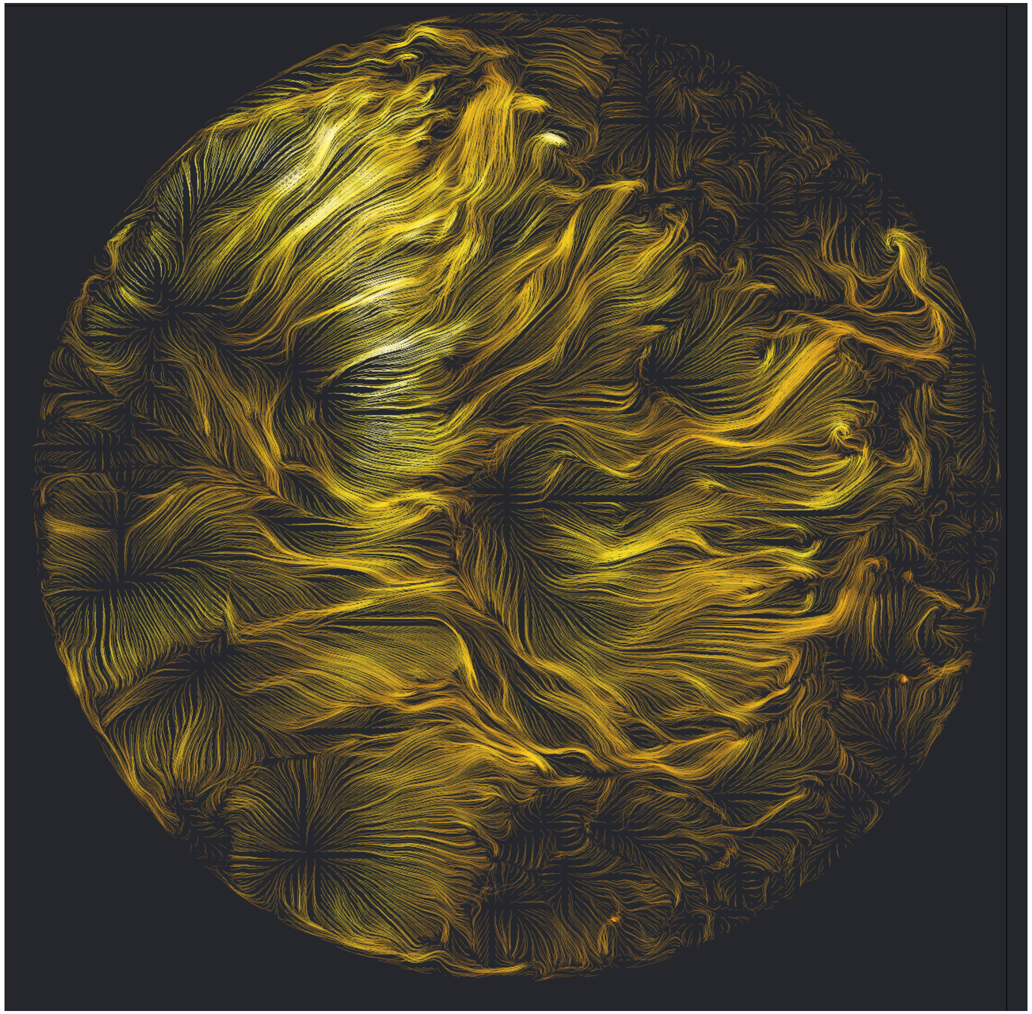


Numerical studies on high-order derivative moments in turbulent shear flows and convective turbulence

