

Effects of shear and blocking in a rapidly distorted boundary layer

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***High Reynolds Number Boundary Layer Turbulence
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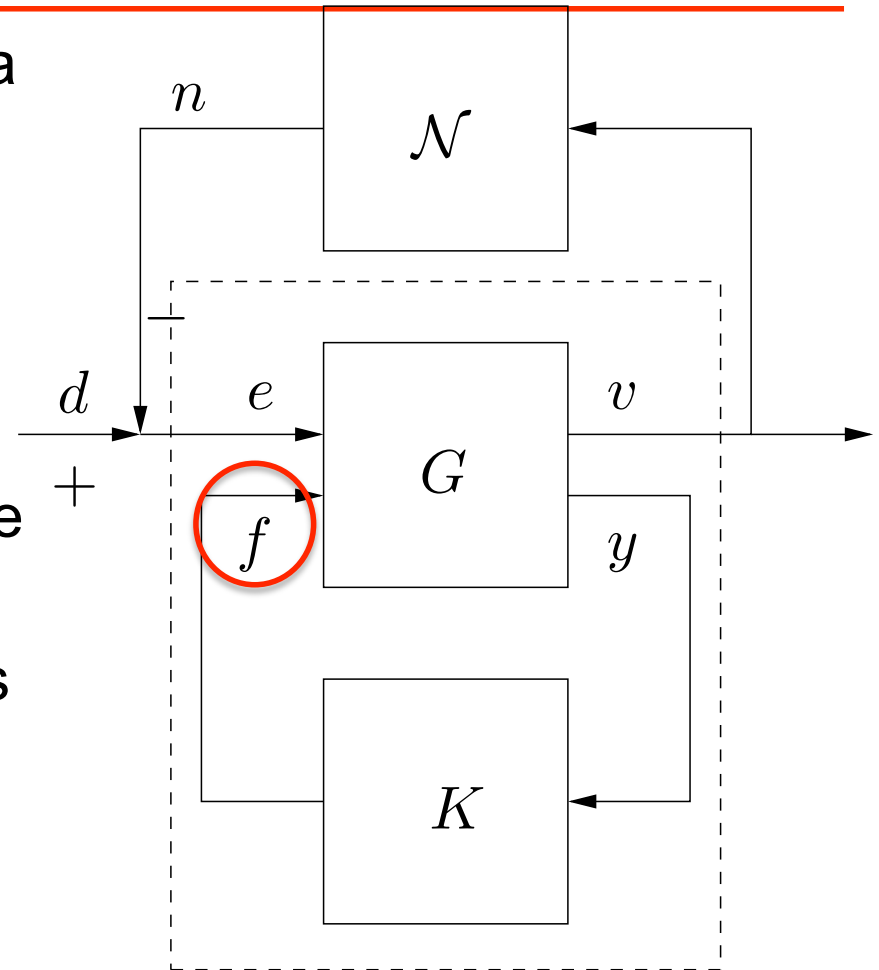
Synopsis

- Motivation
- A full-domain linear controller that relaminarises turbulent channel flow $Re_\tau \leq 400$
- How does this work?
- Importance of pressure fluctuations – Batchelor, Landahl & Townsend (BLT)
- Comparison of timescales
- Measurements in a rapidly distorted boundary layer

Linear globally stabilising controller

- Navier-Stokes equations written as a linear system G with control K and nonlinear forcing, f
- Nonlinear term N is conservative w.r.t. disturbance energy
- Turbulent shear stresses treated as part of perturbations we wish to force
- Linear controller works in presence of nonlinearity by characterising it as positive real i.e. passive.
- Stability: choose K such that linear part of closed loop is passive
- The linear terms always dissipate disturbance energy

Sharma *et al.* 2010

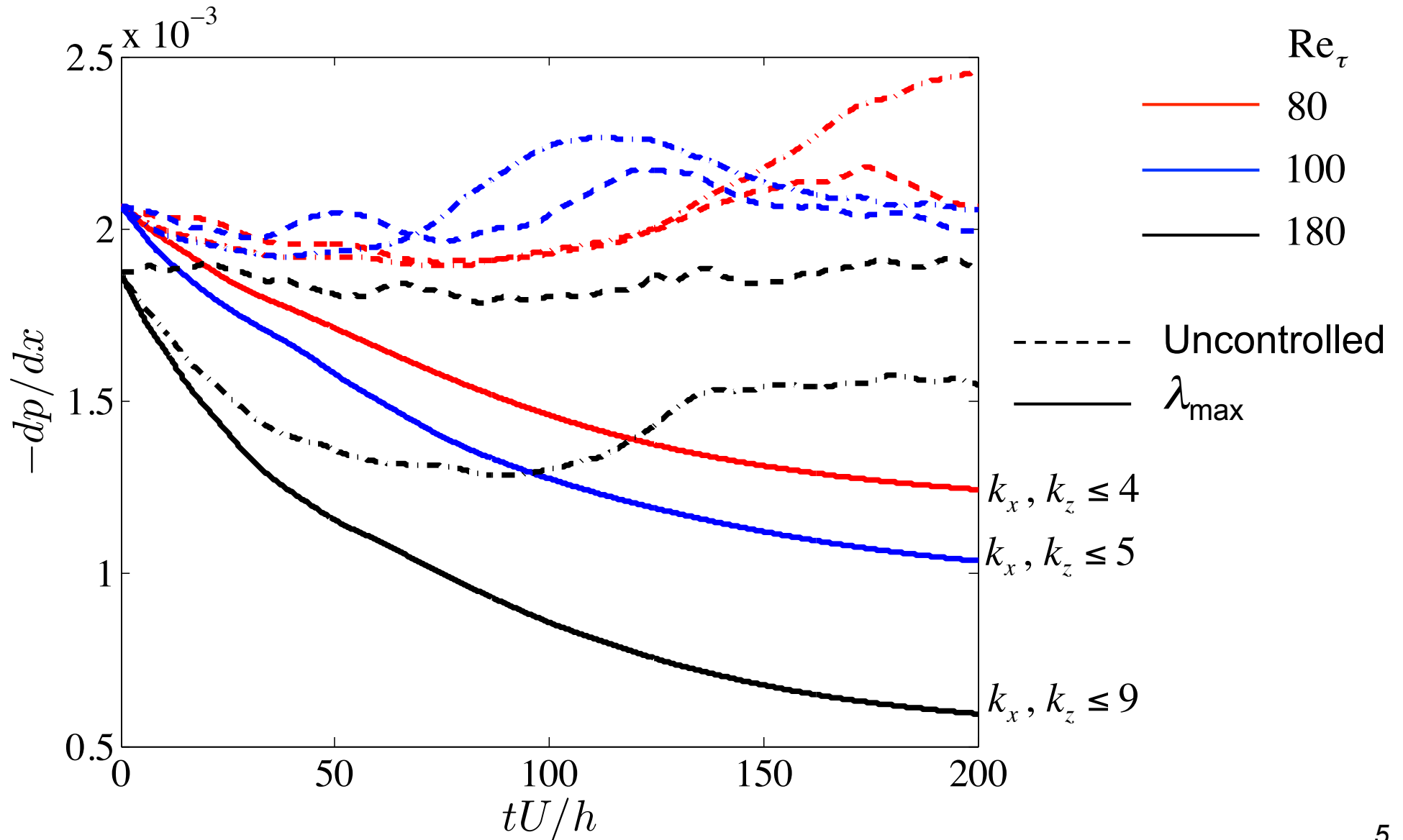


$$\frac{dE}{dt} = \oint_{x \in \Omega} \left(U'_L(y) uv + \frac{\varepsilon}{\text{Re}} \right) \leq 0$$

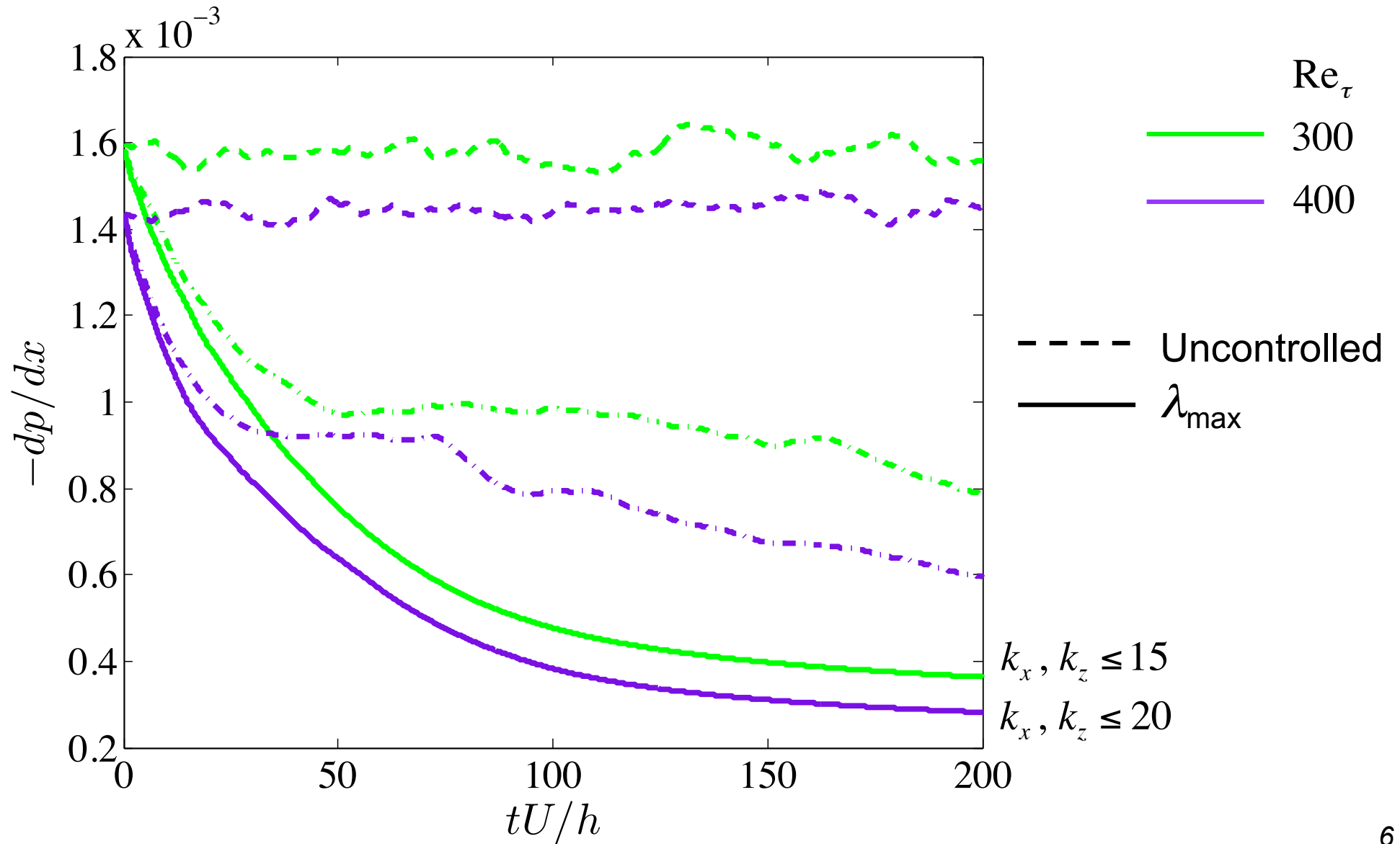
Turbulent channel flow

- $Re_\tau = 80, 100, 180, 300$: Domain $4\pi h \times 2h \times 2\pi h$
- $Re_\tau = 400$: Domain $2.5\pi h \times 2h \times \pi h$
- Channelflow 0.9.15 (Gibson *et al.* '08)
- Full-domain sensing, actuation on v
- Control focuses on vU'
- Forcing bandwidth progressively increased
- Details for $Re_\tau = 180, k_x, k_z \leq 9$
- & at $y^+ = 20|_{\text{init}}$

