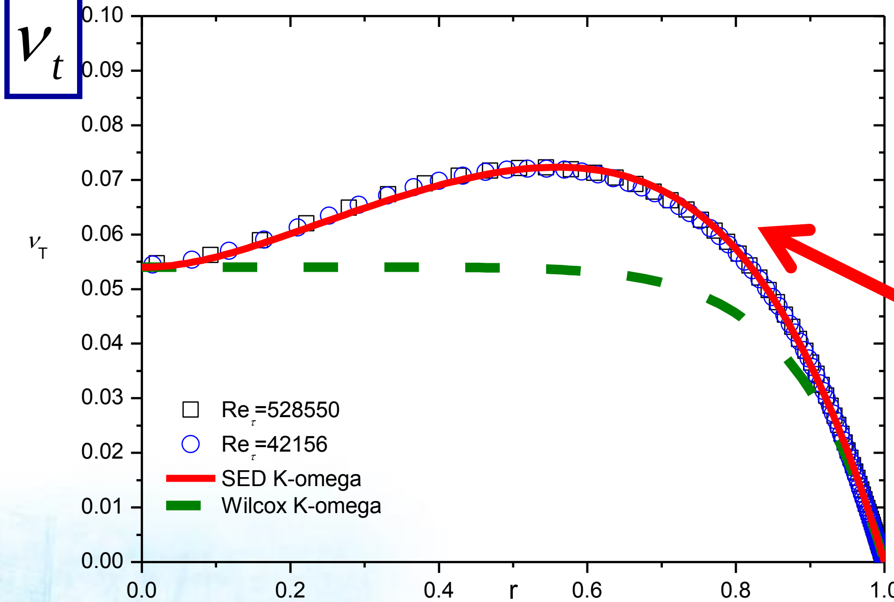
One single parameter r_c describe transition in Θ_ν

SED transition ansatz 2:

$$\nu_t = \frac{W}{S} \approx \nu_0 \left(1 - r^5\right)\left(1 + \left(r / r_c\right)^2\right)^a$$

⇨ **This is an analytic solution to K-omega equations!**

ν_t



r

Wilcox model parameters are such that $a \equiv 0$

DNS suggests for pipe that $a \approx 1/6$

We are inspired to modify K-omega-Wilcox model so that ν_t do displays proper transition as above.

SED Solutions in terms of ν_t and Θ_ν

$$S^+ = \frac{W^+}{\mathrm{Re}_\tau \nu_T} \approx \frac{r}{\mathrm{Re}_\tau \nu_T}$$

$$P^+ = S^+W^+ \approx \frac{r^2}{\mathrm{Re}_\tau \nu_T}$$

$$\varepsilon^+ = S^+W^+\Theta_\nu \approx \frac{r^2}{\mathrm{Re}_\tau \nu_T}\Theta_\nu$$

$$k^+ = \sqrt{\frac{\varepsilon^+ \mathrm{Re}_\tau \nu_T}{\beta^*}} \approx r\sqrt{\frac{\Theta_\nu}{\beta^*}}$$

$$\omega^+ = \frac{k^+}{\mathrm{Re}_\tau \nu_T} = \frac{r}{\mathrm{Re}_\tau \nu_T}\sqrt{\frac{\Theta_\nu}{\beta^*}}$$

$$\nu_t = \frac{\mathrm{W}}{S} = \alpha^* \frac{\mathrm{k}}{\varpi}$$

Outer approximation:

$$W^+ \approx r, \quad S^+ \approx r/\left(1+\nu_t^+\right)$$

$$S^+ + W^+ = r$$

$$S^+W^+ - \varepsilon + \frac{d^2k^+}{dy^{+2}} + T.T = 0$$

All quantities are expressed in terms of ν_t and Θ_ν

K-omega-SED:

ω **Equation:**

$$\alpha \frac{\omega^+}{k^+} S^+ W^+ - \beta \omega^{+2} + \frac{d^2 \omega^+}{dy^{+2}} + \frac{d}{dy^+}\left(\sigma_0 \left(1 + \left(\frac{\gamma k^+}{\omega^+ \mathrm{Re}_\tau} \right)^2 \right) \alpha^* \frac{k^+}{\omega^+} \frac{d\omega^+}{dy^+} \right) = 0$$

Parameter changes:

One layer near the outer edge

1） Karman constant

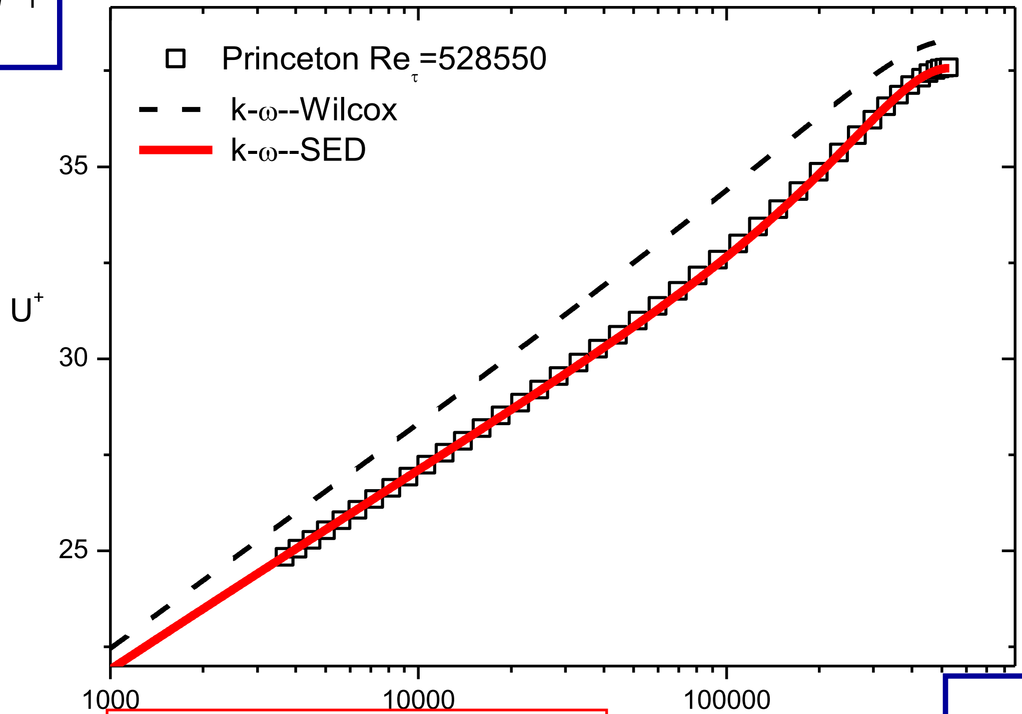
For pipe:	K-omega -Wilcox	κ=0.40
For all three:	K-omega-SED	κ=0.45