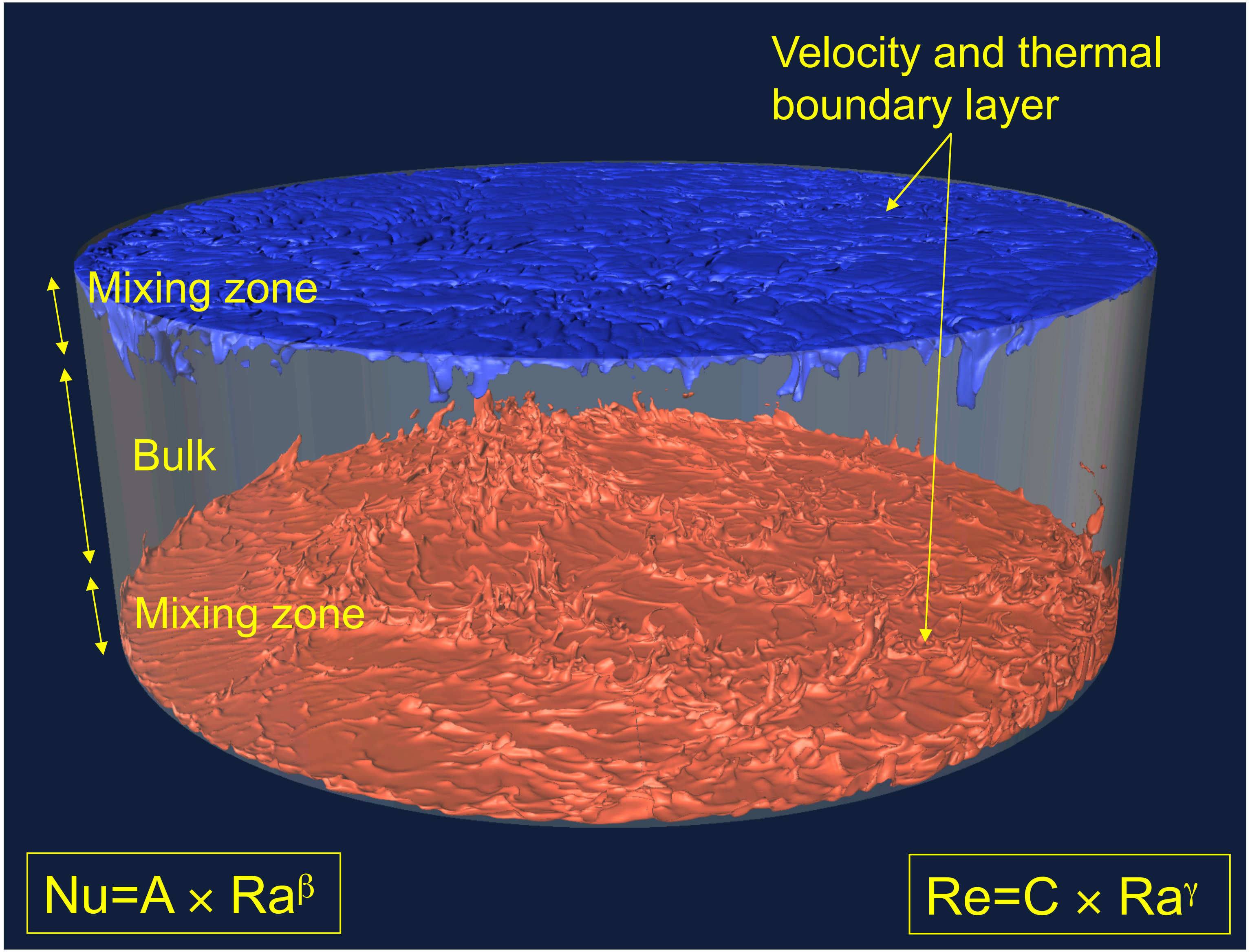
Jörg Schumacher

Institute of Thermodynamics and Fluid Mechanics, TU Ilmenau, Germany

Outline

- Boundary layer structure in Rayleigh-Bénard convection
  with R. du Puits, R. Kaiser, C. Resagk, A. Thess (TU Ilmenau)
- Large-scale order in fully developed convective turbulence
  with J.D. Scheel (Occidental College, Los Angeles), M.S. Emran (TU Ilmenau)
- Universality of velocity gradient statistics
  with J.D. Scheel, V. Yakhot (Boston University), K.R. Sreenivasan (New York University)

How is heat and momentum transported?

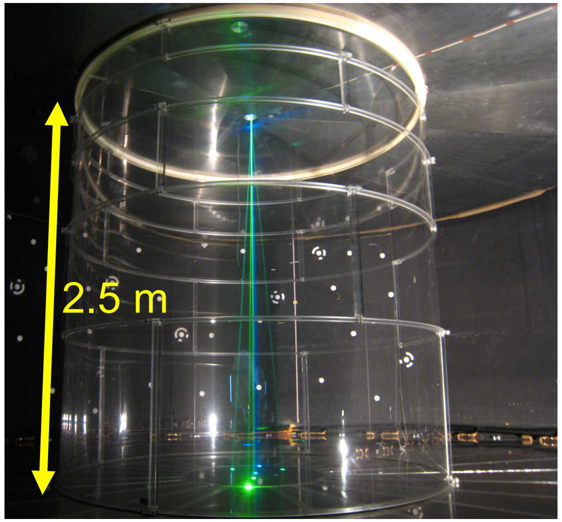


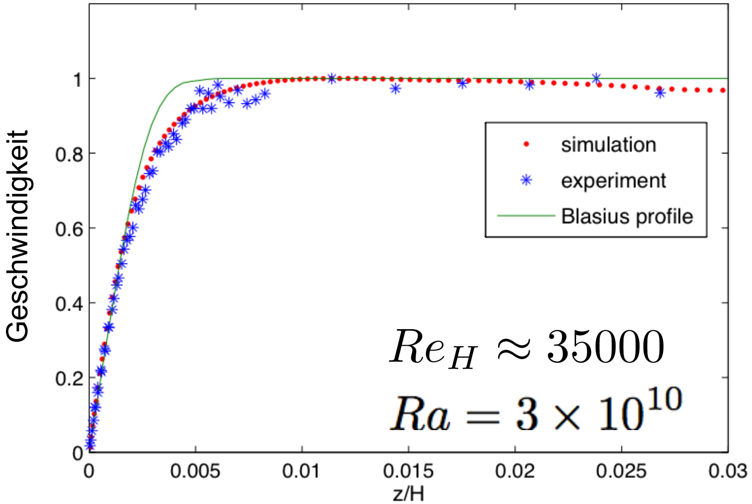
Chillà & Schumacher, Eur. J. Phys. (2012)

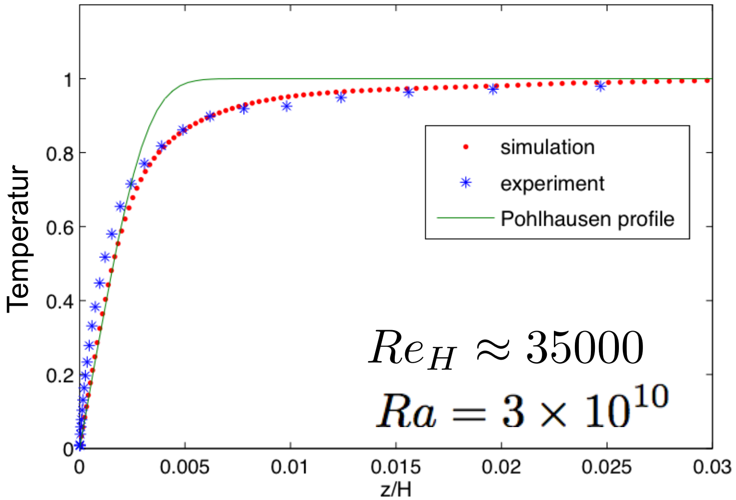
Boundary layer analysis

Li, Shi, du Puits, Resagk, Schumacher & Thess, Phys. Rev. E (2012); Shi, Emran & Schumacher, J. Fluid Mech. (2012)