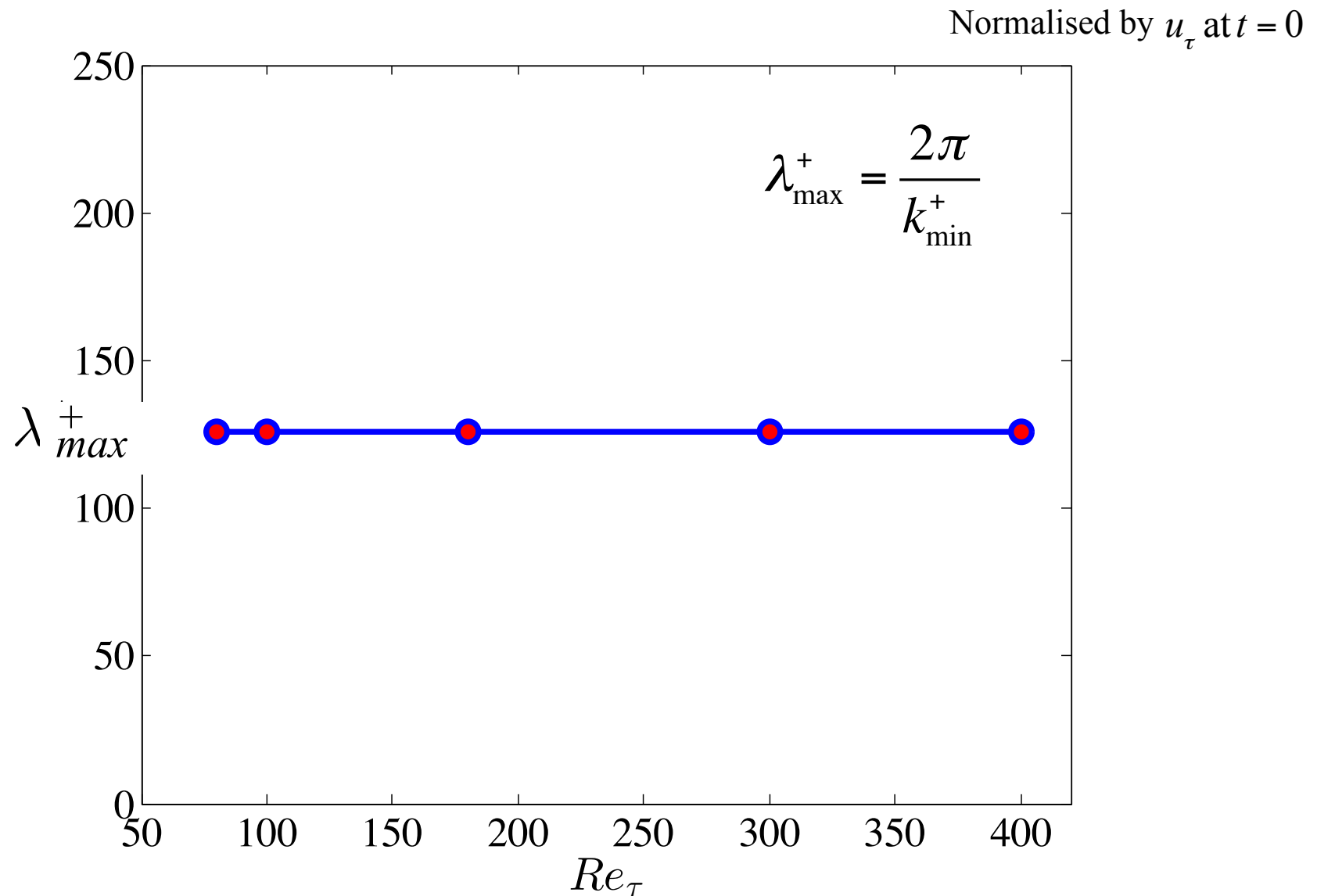
$$Re_\tau \leq 180$$



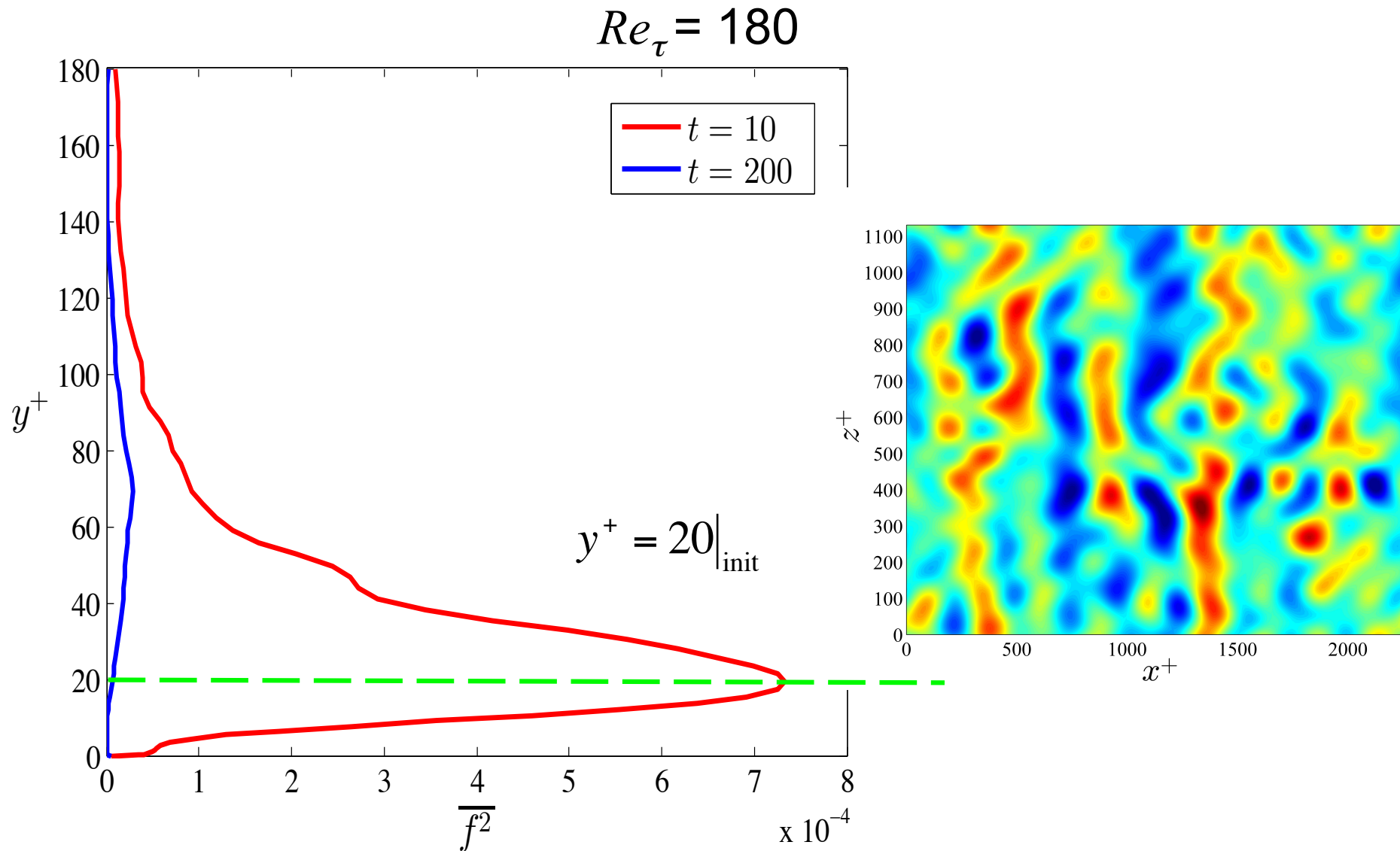
$$Re_\tau \leq 400$$



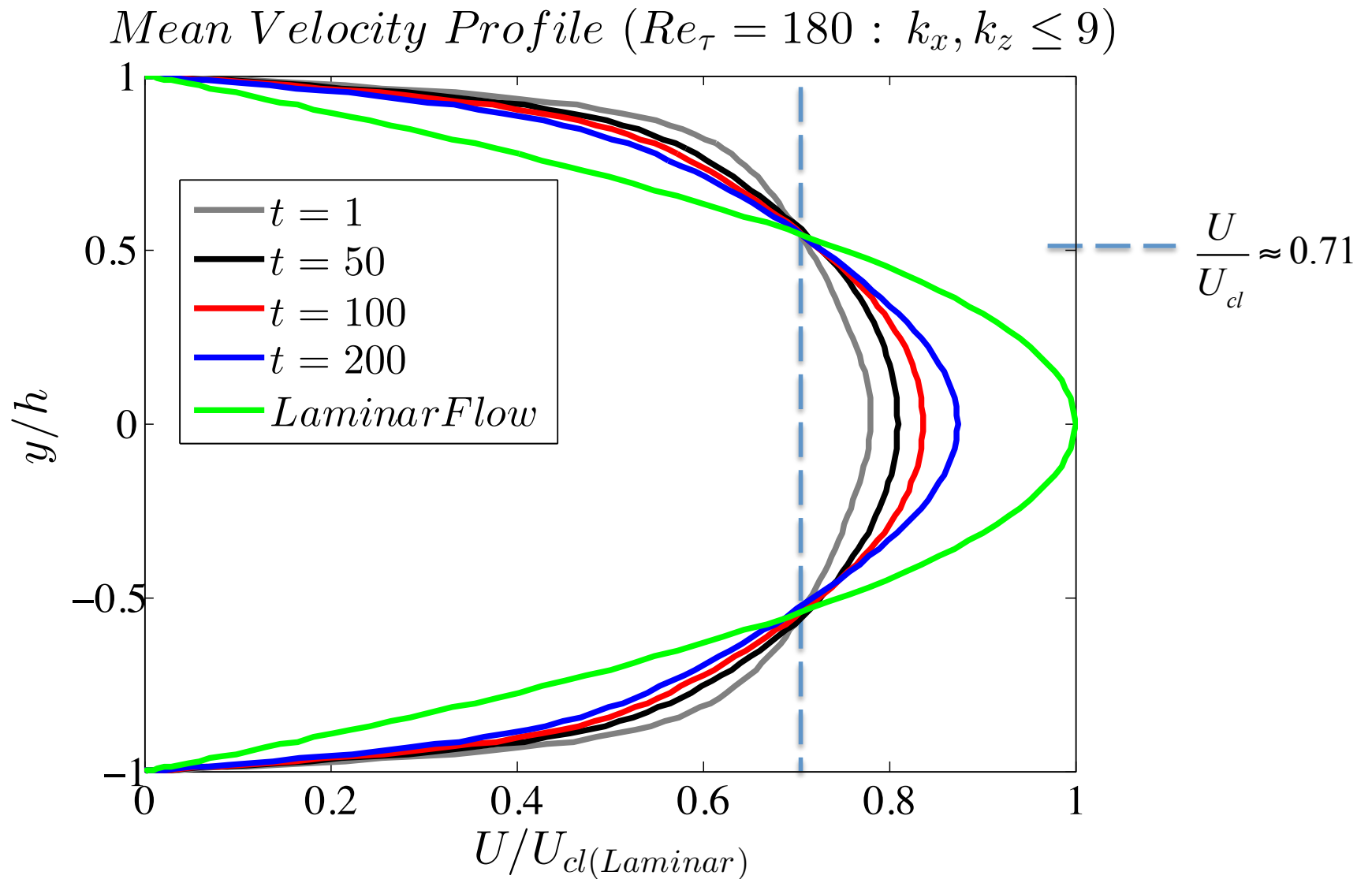
Minimum wavenumber for relaminarisation



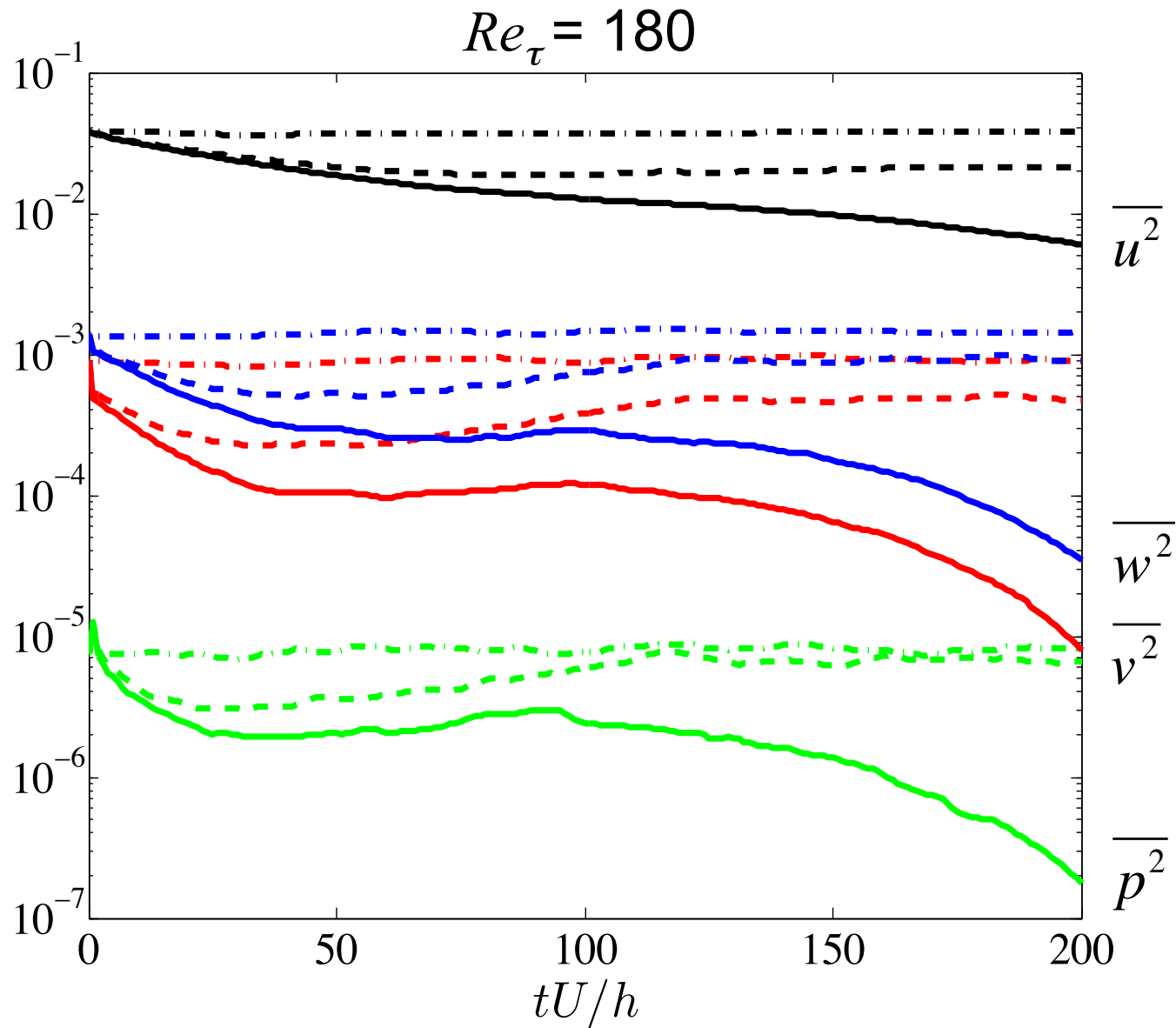
Mean square forcing: $\overline{f^2}(y)$