K-omega-SED:

ω **Equation:**

$$\alpha \frac{\omega^+}{k^+} S^+ W^+ - \beta \omega^{+2} + \frac{d^2 \omega^+}{dy^{+2}} + \frac{d}{dy^+}\left(\sigma_0 \left(1 + \left(\frac{\gamma k^+}{\omega^+ \mathrm{Re}_\tau}\right)^2\right) \alpha^* \frac{k^+}{\omega^+} \frac{d\omega^+}{dy^+}\right) = 0$$

U^+



y^+

$Re_\tau \approx 528{,}550$

K-omega -Wilcox

$\kappa = 0.40$

K-omega-SED

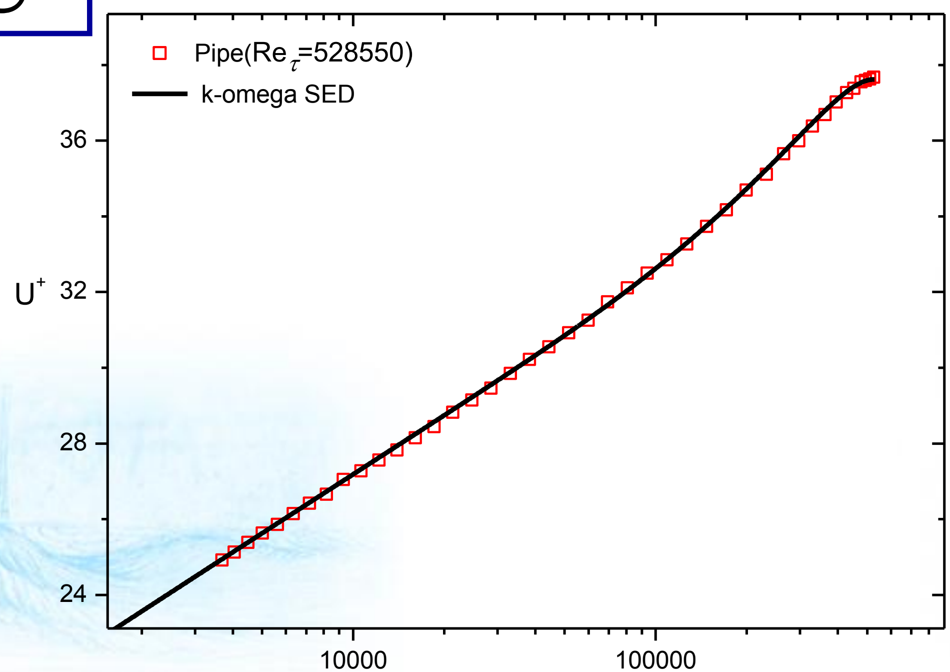
$\kappa = 0.45$

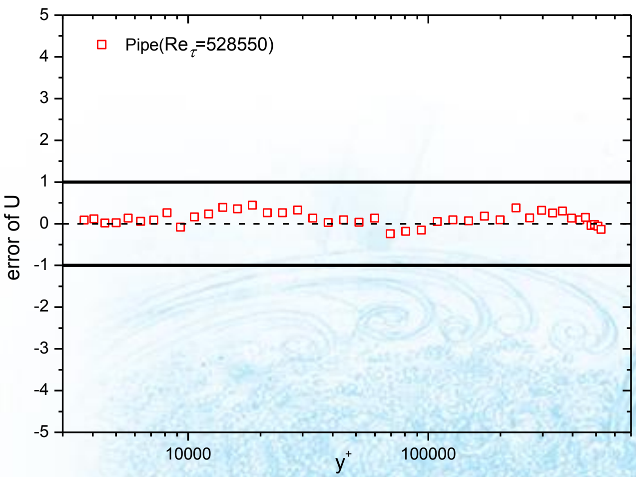
K-omega-SED:

ω Equation:

$$\alpha \frac{\omega^+}{k^+} S^+ W^+ - \beta \omega^{+2} + \frac{d^2\omega^+}{dy^{+2}} + \frac{d}{dy^+}\left(\sigma_0\left(1+\left(\frac{\gamma k^+}{\omega^+ \mathrm{Re}_\tau}\right)^2\right)\alpha^* \frac{k^+}{\omega^+}\frac{d\omega^+}{dy^+}\right) = 0$$

U^+





$Re_\tau \approx 528,550$

y^+

K-omega-SED:

ω Equation:

$$\alpha\frac{\omega^+}{k^+}S^+W^+ - \beta\omega^{+2} + \frac{d^2\omega^+}{dy^{+2}} + \frac{d}{dy^+}\left(\sigma_0\left(1+\left(\frac{\gamma k^+}{\omega^+ \mathrm{Re}_\tau}\right)^2\right)\alpha^*\frac{k^+}{\omega^+}\frac{d\omega^+}{dy^+}\right)=0$$

One layer near the outer edge

Parameter changes:

1） Karman constant

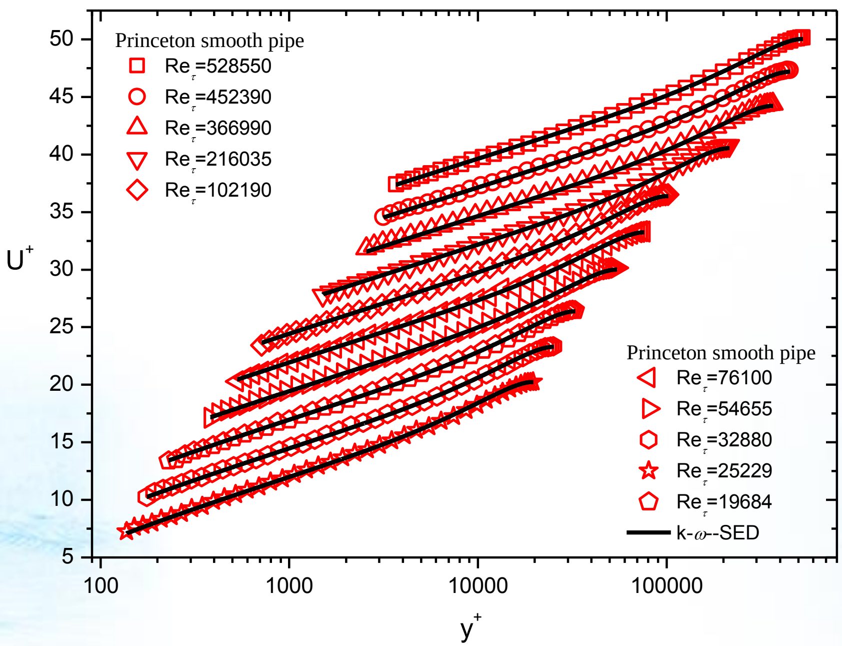
2） The new parameter: γ displays CH, pipe, TBL.

γ describes a transition of balance mechanism in actions in omega, which is to be elucidated!

K-omega-SED:

ω Equation:

$$\alpha\frac{\omega^+}{k^+}S^+W^+-\beta\omega^{+2}+\frac{d^2\omega^+}{dy^{+2}}+\frac{d}{dy^+}\left(\sigma_0\left(1+\left(\frac{\gamma k^+}{\omega^+\mathrm{Re}_\tau}\right)^2\right)\alpha^*\frac{k^+}{\omega^+}\frac{d\omega^+}{dy^+}\right)=0$$



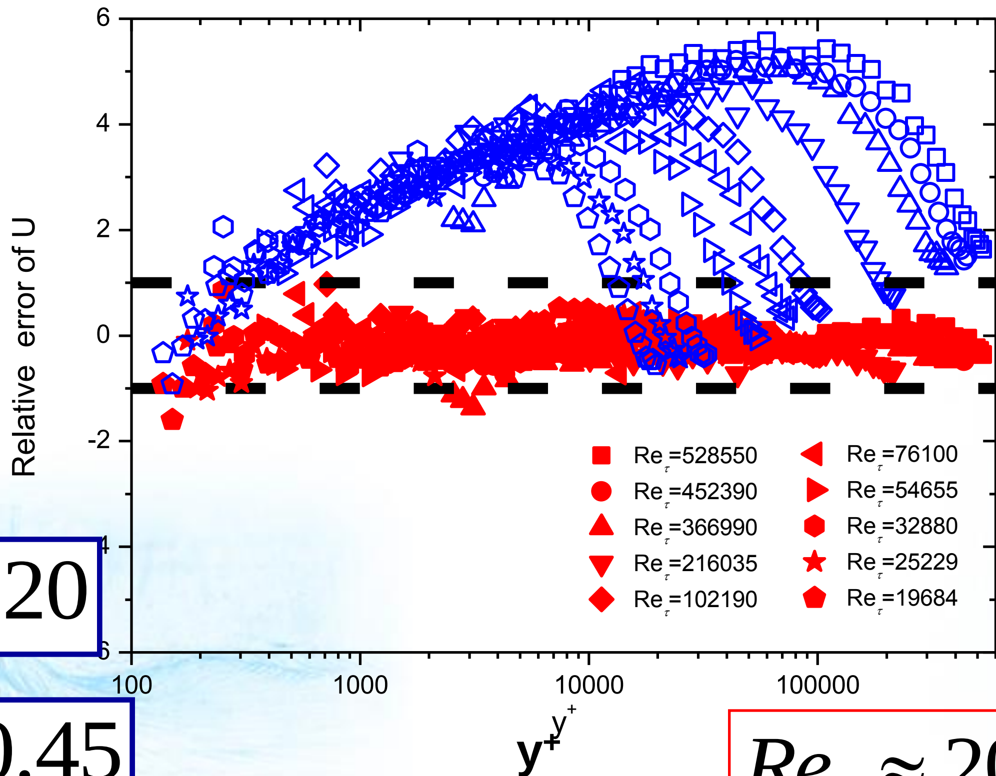
$\gamma = 22$

$\kappa = 0.45$

K-omega-SED:

Errors in comparison to Princeton Superpipe data

Lines: 1% error



K-omega -Wilcox

K-omega-SED

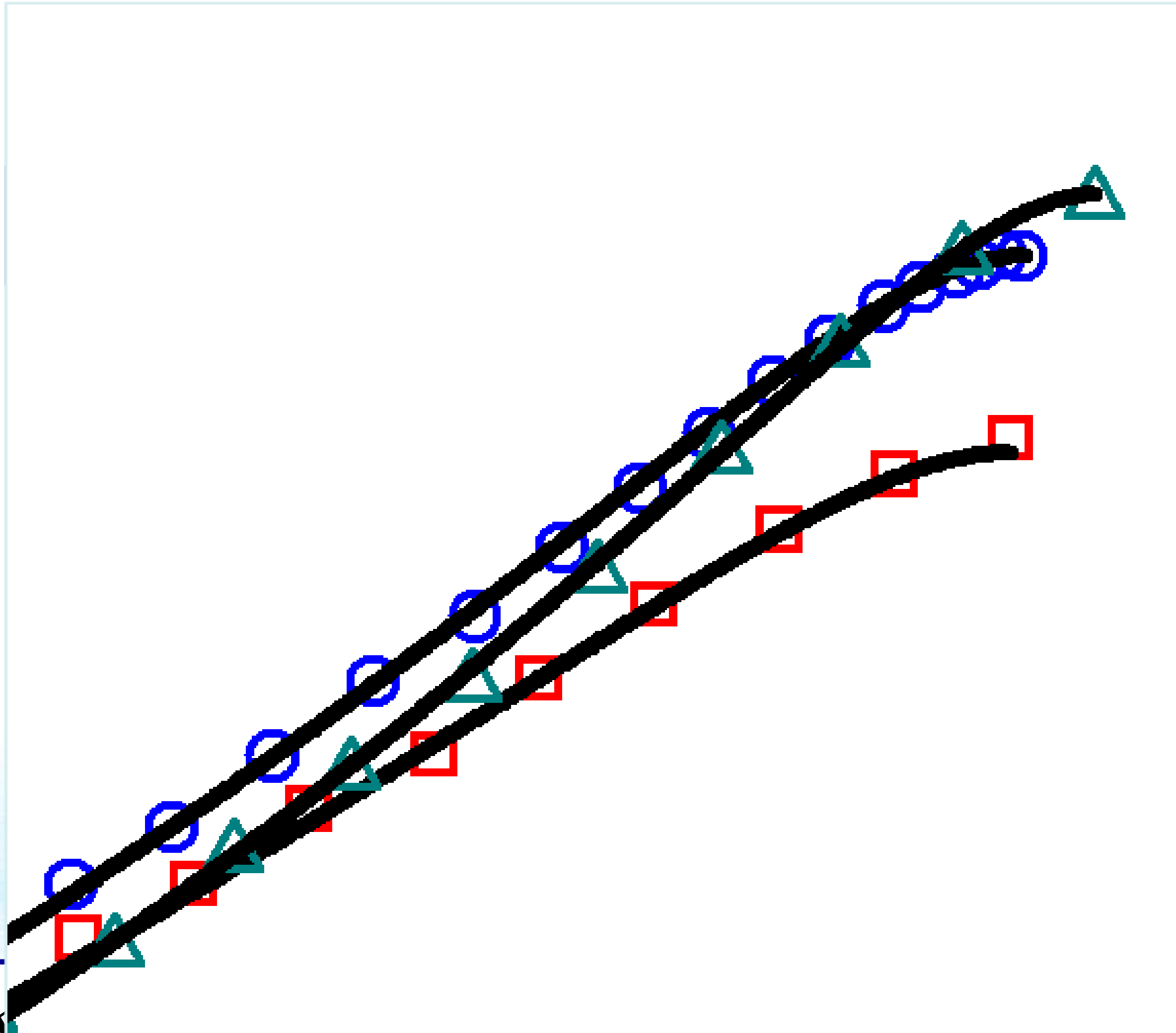
$\gamma = 20$

$\kappa = 0.45$

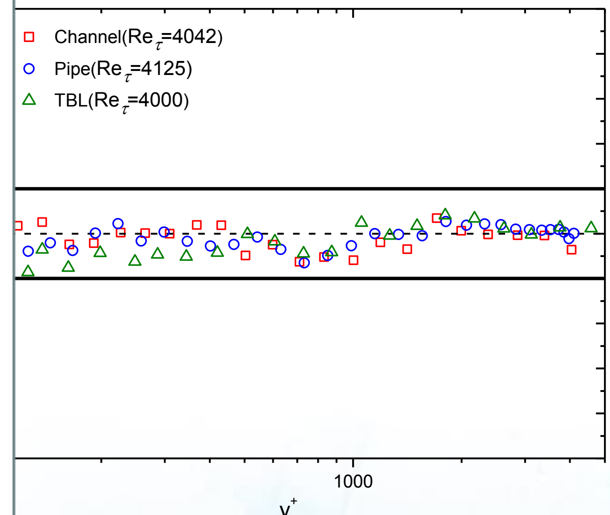
From 6% to 1%

$$Re_\tau \approx 20,000 \rightarrow 500,000$$

K-omega-SED for all three flows:



Errors < 1%



$$\gamma^{CH} = 10$$

$$\gamma^{Pipe} = 22$$

$$\gamma^{TBL} = 26$$

Conclusions:

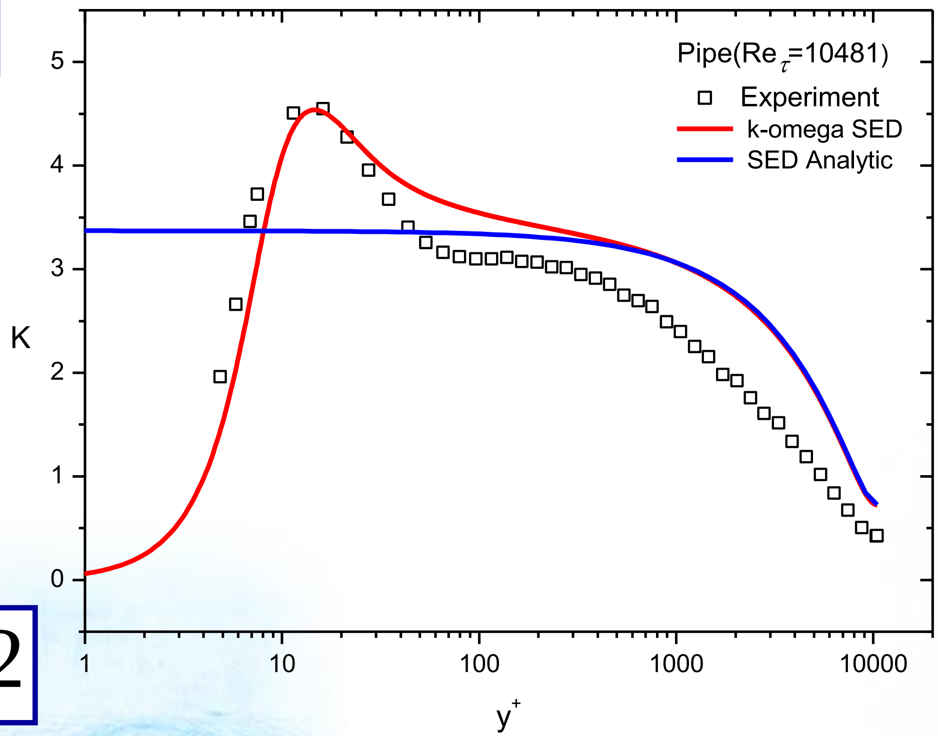
- Ansatz for ν_t and Θ_ν yields analytic (outer) solution to K-omega model equations.
- Modification to K-omega-Wilcox is introduced, to yield better solutions when compared with Princeton pipe experiments
- New K-omega-SED is capable of developing a unified mean velocity description of channel, pipe and TBL, with a single parameter. γ
- K-omega model parameterizes multi-layer structures, which are revealed by SED.
- This understanding is being extended to the description of kinetic energy distribution.

K-omega-SED:

Prediction about K

More work needed for energy distribution!