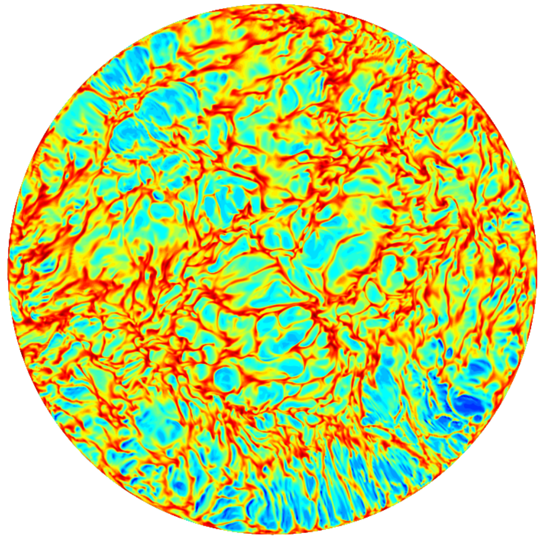
Barrel of Ilmenau





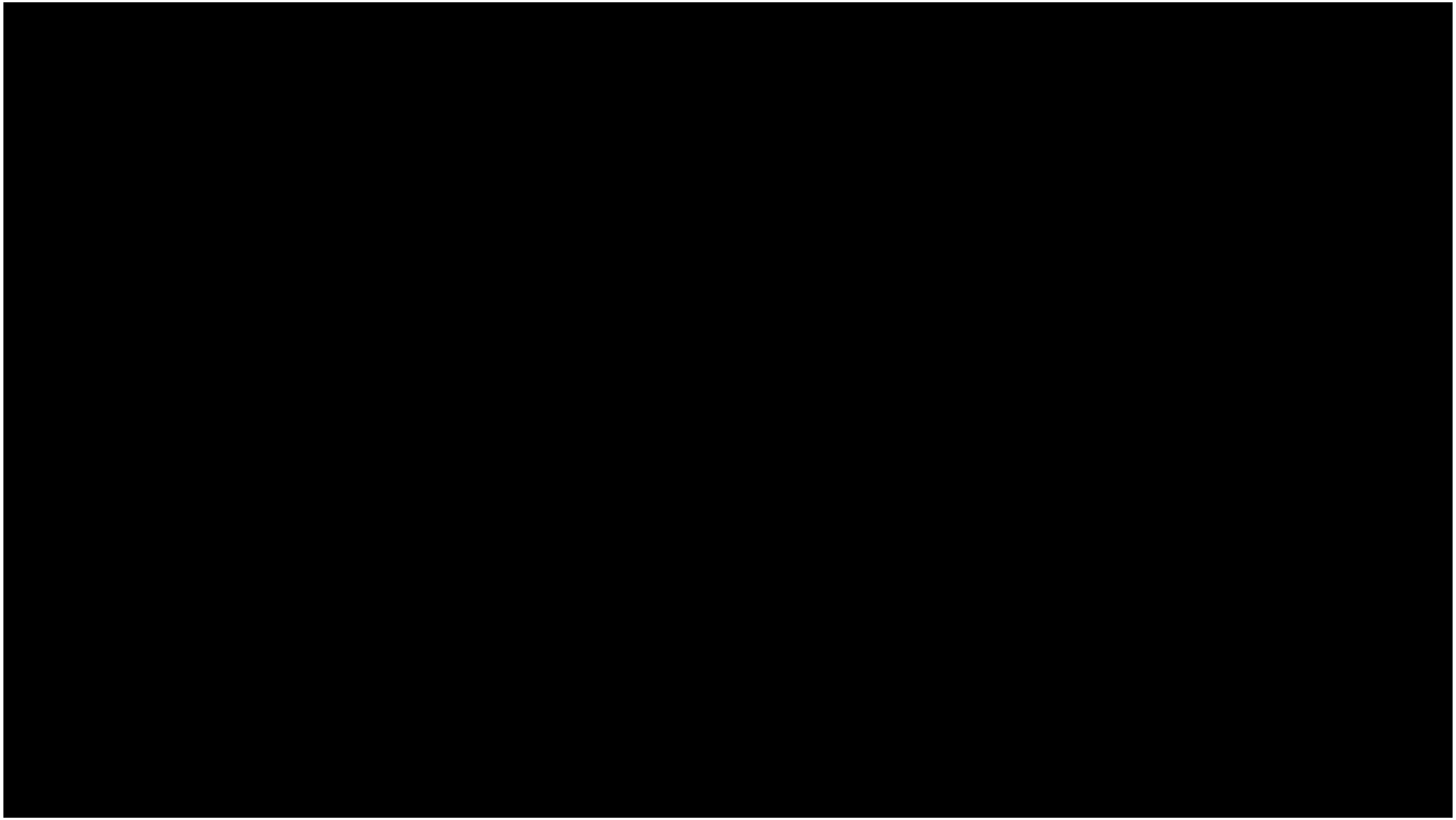


DNS



Velocity and temperature profiles deviate significantly from Prandtl-Blasius-Pohlhausen solution which enters mean field theory of turbulent transport

2d PIV study of boundary layer dynamics

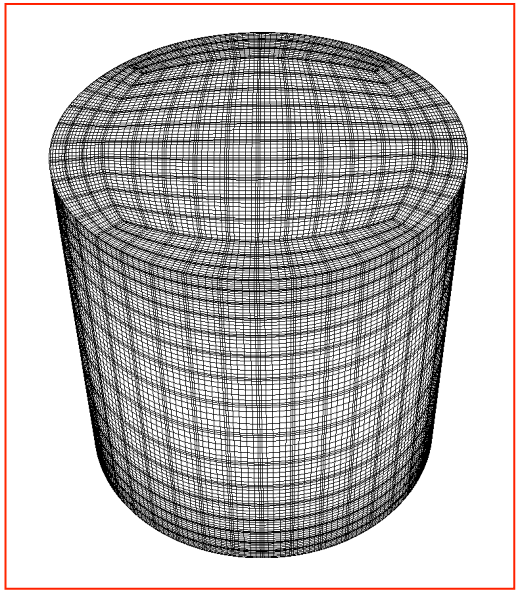


$Ra = 10^{10} \quad Pr = 0.7$

du Puits, Willert, Resagk & Thess, submitted (2013)

Numerical solution of Boussinesq equations

Verzicco & Orlandi, J. Comp. Phys. 1996; Fischer, J. Comp. Phys. 1997



- 2nd-order finite difference method in cylindrical coordinates

  Poisson solver with Fishpack
  3rd order RK scheme
  MPI/OpenMP hybrid version

- Spectral element method Nek5000

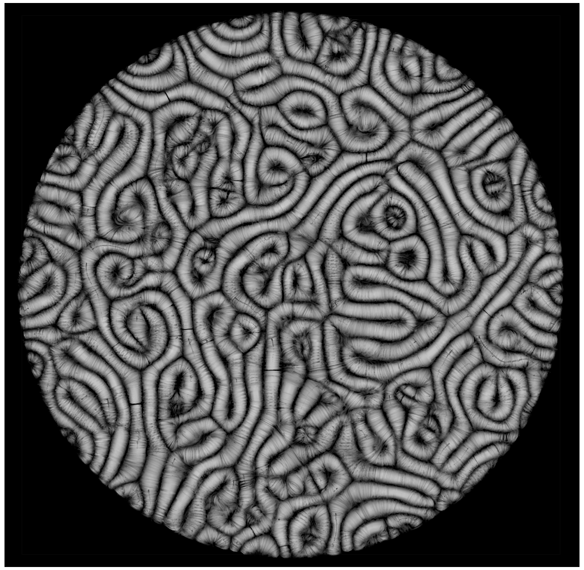
  Poisson solver with AMG preconditioner
  Lagrangian interpolation polynomials
  BDF2 scheme
  pure MPI version
  Strong scaling tests on up to 131072 cores

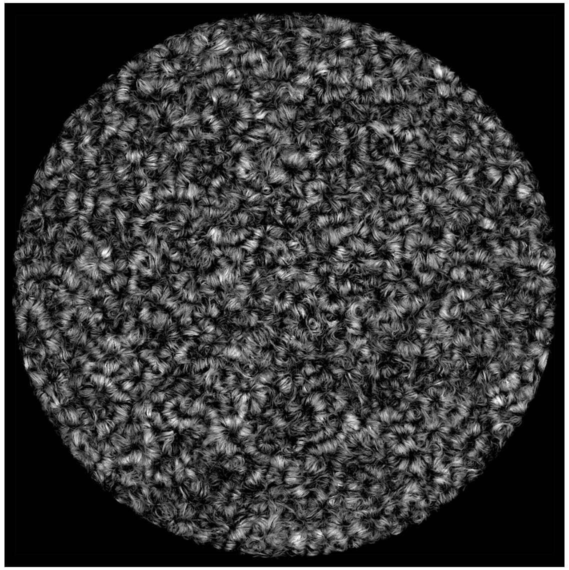
  $Ra=10^{10}$: 32768 BGQ cores, 2.37 million elements, polynomial order N=11