Mean velocity profile



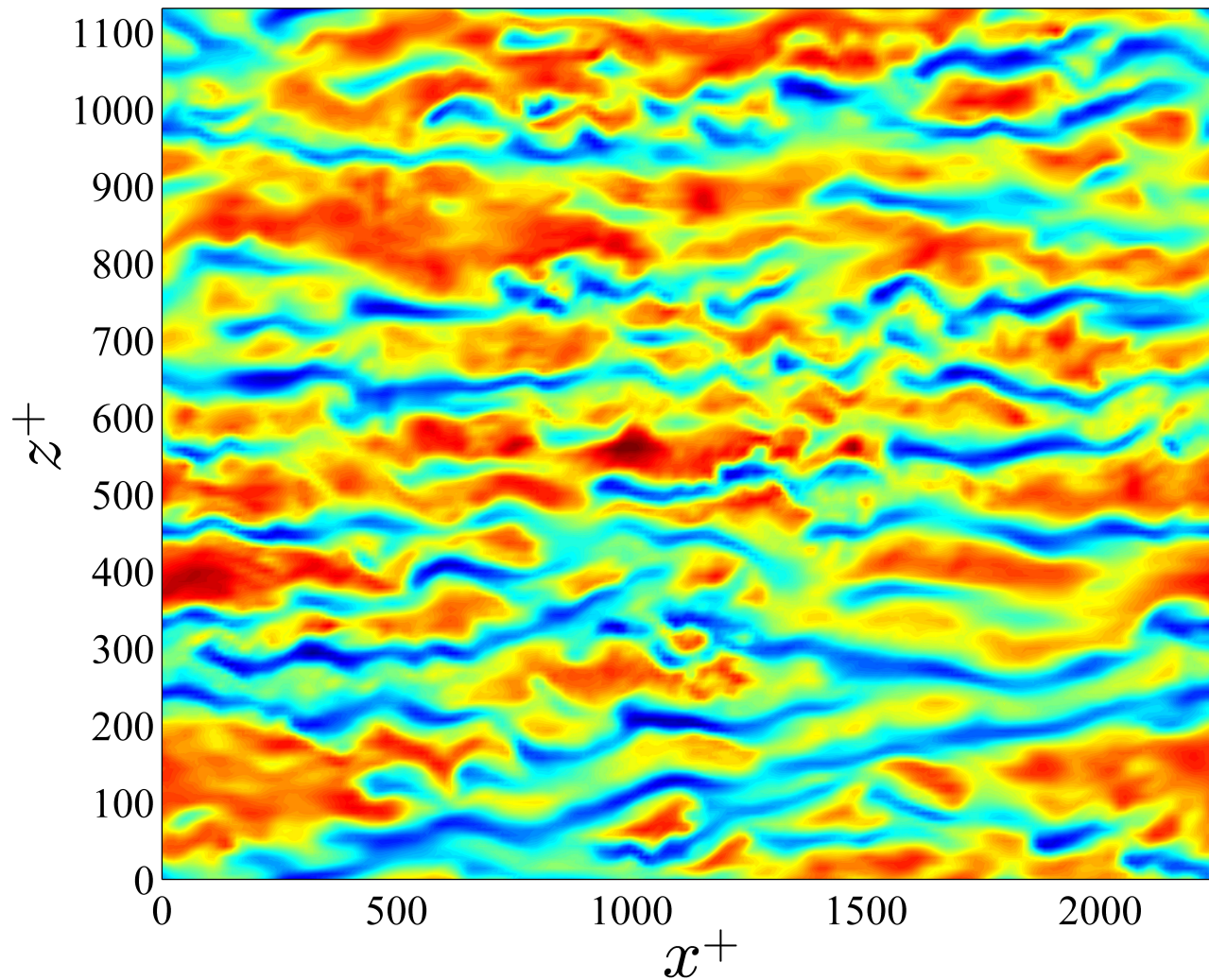
Fluctuations about target laminar profile



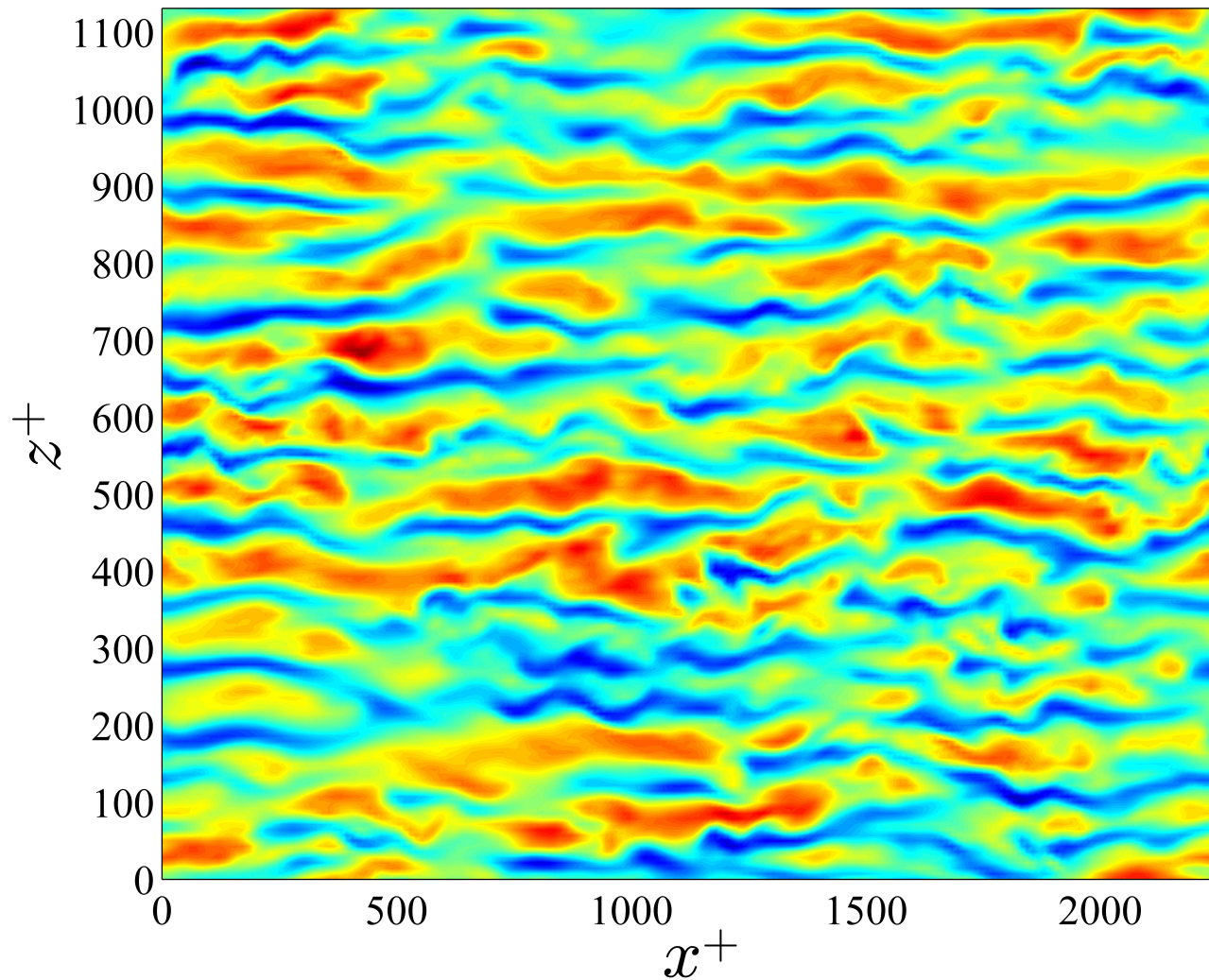
Controlled u : $y^+ = 20|_{\text{init}}$