K^+



K-omega -SED

SED analytic

$k^+ \approx r\sqrt{\frac{\Theta_\nu}{\beta^*}}$

$\gamma = 22$

κ=0.45

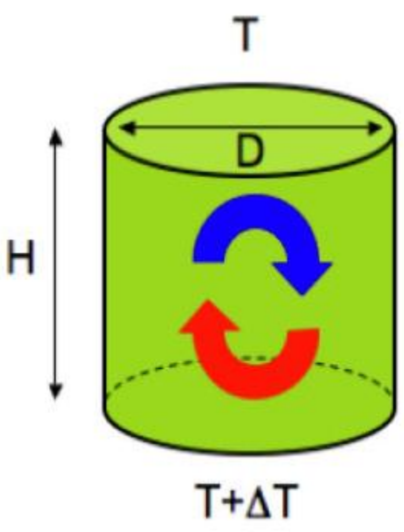
y+

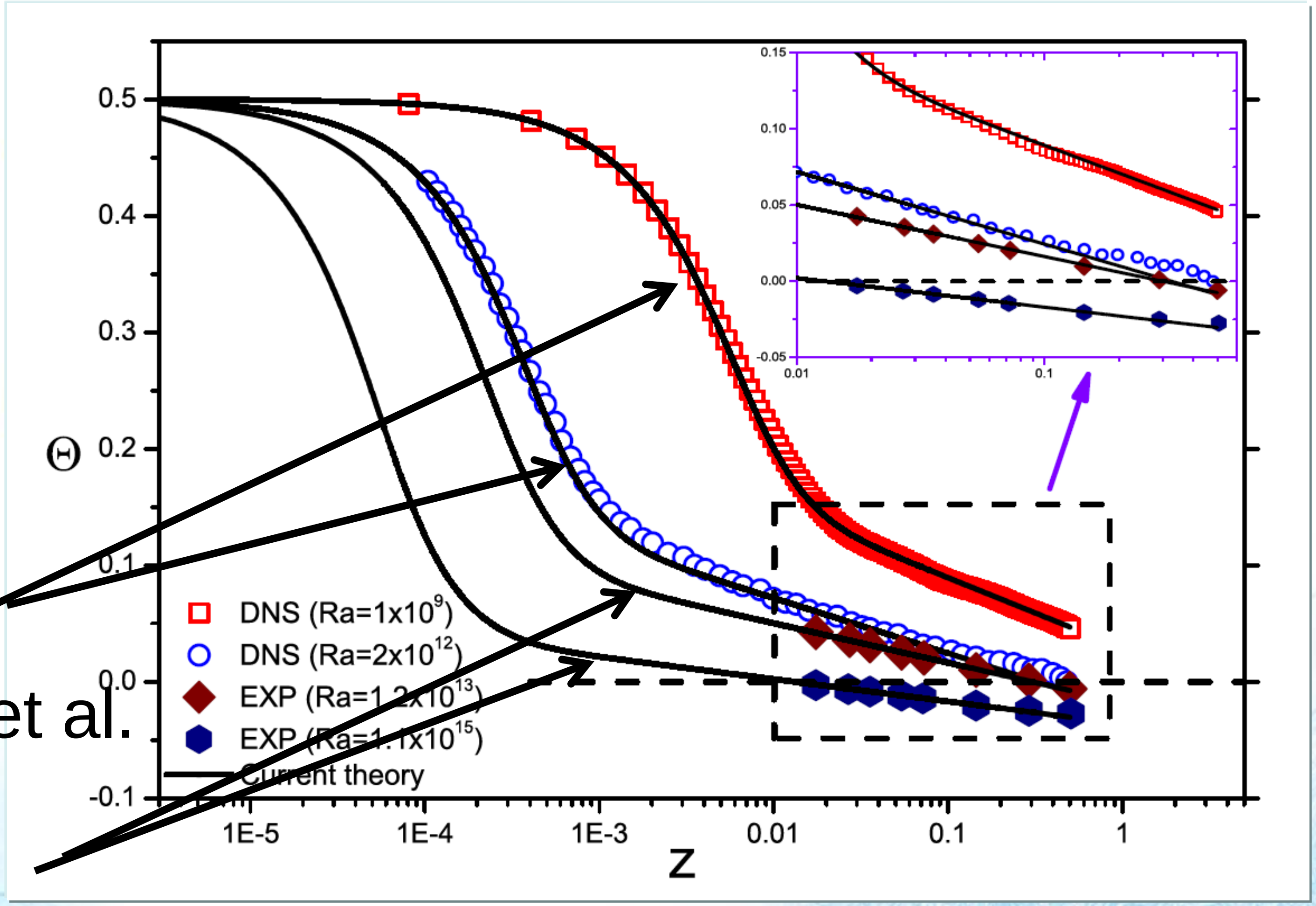
$Re_\tau \approx 14000$

Discussions:

- **Is it final? A new philosophy says that for a complex system, good solutions are not unique. SED finds more rational in RANS modeling.**
- **More interesting questions:**
  1. temperature and velocity distribution is RB convection?
  2. MVP in compressible flows with strong pressure gradients, separation and shocks?
  3. Implications of universal Karmen constant?
  - Etc.

RB convections:





Ra:10^{9-12}
DNS(Lohse et al.

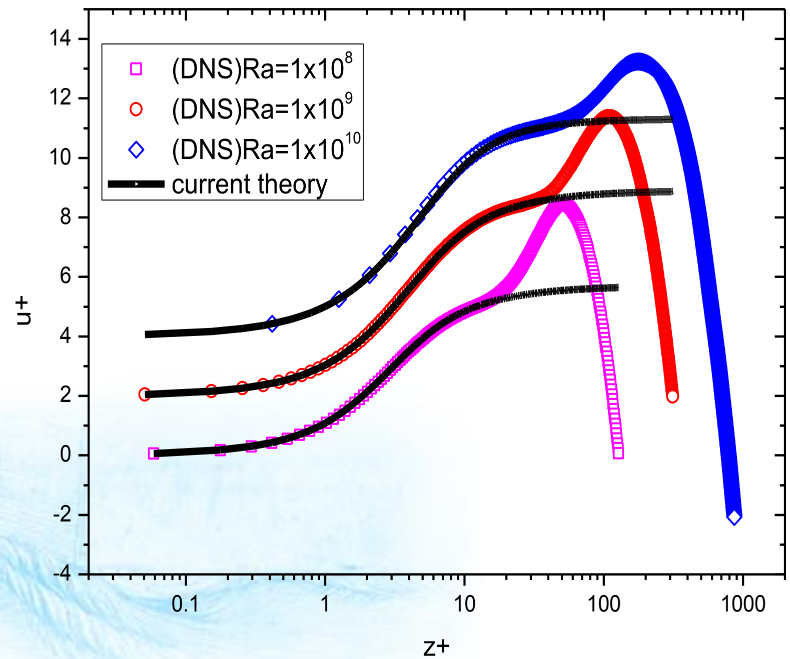
Ra: 10^{13-15}
EXP(Ahlers et al.)

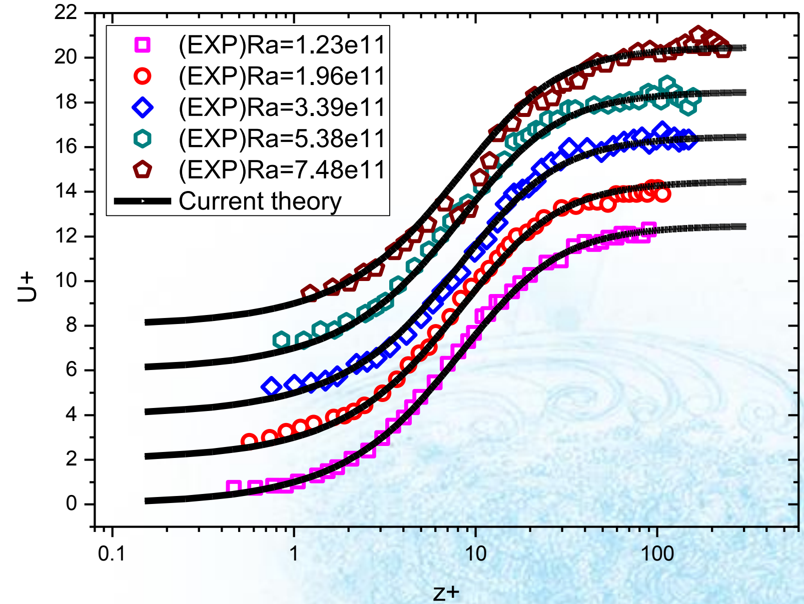
Lines: SED expressions (one parameter, mixing zone thickness)