Large-scale organization

Busse, Surv. Geophysics (2003); Bailon-Cuba et al., J. Fluid Mech. (2010)

Snapshot





Snapshot

$Ra = 5 \times 10^3$

$\Gamma = 50$

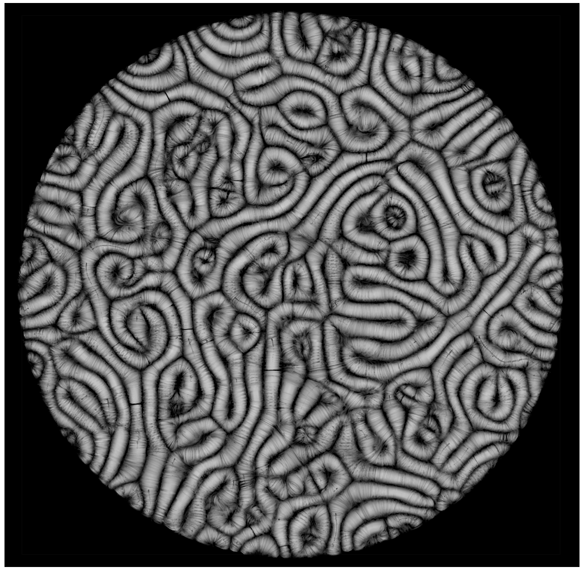
$Ra = 5 \times 10^5$

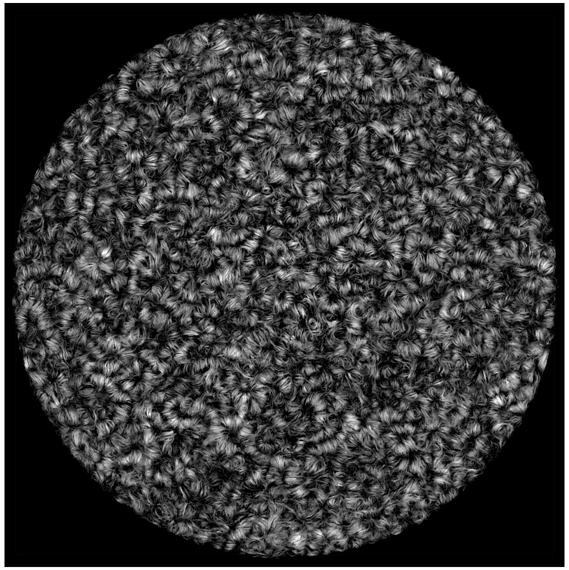
$\Gamma = 50$

Large-scale organization

Busse, Surv. Geophysics (2003); Bailon-Cuba et al., J. Fluid Mech. (2010)