$Re_\tau = 180$

$t = 2$

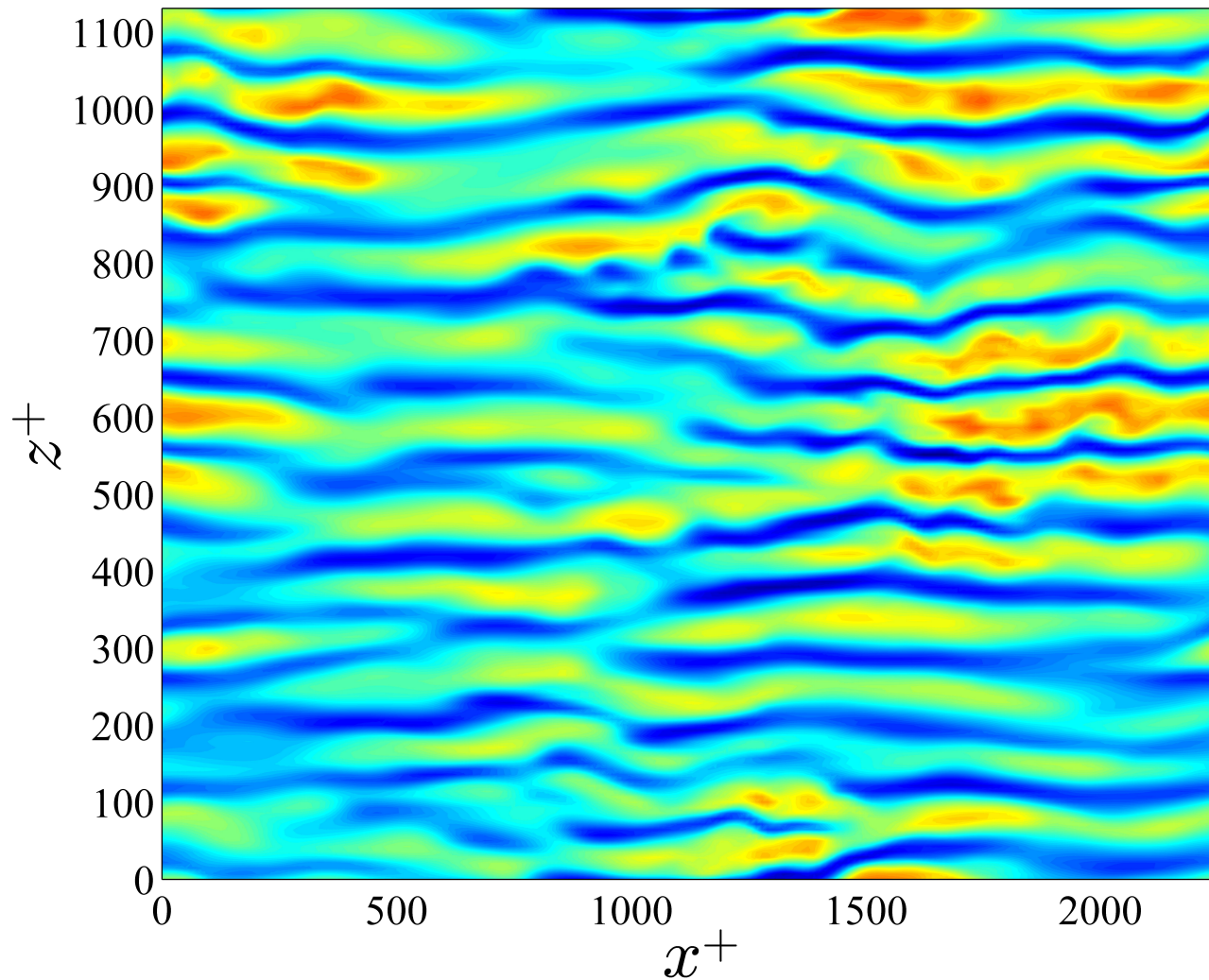


Controlled u : $y^+ = 20|_{\text{init}}$



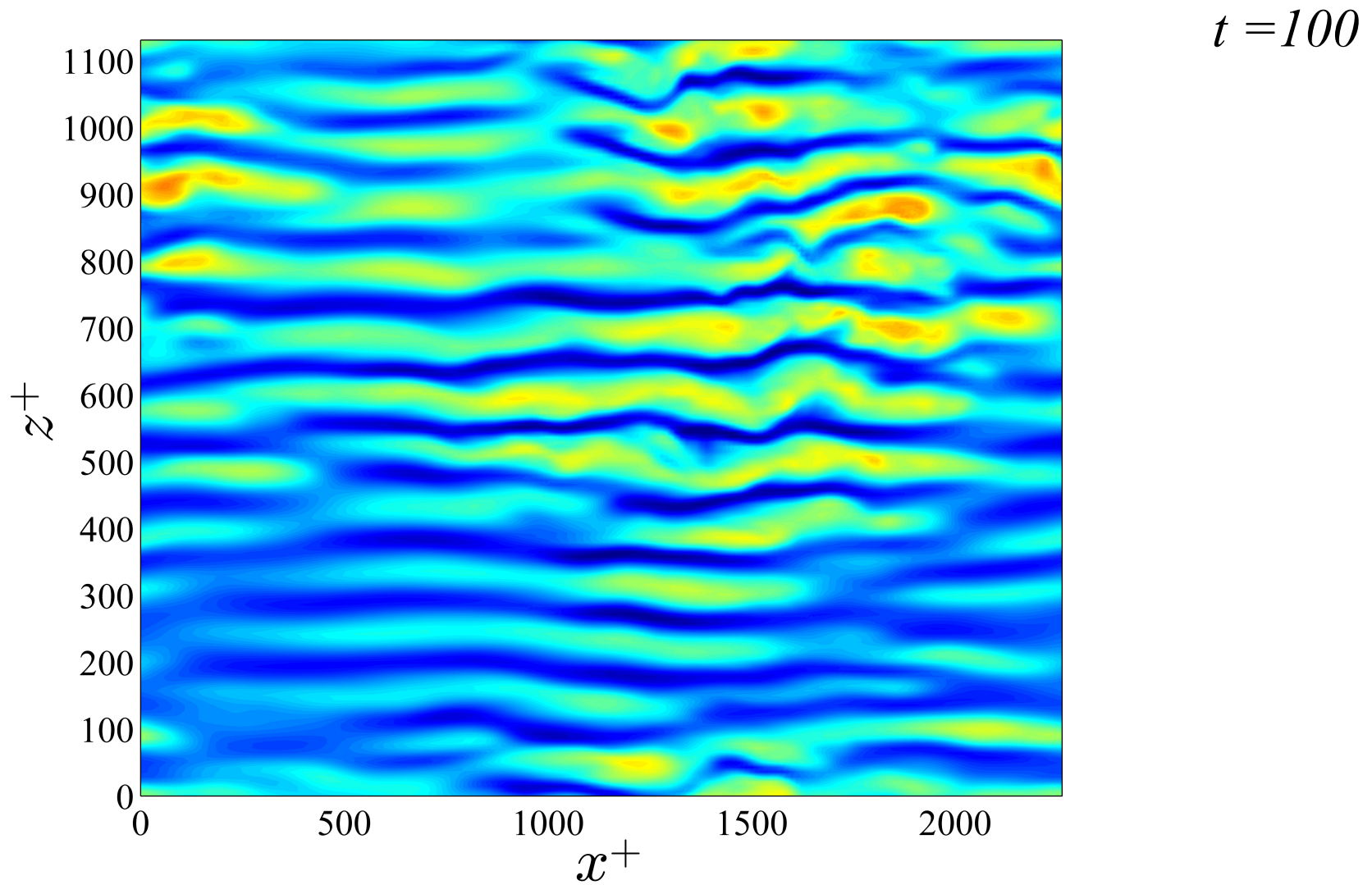
$t = 10$

Controlled u : $y^+ = 20|_{\text{init}}$



$t = 50$

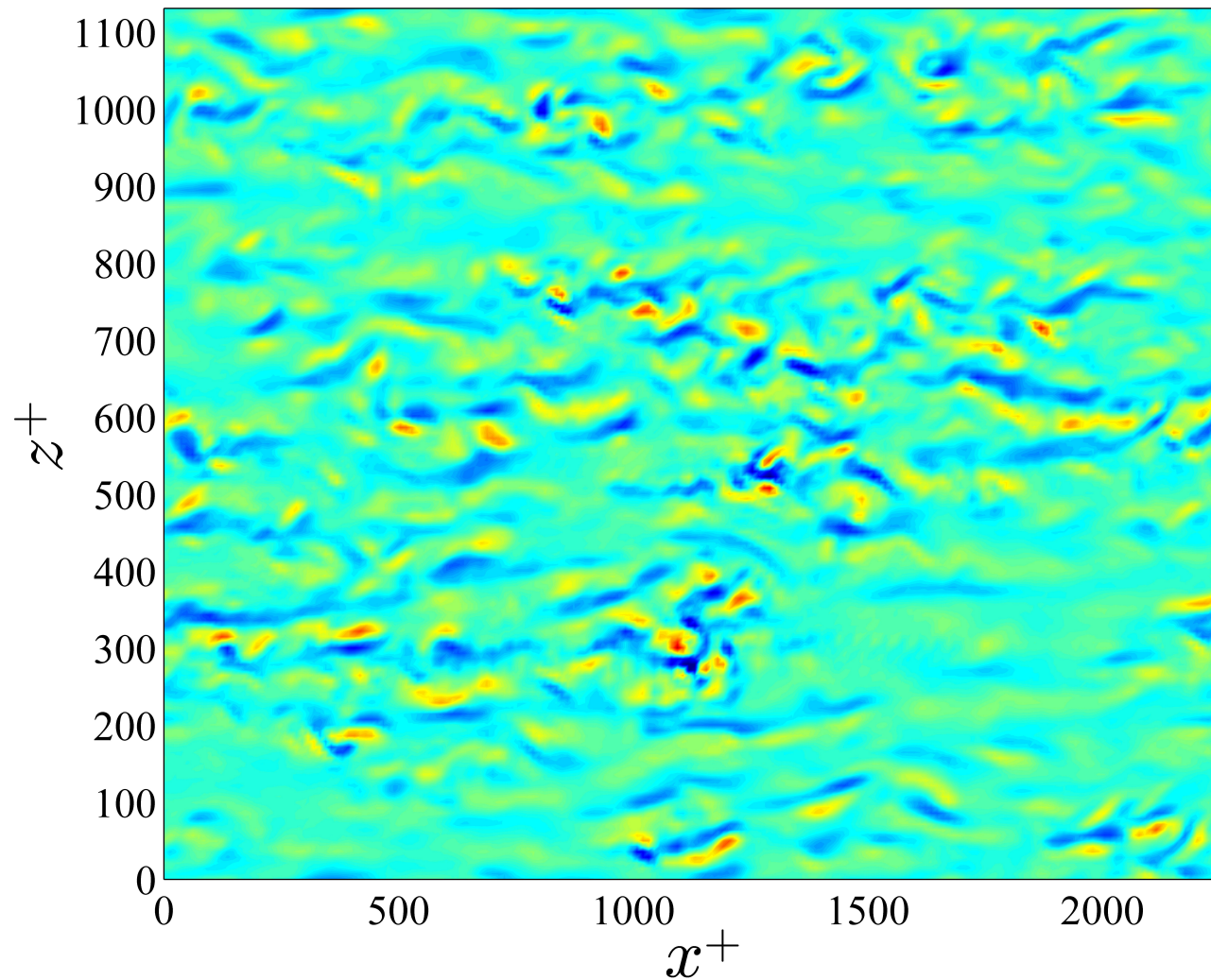
Controlled u : $y^+ = 20|_{\text{init}}$



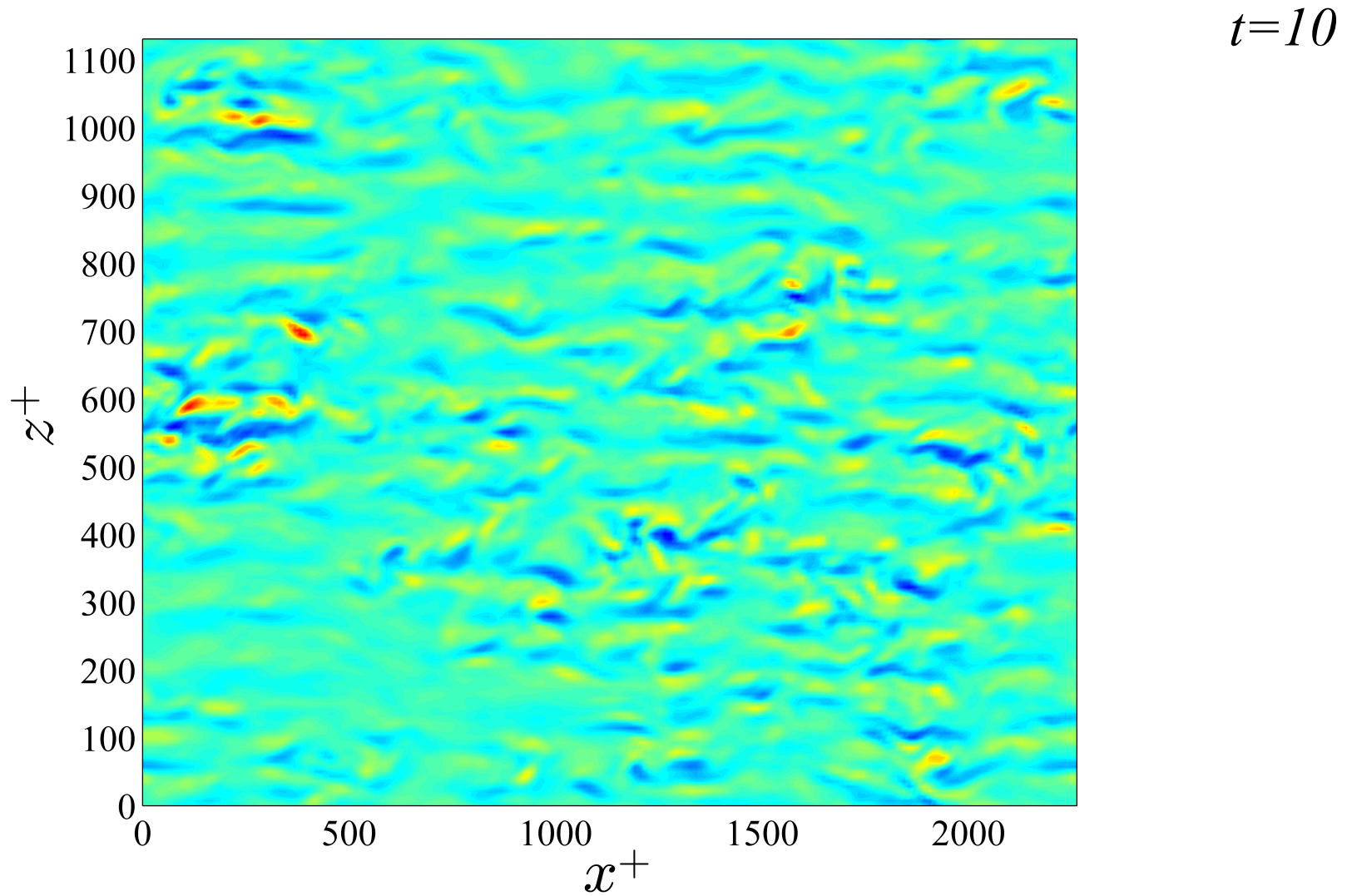
Controlled ν : $y^+ = 20|_{\text{init}}$