Ilmenau Experiment:

$$\ell_M^{+} = 0.054 z^{+3/2}\left(1+\left(\frac{z^{+}}{17.5}\right)^{4}\right)^{1/4}$$

U+ **Our DNS simulations** **du Puits et al, PRL, 2007**





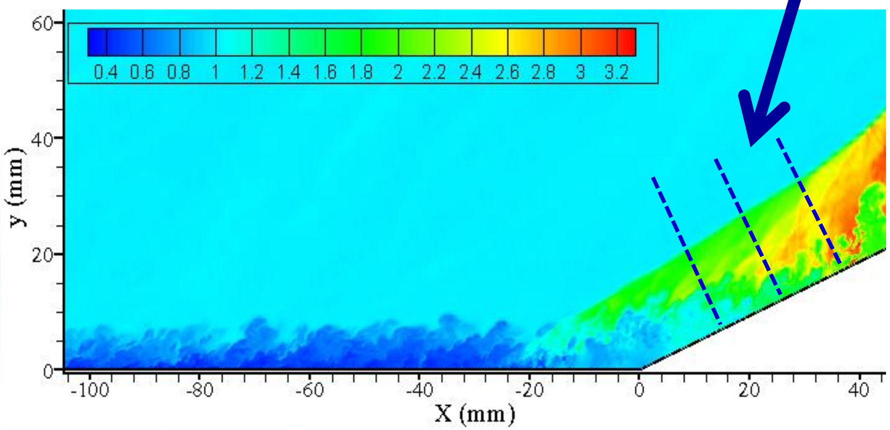
Capture the velocity distribution in the near-wall region

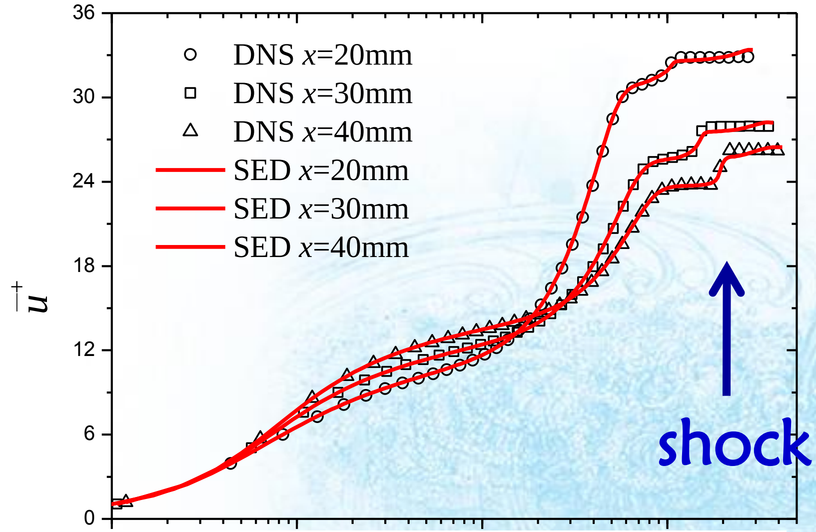
Compressible flow with a ramp

SED develops analytic description of the mean velocities in regions of strong adverse pressure gradients. Comparison with DNS of spatially developing flow passing a ramp.

After reattachment

(Ma=3.0, 25 degree)

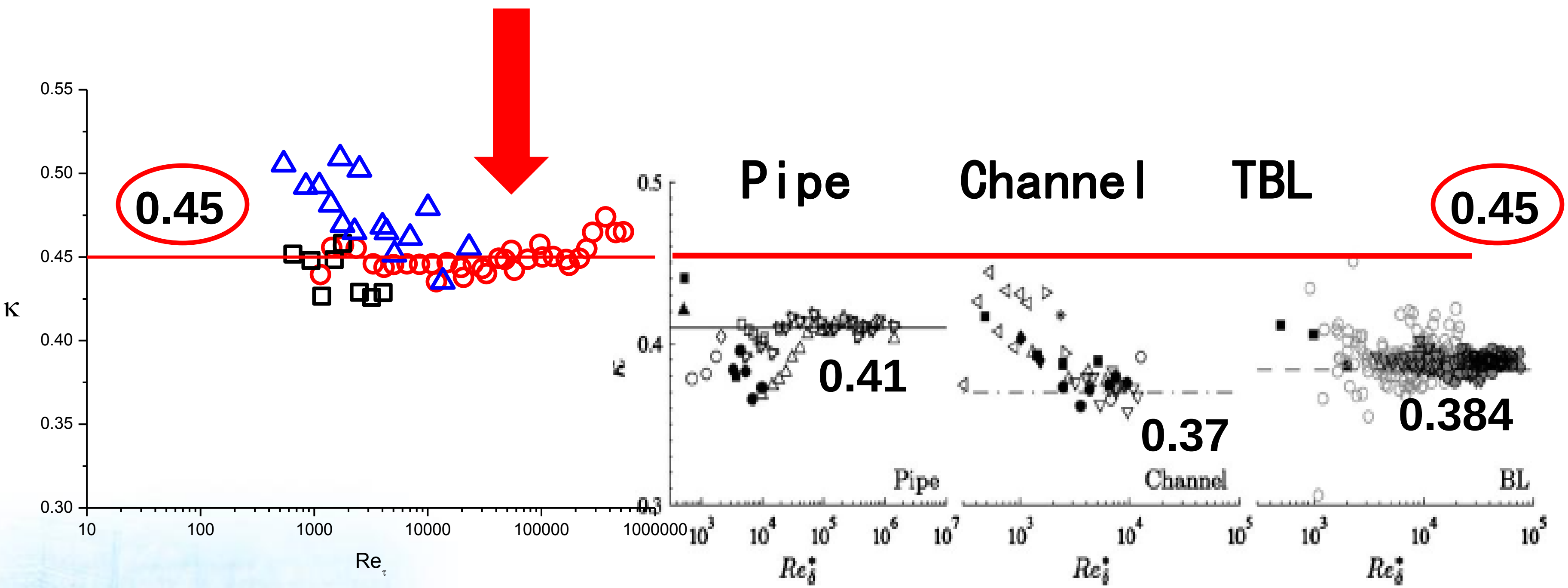




Comparison with DNS

Universal Karman constant:

Three canonical wall-bounded turbulent flow: 0.45



57 sets of data

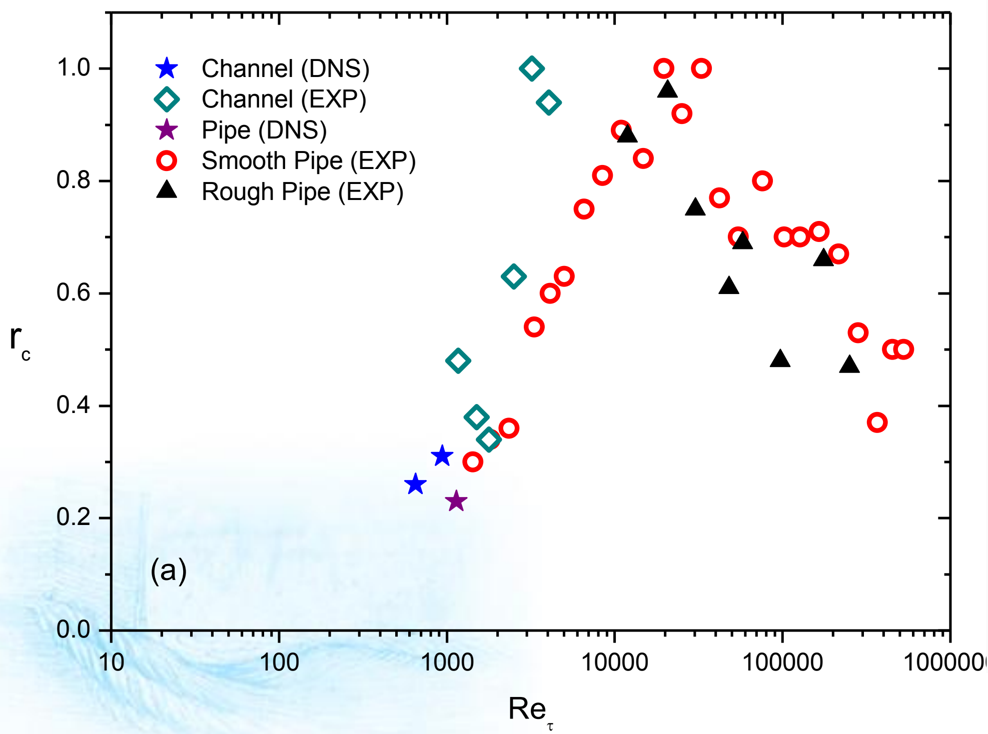
Marusic et al. Phy. Fluid (2010)

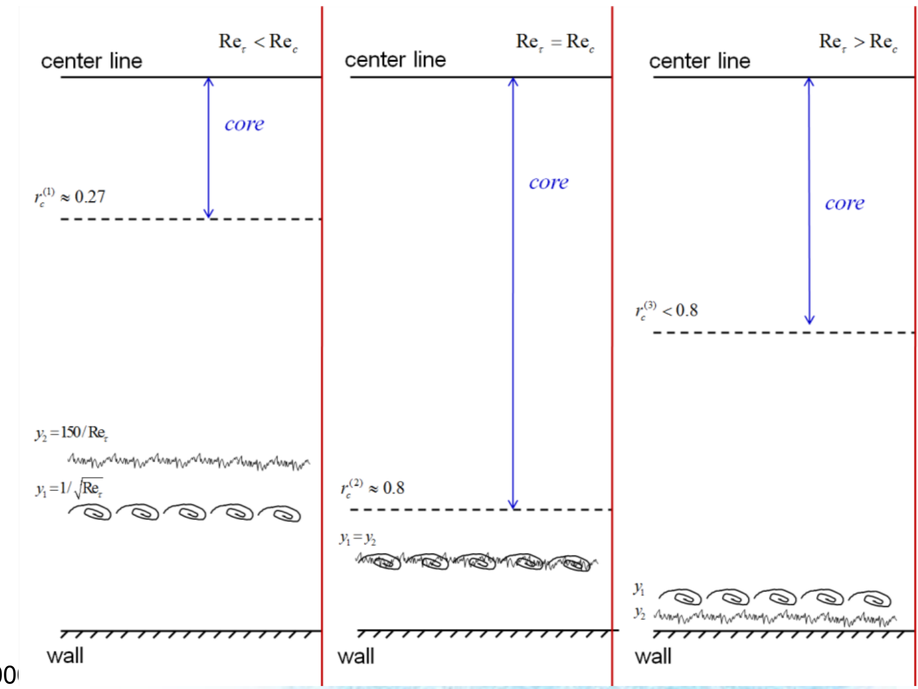
Universal Karman constant:

Measurement of r_c:

A critical Ret around 20,000

$$\sqrt{Re_c} \approx 150$$





Thanks for your attention!