Snapshot





Snapshot

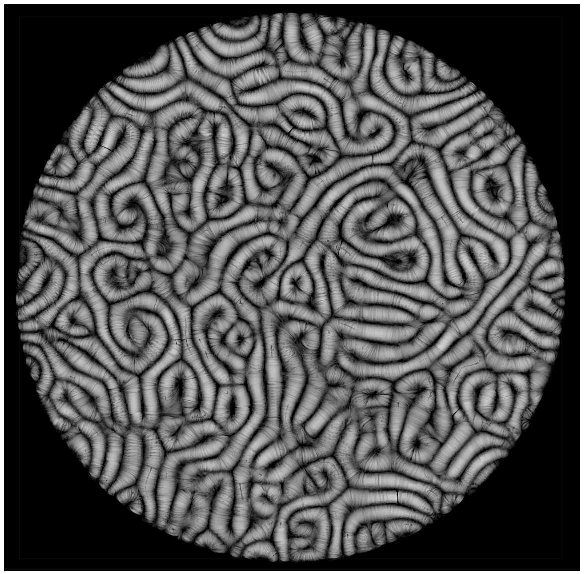
$Ra = 5 \times 10^3$

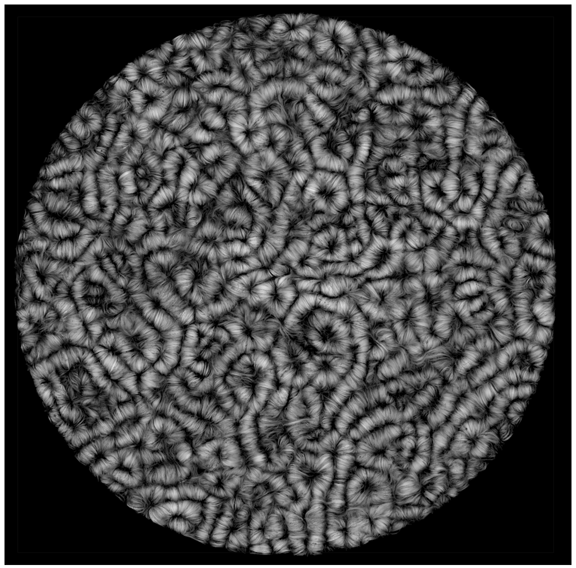
$\Gamma = 50$

$Ra = 5 \times 10^5$

$\Gamma = 50$

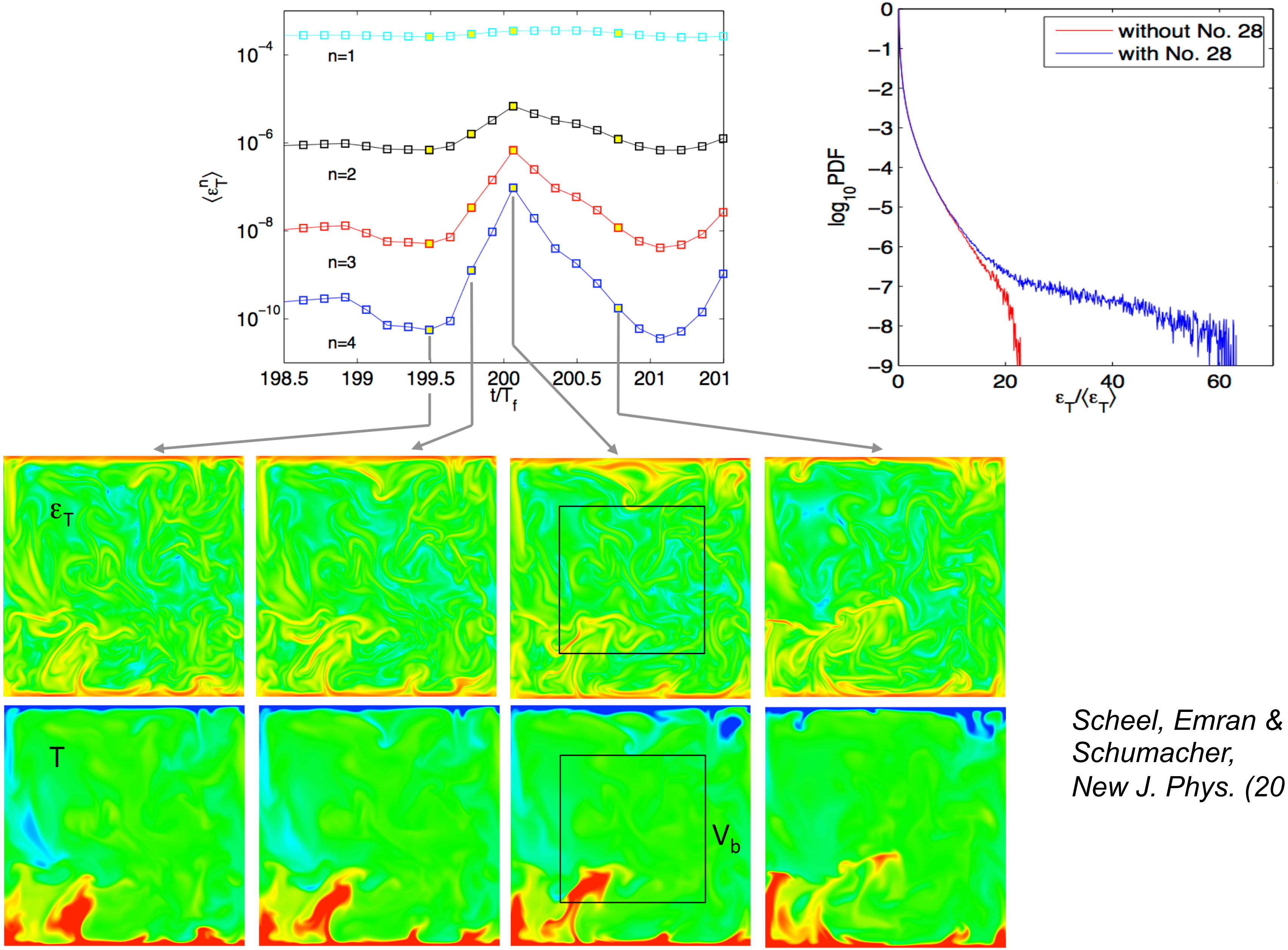
Average





Average

Strong plume events affect bulk statistics

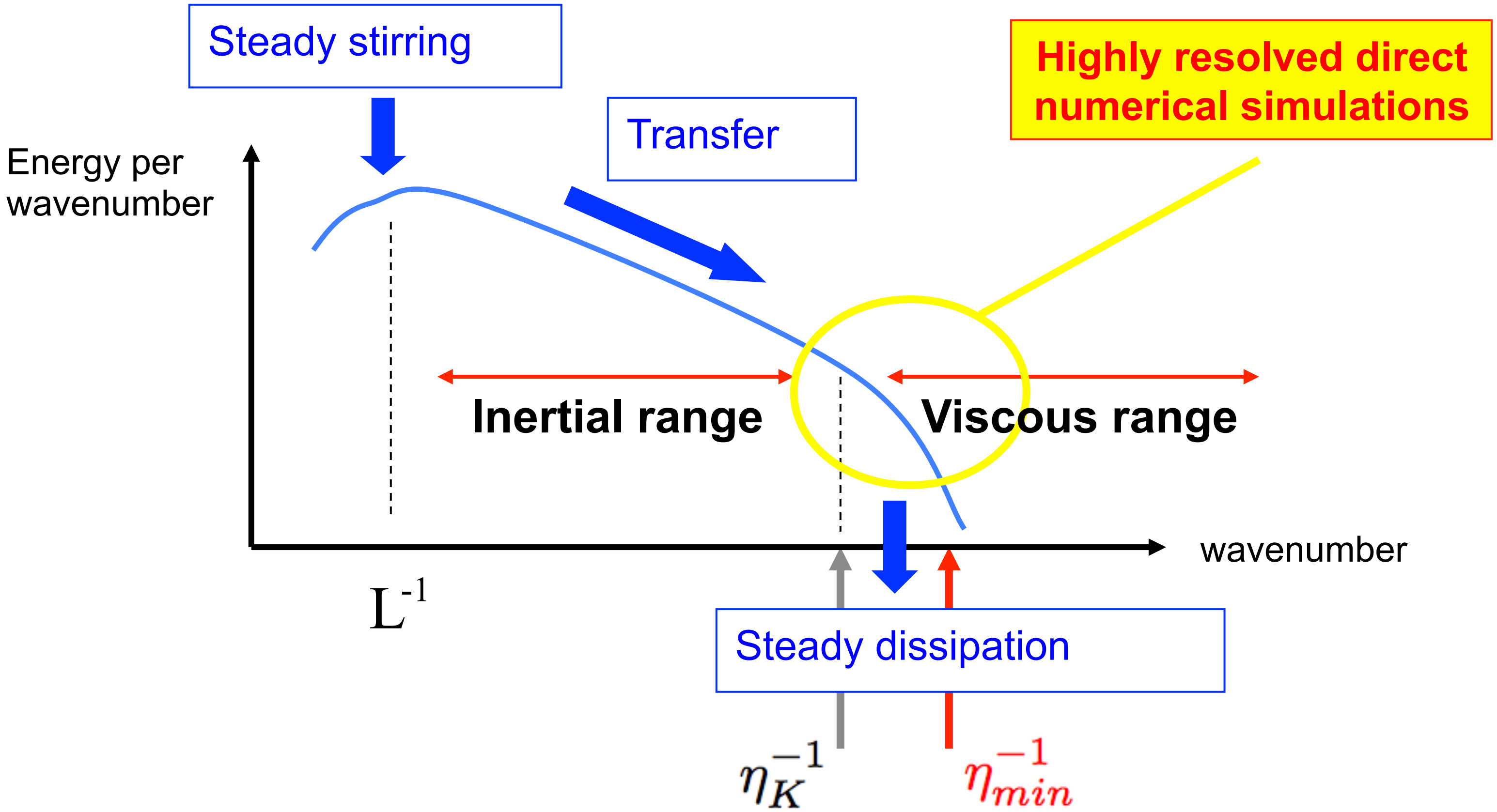


Scheel, Emran & Schumacher, New J. Phys. (2013)