$Re_\tau = 180$

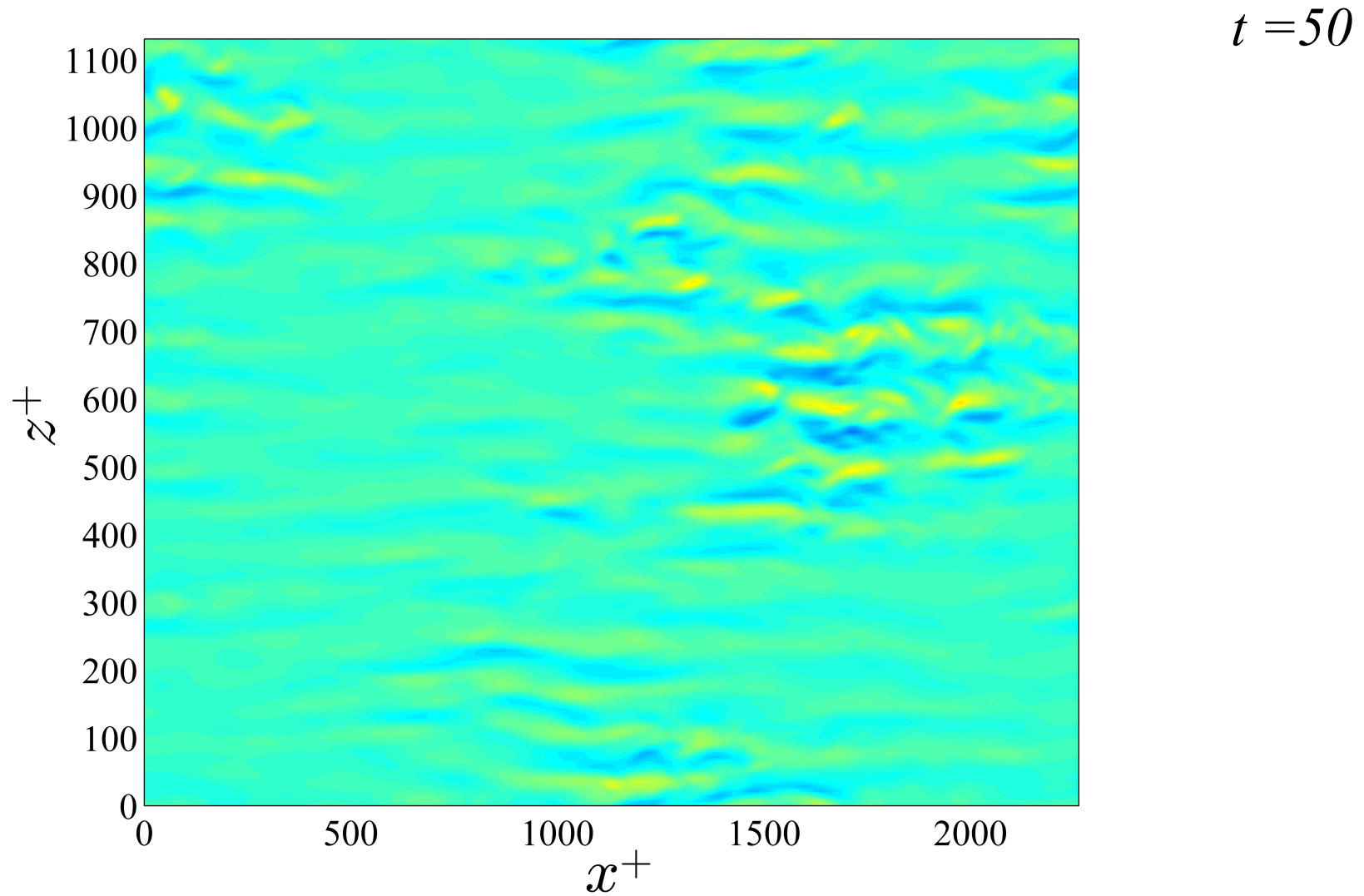
$t = 2$



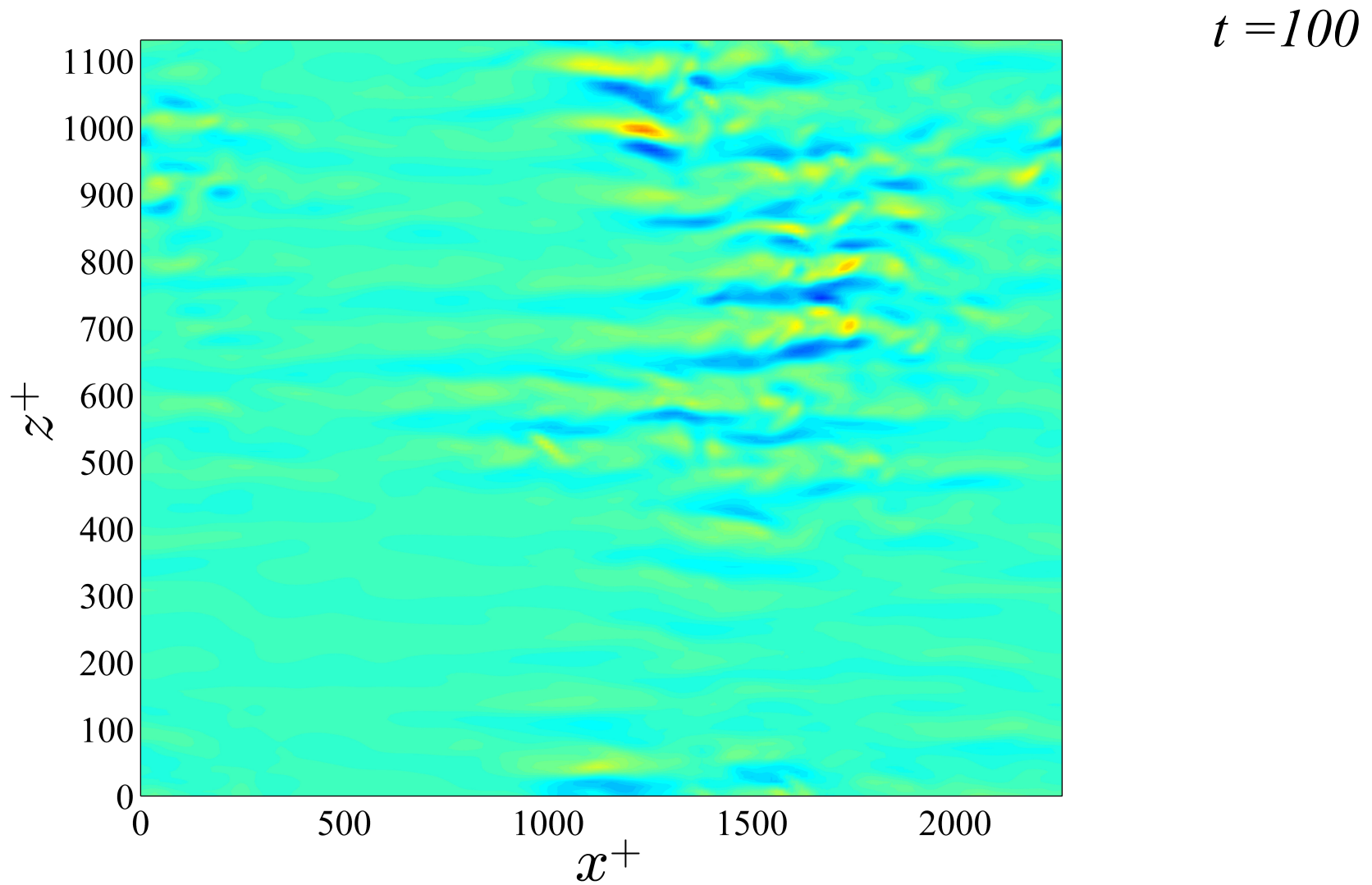
Controlled ν : $y^+ = 20|_{\text{init}}$



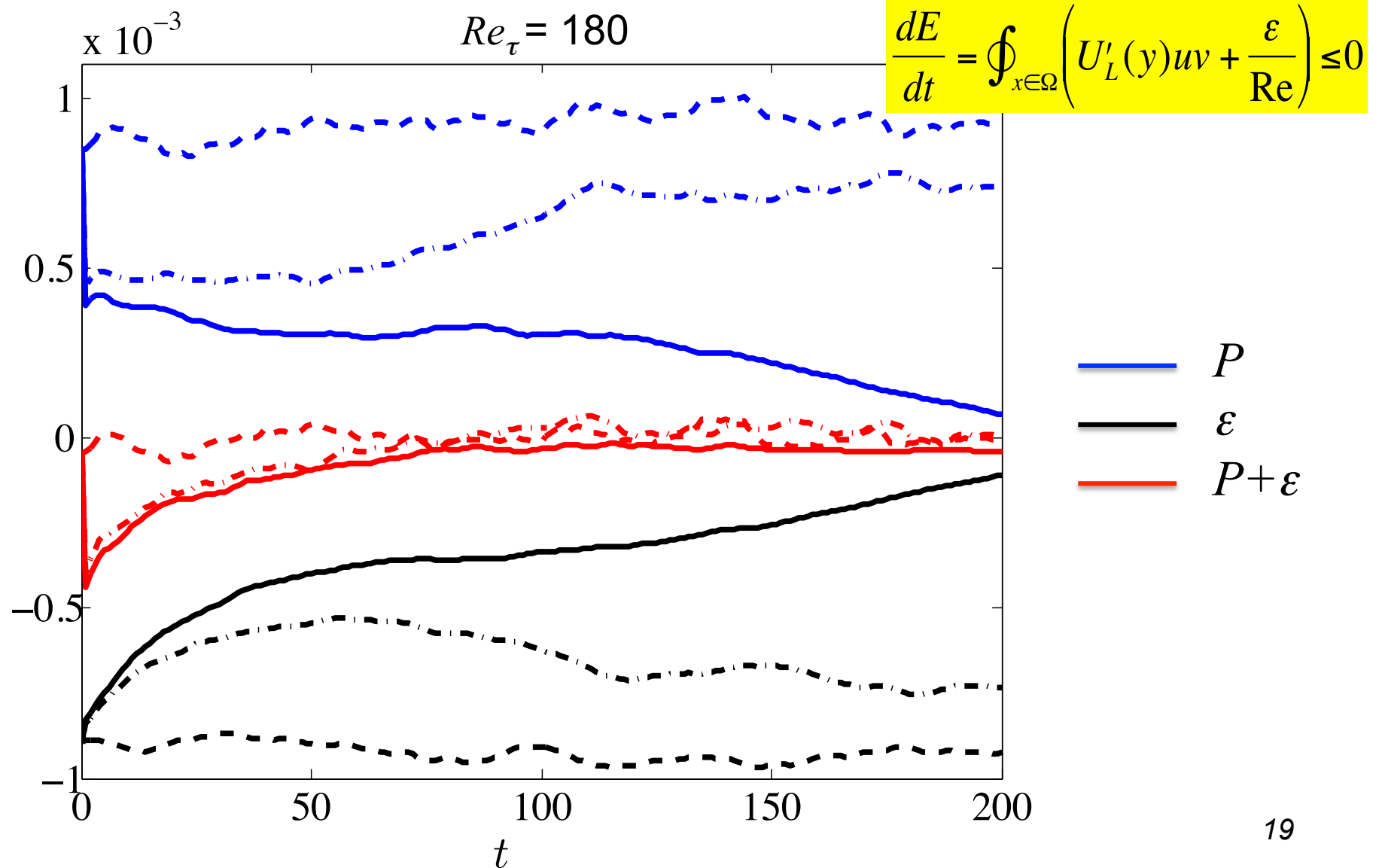
Controlled ν : $y^+ = 20|_{\text{init}}$



Controlled ν : $y^+ = 20|_{\text{init}}$



Fluctuations about target laminar profile

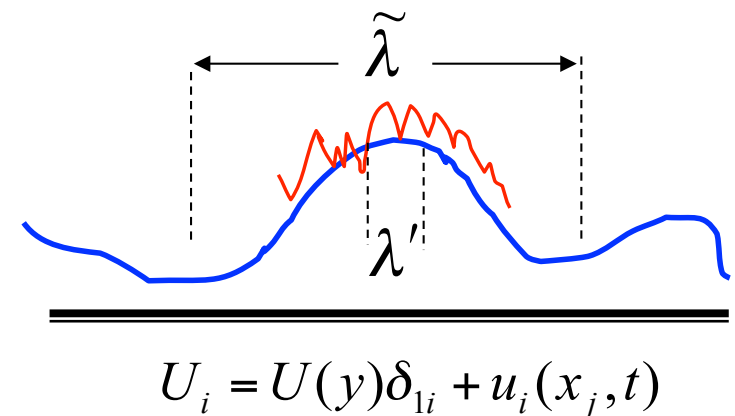


A useful theory for Inner-Outer Interaction?

- Landahl ('93, '90, '75): initial disturbance scales L, u_0 with timescales: shear interaction $\{U'_w\}^{-1} \ll$ viscous $\{L^2/(\nu U'^2)\}^{1/3} \ll$ nonlinear L/u_0 .
- Large and small-scale decomposition: $u_i = \tilde{u}_i + u'_i$
- Small scale, λ' , large scale $\tilde{\lambda}$ where $\lambda'/\tilde{\lambda} = \varepsilon \ll 1$
- To first order in ε , large-scale and small-scale fields may be represented separately by the same equations:

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \nabla^2 v - U'' \frac{\partial v}{\partial x} - \frac{\nabla^4 v}{\text{Re}} = q$$

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \eta + U' \frac{\partial v}{\partial z} - \frac{\nabla^2 \eta}{\text{Re}} = p$$



- p, q nonlinear source terms (turbulent stresses) significant only in local regions: “intense small-scale turbulence of an intermittent nature” interspersed with “laminar-like unsteady motion of a larger scale”.