... a slightly different focus

$$\epsilon(\vec{x},t) = \frac{\nu}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)^2$$

$$S_n(x) = \langle(\delta_x u)^n\rangle \sim x^{\zeta_n}$$



Turbulence = High spatial intermittency of velocity gradients at small scales

Scaling exponents for velocity derivatives

Yakhot, Physica D 2006

$$\eta_{2n} = L\, Re^{\frac{1}{\zeta_{2n}-\zeta_{2n-1}-1}}$$

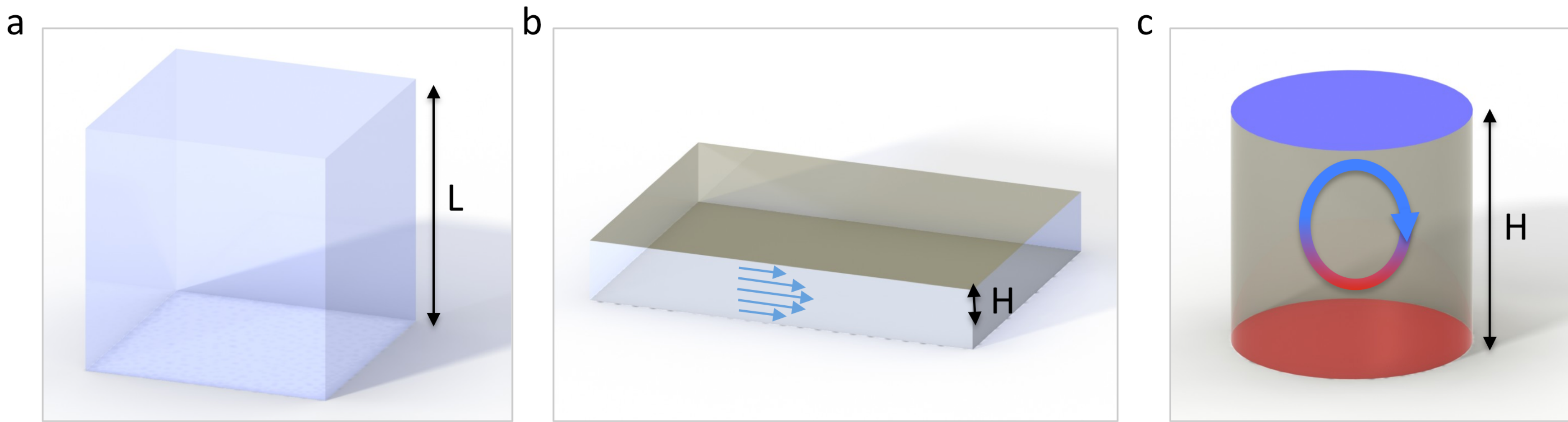
- $\zeta_{2n} = \dfrac{2n}{3} \quad \rightarrow \quad \eta_{2n} = L\, Re^{\frac{3}{4}} = \eta_K$
- $n \rightarrow \infty \ : \quad \eta_{2n} \rightarrow L\, Re^{-1} = \eta_{min}$
- $$\left\langle \left(\frac{\partial u_x}{\partial x} \right)^n \right\rangle \approx \left\langle \left(\frac{\delta_\eta u_x}{\eta} \right)^n \right\rangle \overset{\nu \approx \eta\, \delta_\eta u_x}{=} \frac{\langle (\delta_\eta u_x)^{2n} \rangle}{\nu^n} \approx \frac{S_{2n}(\eta_{2n})}{\nu^n}$$

$$\sim Re^n \eta_{2n}^{\zeta_{2n}} \sim Re^{n + \frac{\zeta_{2n}}{\zeta_{2n}-\zeta_{2n+1}-1}}$$

$$\langle \epsilon^n \rangle \sim Re^{n + \frac{\zeta_{4n}}{\zeta_{4n}-\zeta_{4n+1}-1}}$$

Differences to the refined similarity hypothesis (K62)