Pressure gradient fluctuations

- High Reynolds numbers: local isotropy and negligible viscous diffusion
- Mean-square acceleration becomes

$$\overline{\left(\frac{Du_i}{Dt}\right)^2} \approx \overline{\left(\frac{\partial p}{\partial x_i}\right)^2} + \nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$$

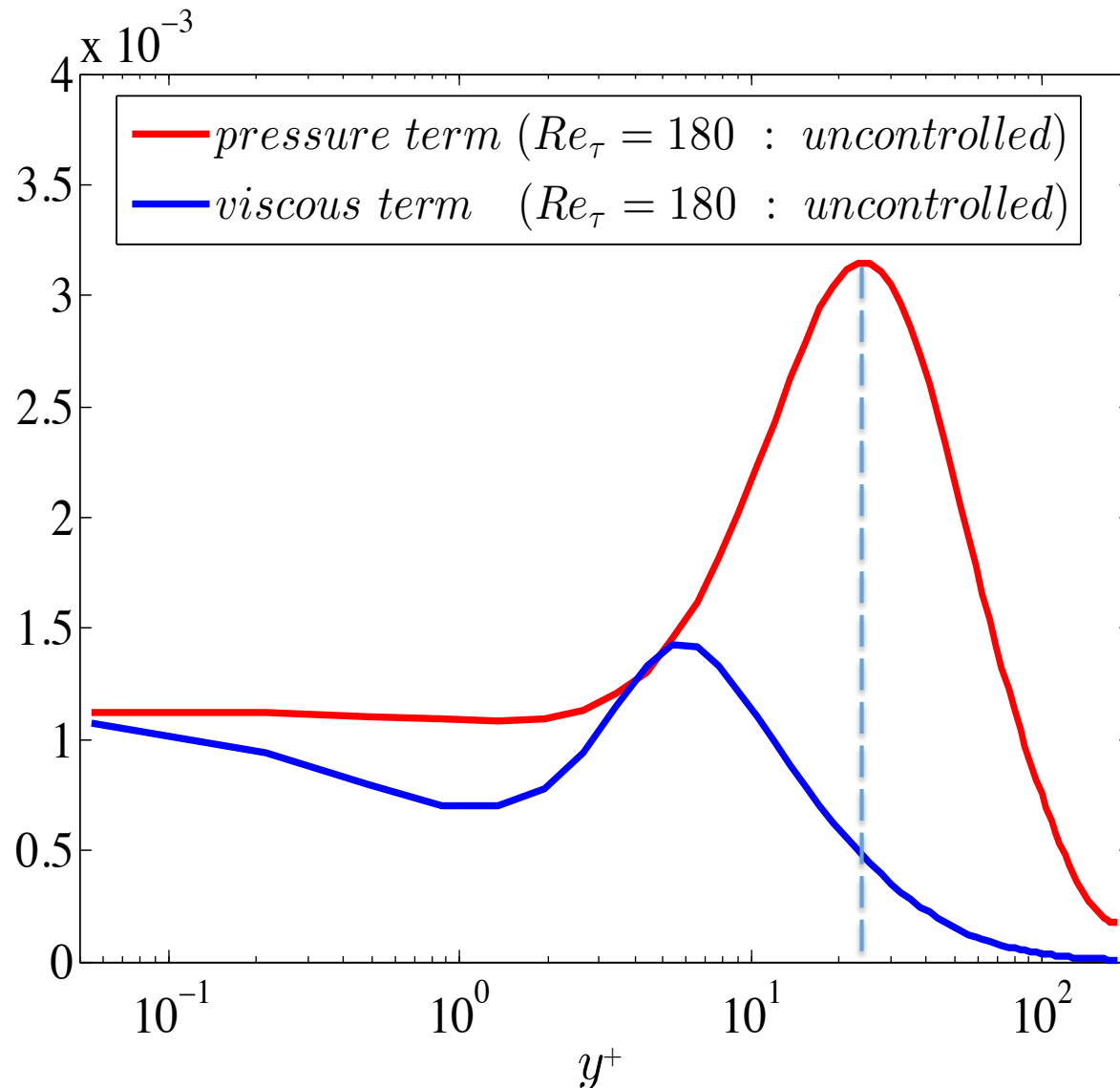
- where

$$\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} \approx 20\nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$$

- Therefore, even the smallest scale motion is driven by pressure gradients and not by viscous forces.

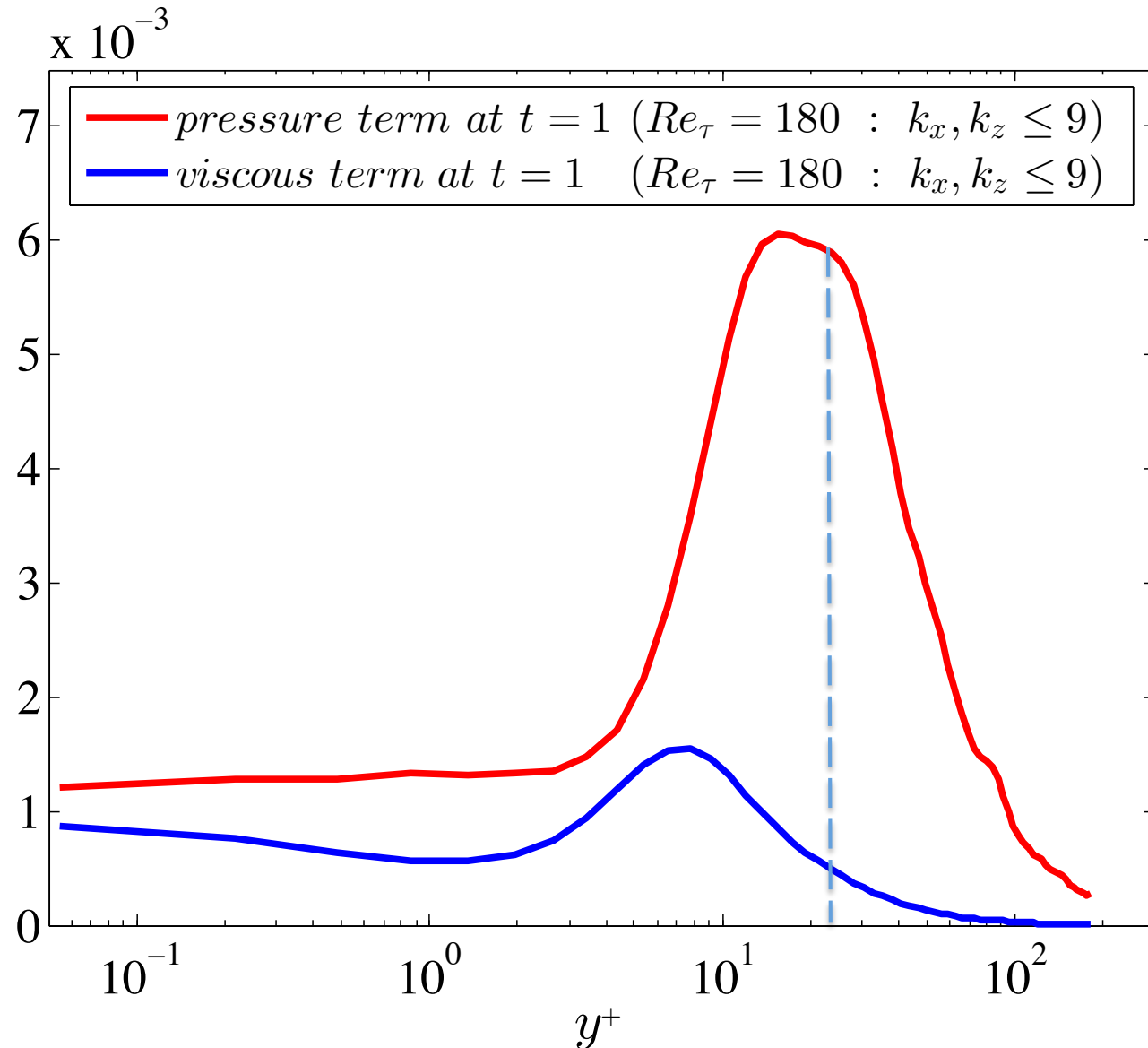
Batchelor & Townsend 1956

Batchelor & Townsend: uncontrolled turbulent channel flow

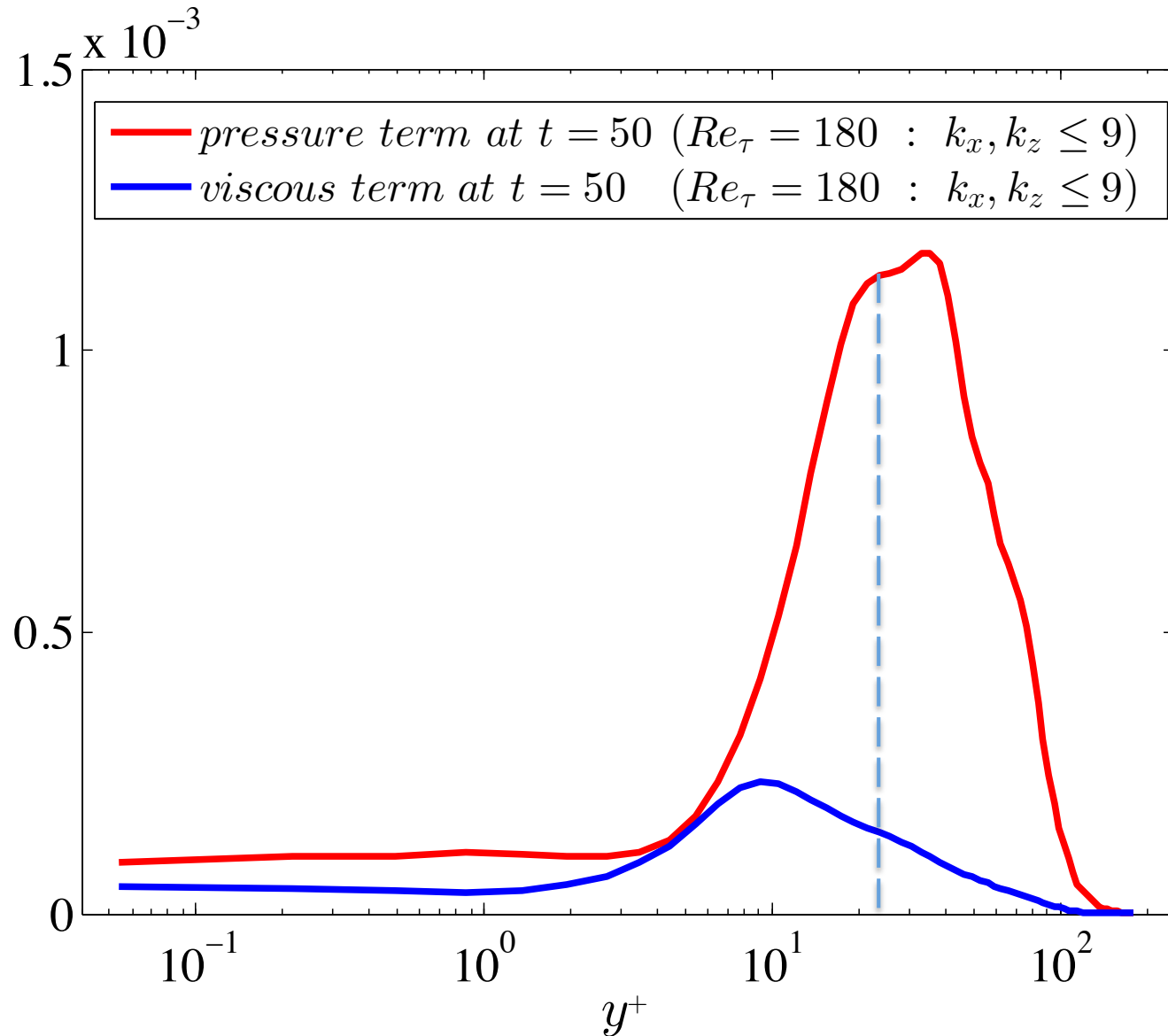


$$\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} \approx 20\nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$$

Fluctuations about target laminar profile



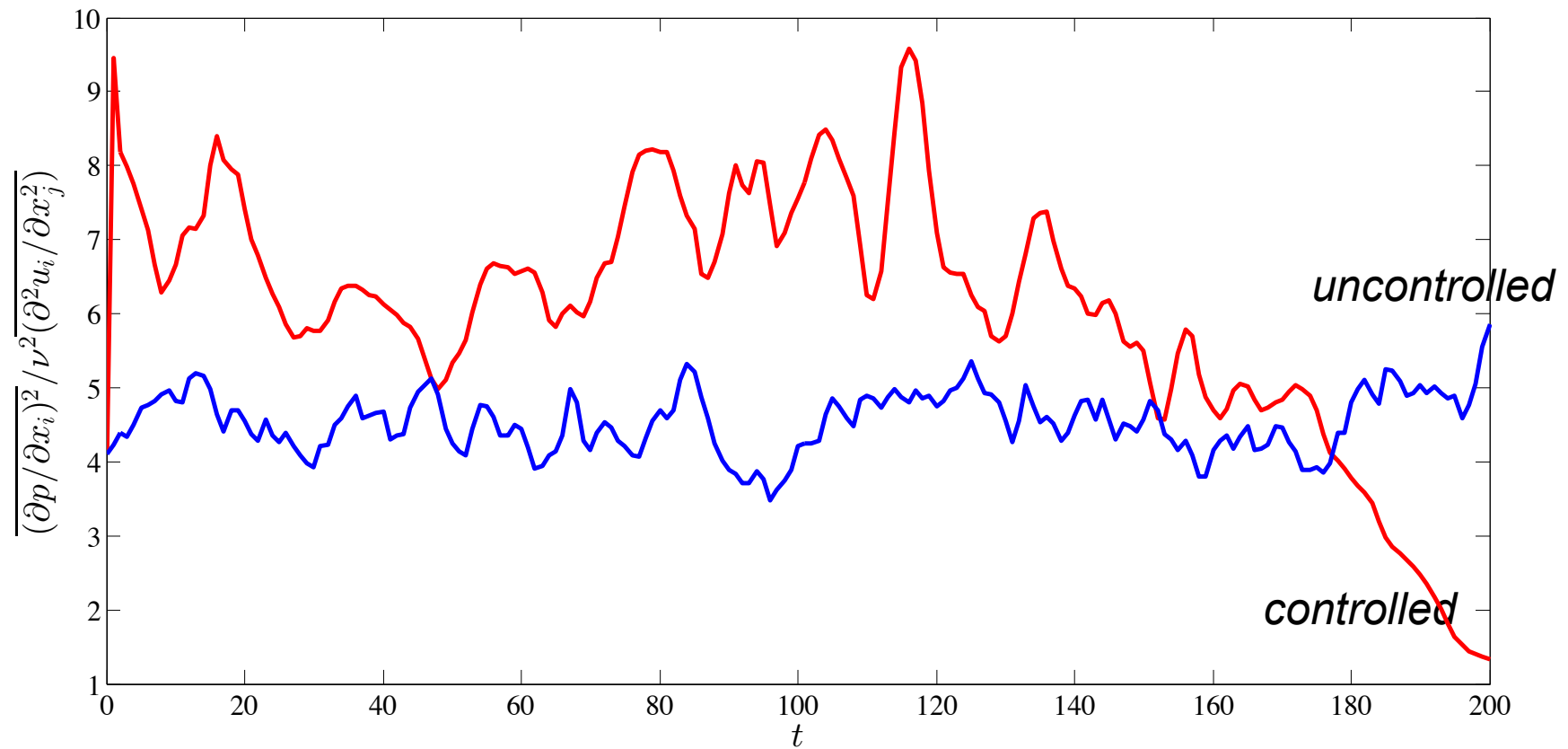
Fluctuations about target laminar profile