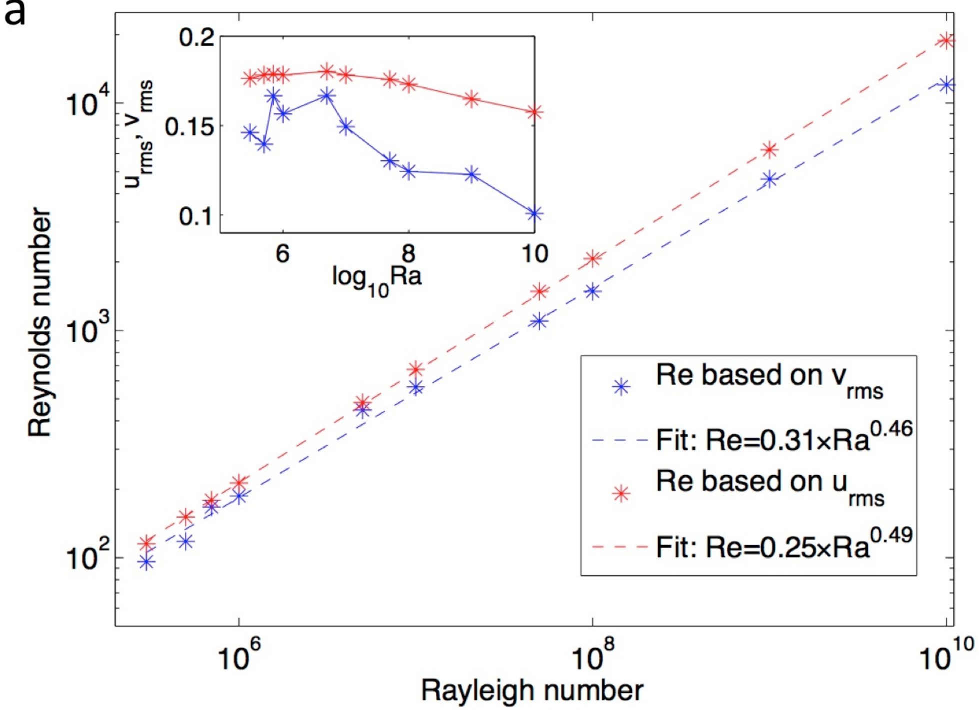
Three turbulent flows



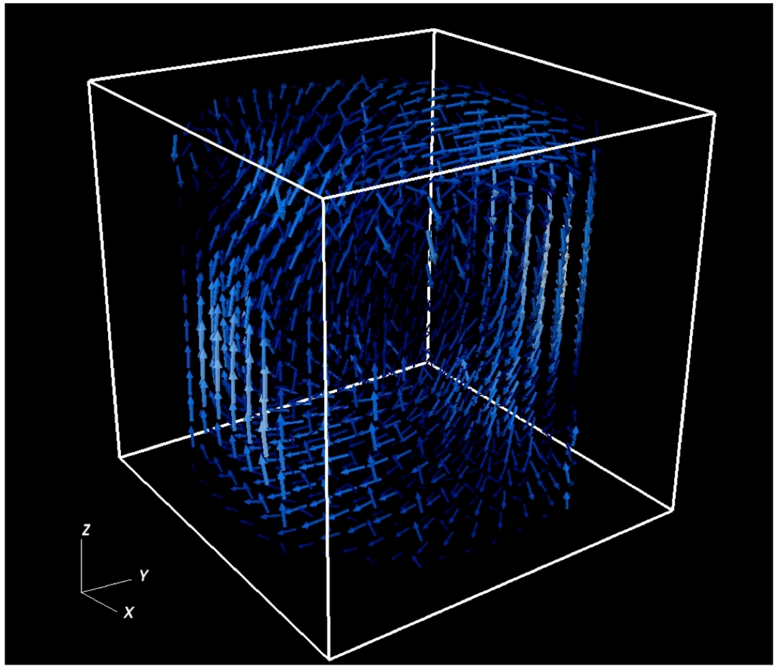
Homogeneous isotropic box turbulence	Turbulent shear flow turbulence	Turbulent Rayleigh-Bénard convection
Periodic boundaries in all three spatial directions	Walls in vertical directions and periodic otherwise	Closed cell with walls
Three homogeneous directions	Two homogeneous directions	One homogeneous direction
$u_i(\vec{x},t) = v_i(\vec{x},t)$	$u_i(\vec{x},t) = \overline{u}_x(z)\delta_{xi} + v_i(\vec{x},t)$	$u_i(\vec{x},t) = \overline{u}_i(\vec{x}) + v_i(\vec{x},t)$

Large-scale flow and Re-Ra scaling

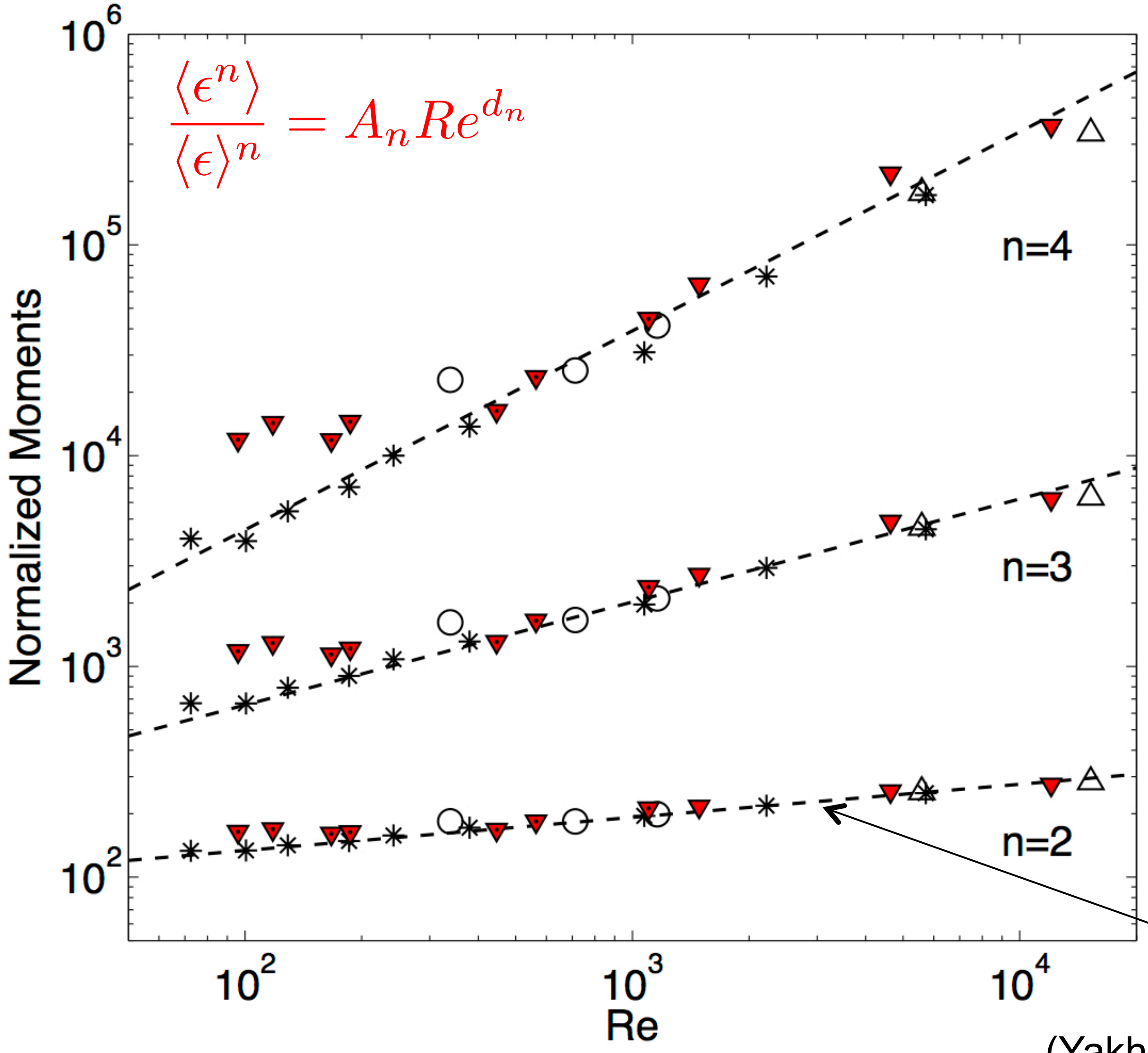
a



b



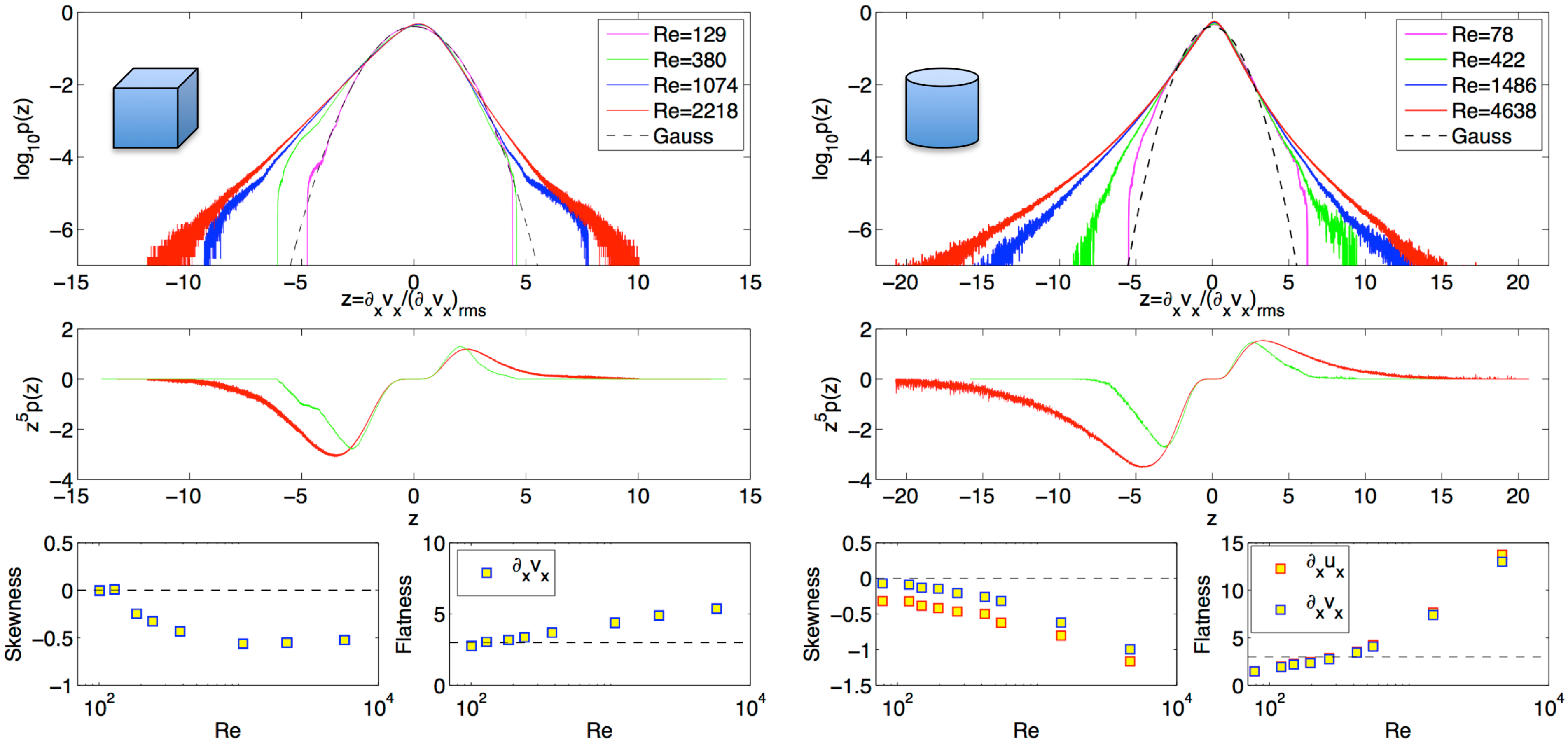
Normalized moments of energy dissipation rate



Theory
(Yakhot, Physica D 2006)

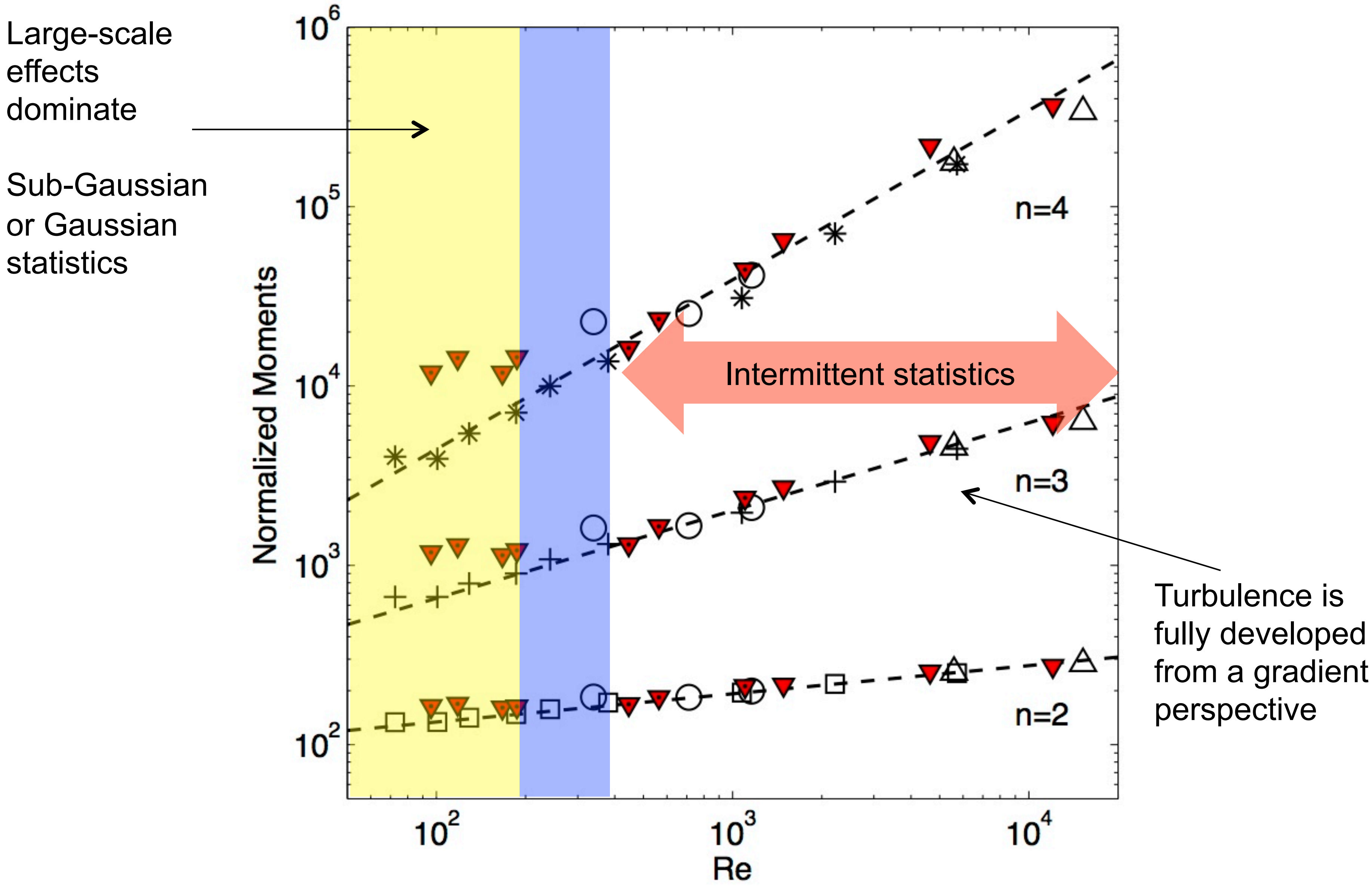
Gaussian velocity gradients in DNS

Schumacher, Sreenivasan & Yakhot, New J. Phys. 2007



Transition point at Re≈100 can be predicted by renormalization group theory

Transition to intermittency



Summary

- Boundary layer dynamics is affected by a large-scale circulation with an increasingly intermittent space-time dynamics for growing Rayleigh number
- Large-scale circulation in turbulent convection is organized in patterns which are similar to the onset of convection
- Transition from Gaussian to intermittent non-Gaussian gradient statistics proceeds at about the same Reynolds number in different turbulent flows
- Universal fluctuations of the velocity gradients for Reynolds numbers above the transition point

Research Centre Jülich HIL07

Research Unit 1182