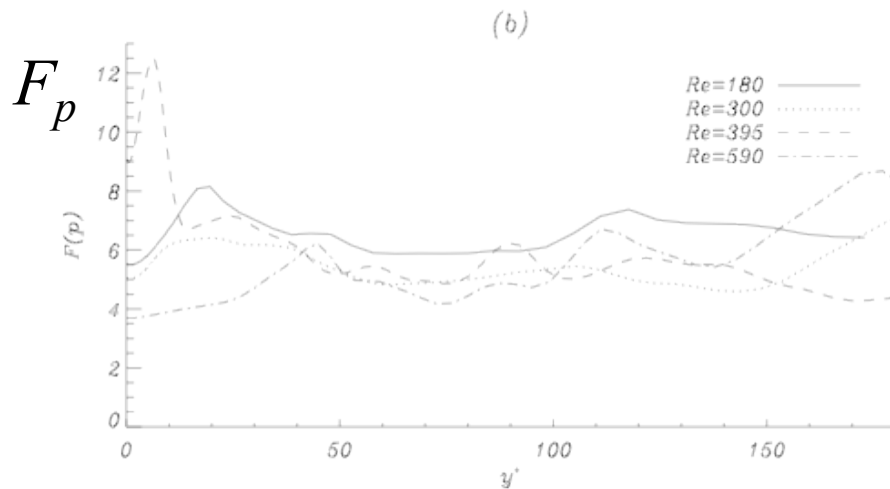
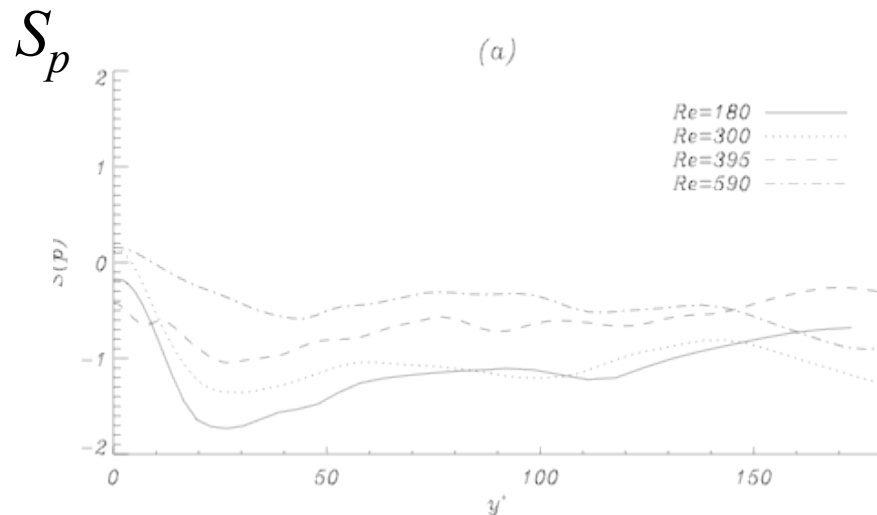
$$\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} / \nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2} \text{ at } y^+ = 20$$

$\text{Re}_\tau = 180$

*U based on
mean profile
U(t)*



Turbulent channel flow pressure statistics

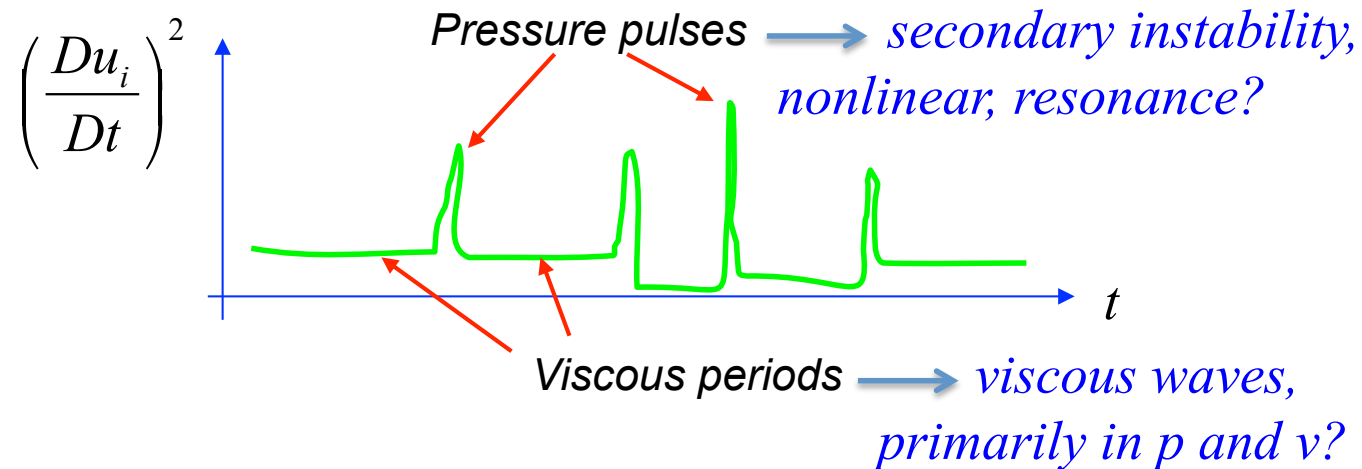


- Spatial intermittency: pressure has small skewness, large flatness (>6)
- Green's function integral shows that contribution to wall pressure comes mostly from near-wall velocity field

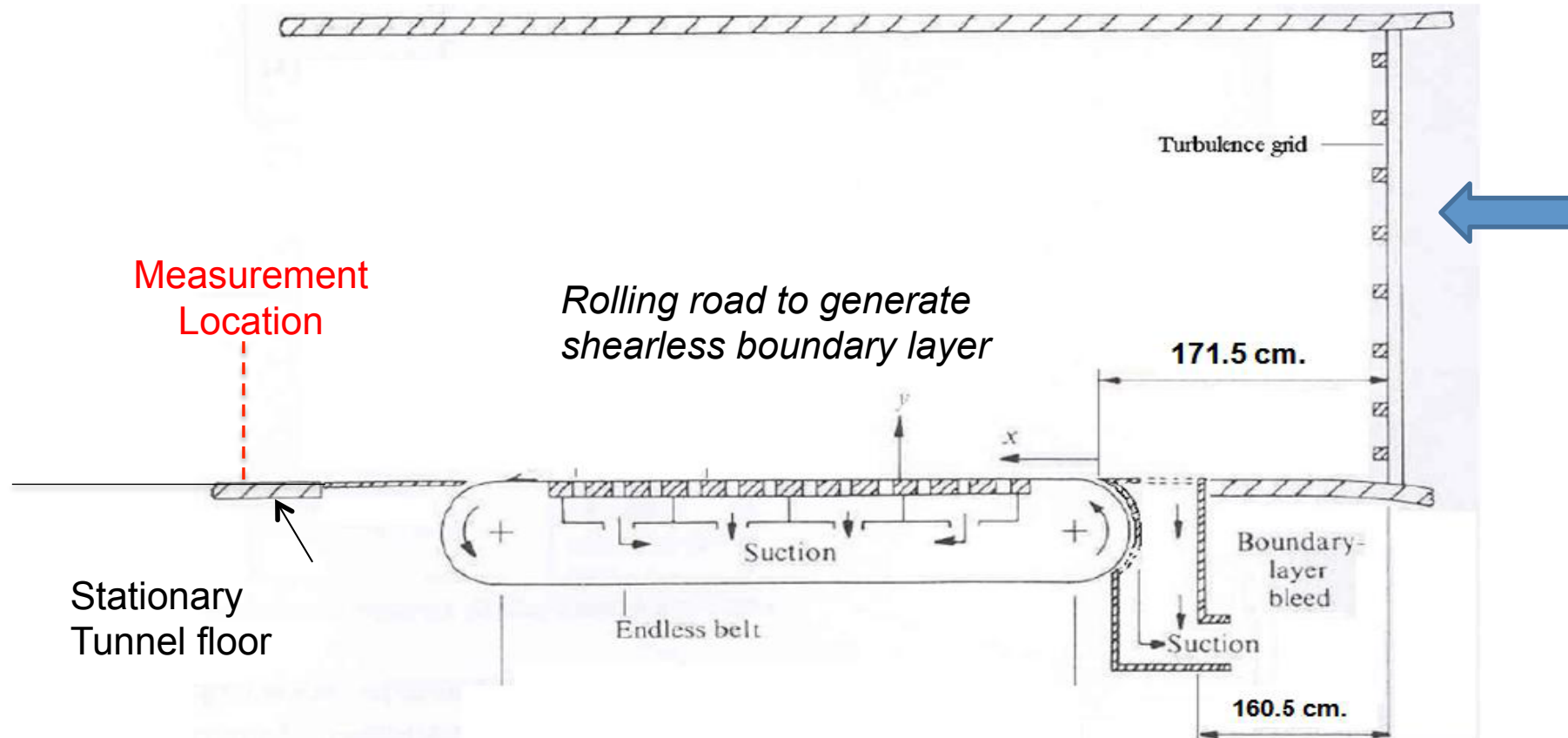
$$-\nabla^2 p = 2U' \frac{\partial v}{\partial x} - \frac{\partial^2}{\partial x \partial y} [uv - \overline{uv}]$$

BLT theory

- Sublayer as a waveguide: primarily for p and v
- u and w also wave-like but including convected eddy behaviour
- Description of both large & small scales – Inner-outer interaction?
- Pressure sources can ‘trigger’ bursts near wall = short shear – interaction timescale



Rapidly distorted boundary layer

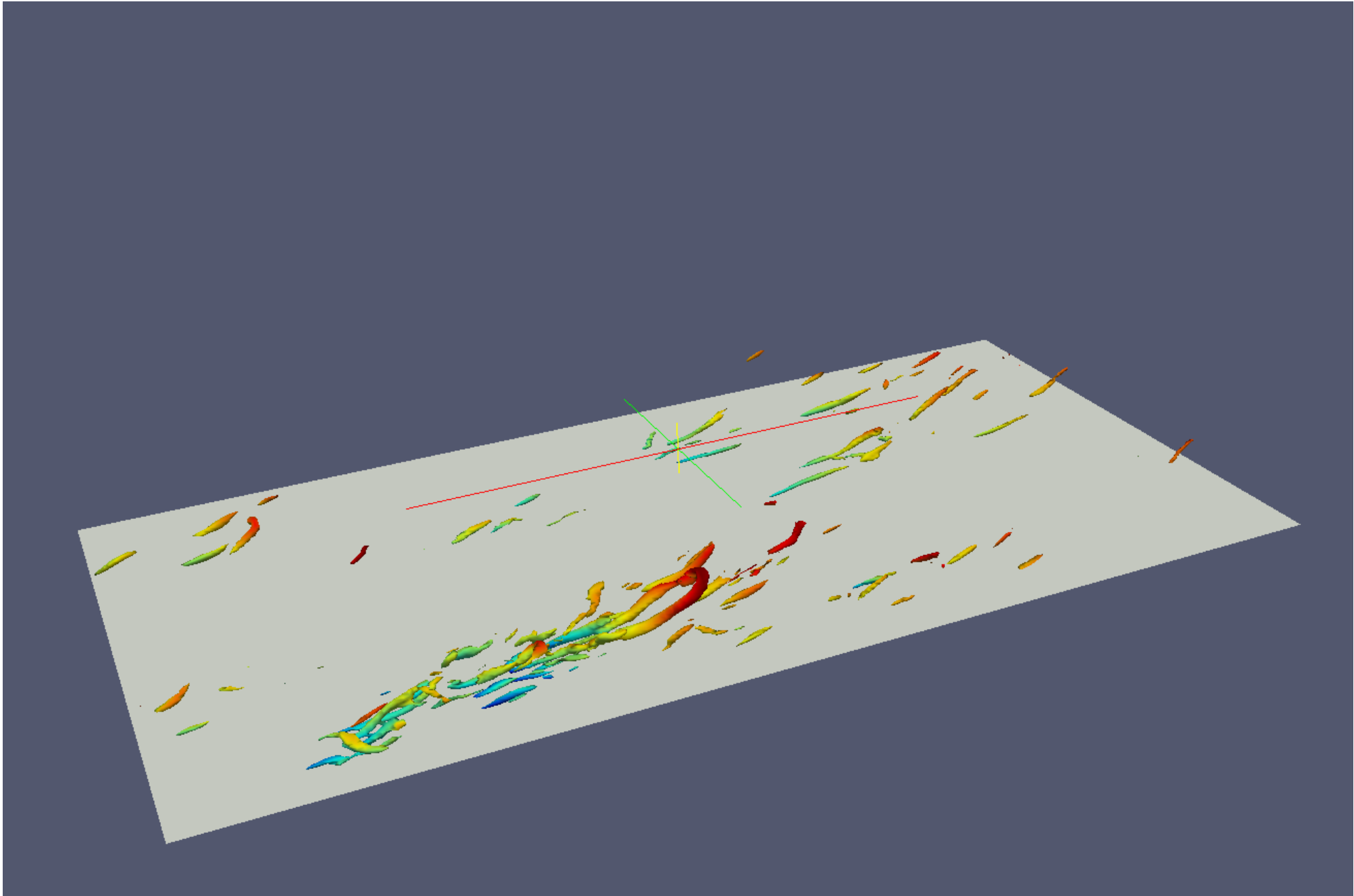


Aims:

1. Experimental study of linear mechanisms in wall turbulence
2. Variable ratio of shear timescale to turbulence timescale

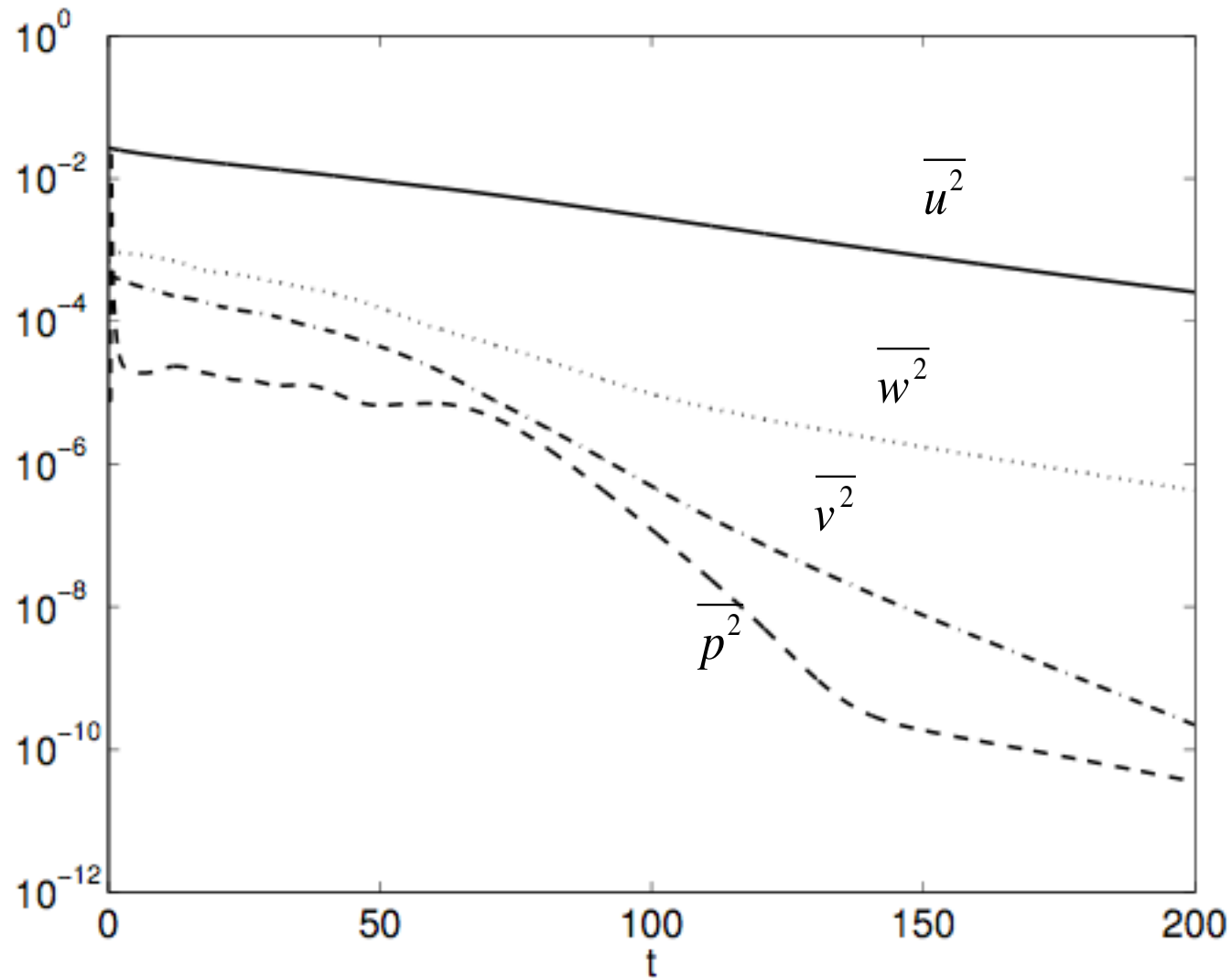
Conclusions

- Linear full-domain forcing via $\nu U'$ at low wavenumbers effective in attenuating turbulent channel flow
- Control-theoretic approach (“passivity”) explained by conservative nature of nonlinear terms contributing to Reynolds-Orr equation
- Control acts on ν -component field and hence pressure field via rapid source term of Poisson equation
- Qualitative support for Landahl’s theory: inner-outer interaction effected by linear shear-interaction on short timescales
- Relevance of Landahl’s theory for linear control lies in the fact that, over the short time for which the controller is effective, the longer turbulence time scale is not significant
- Shear timescale effective because of pressure – linear source term is an RDT approximation



Fluctuations about target laminar profile

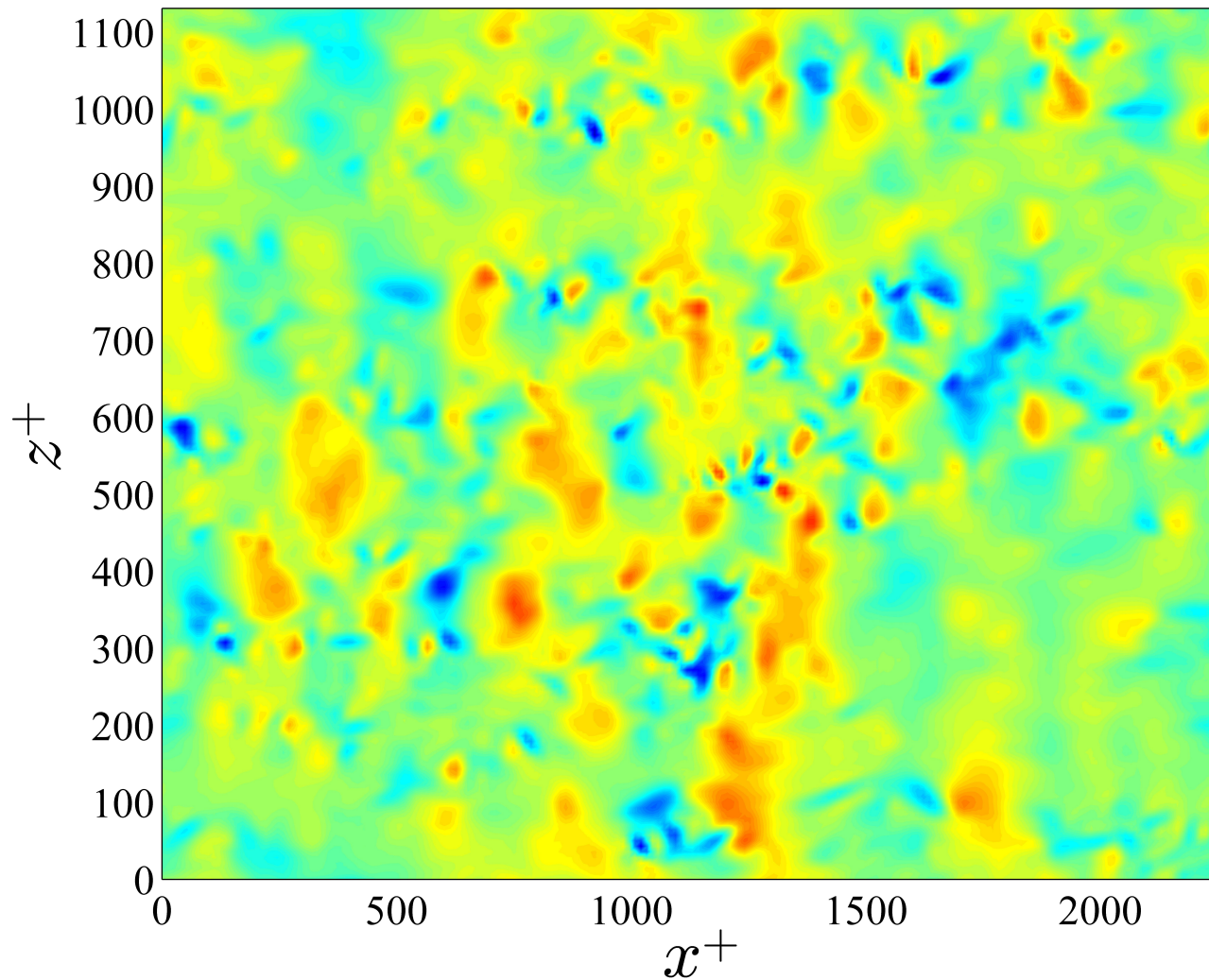
$Re_\tau = 80$



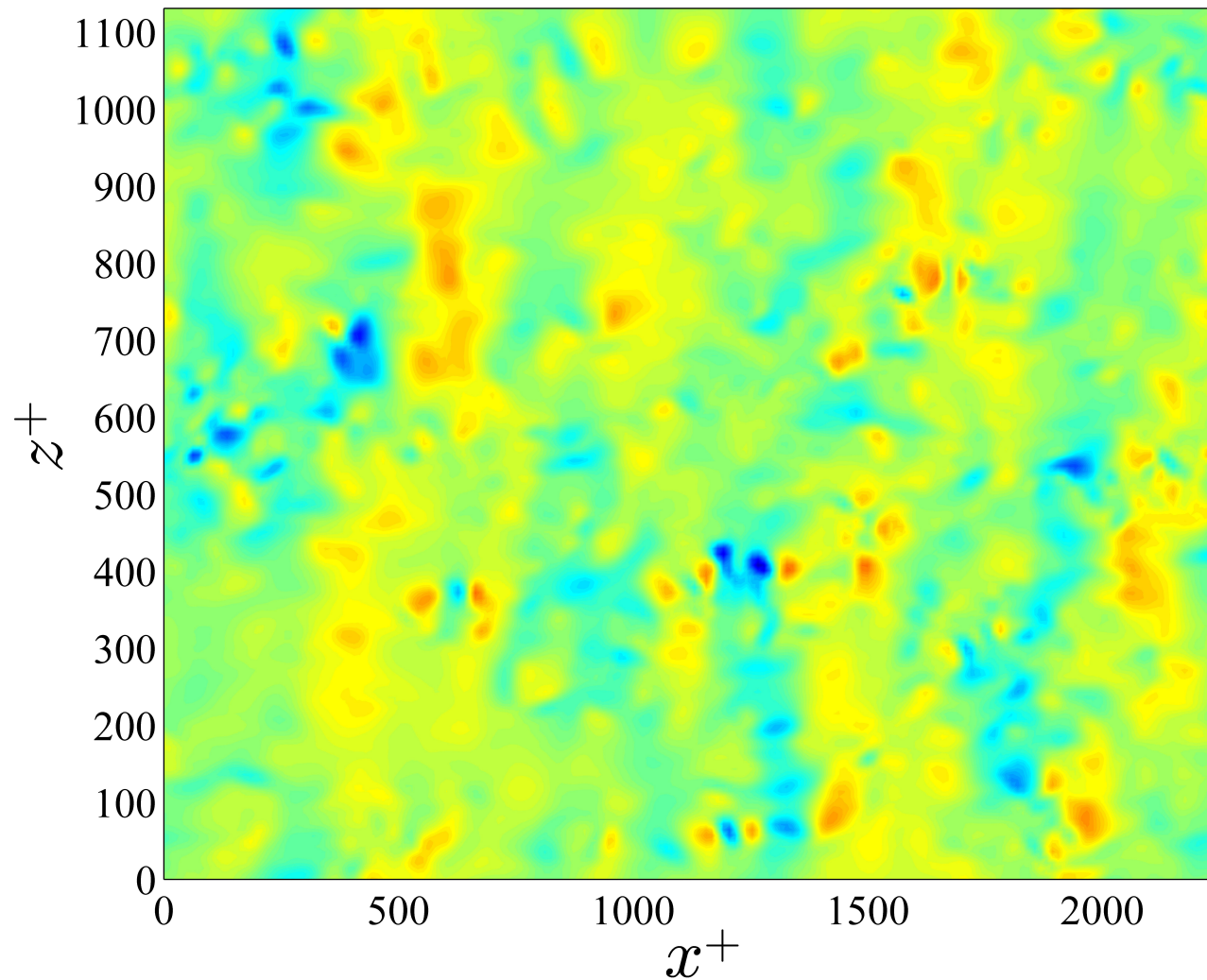
Controlled p : $y^+ = 20|_{\text{init}}$

$Re_\tau = 180$

$tU/h = 2$

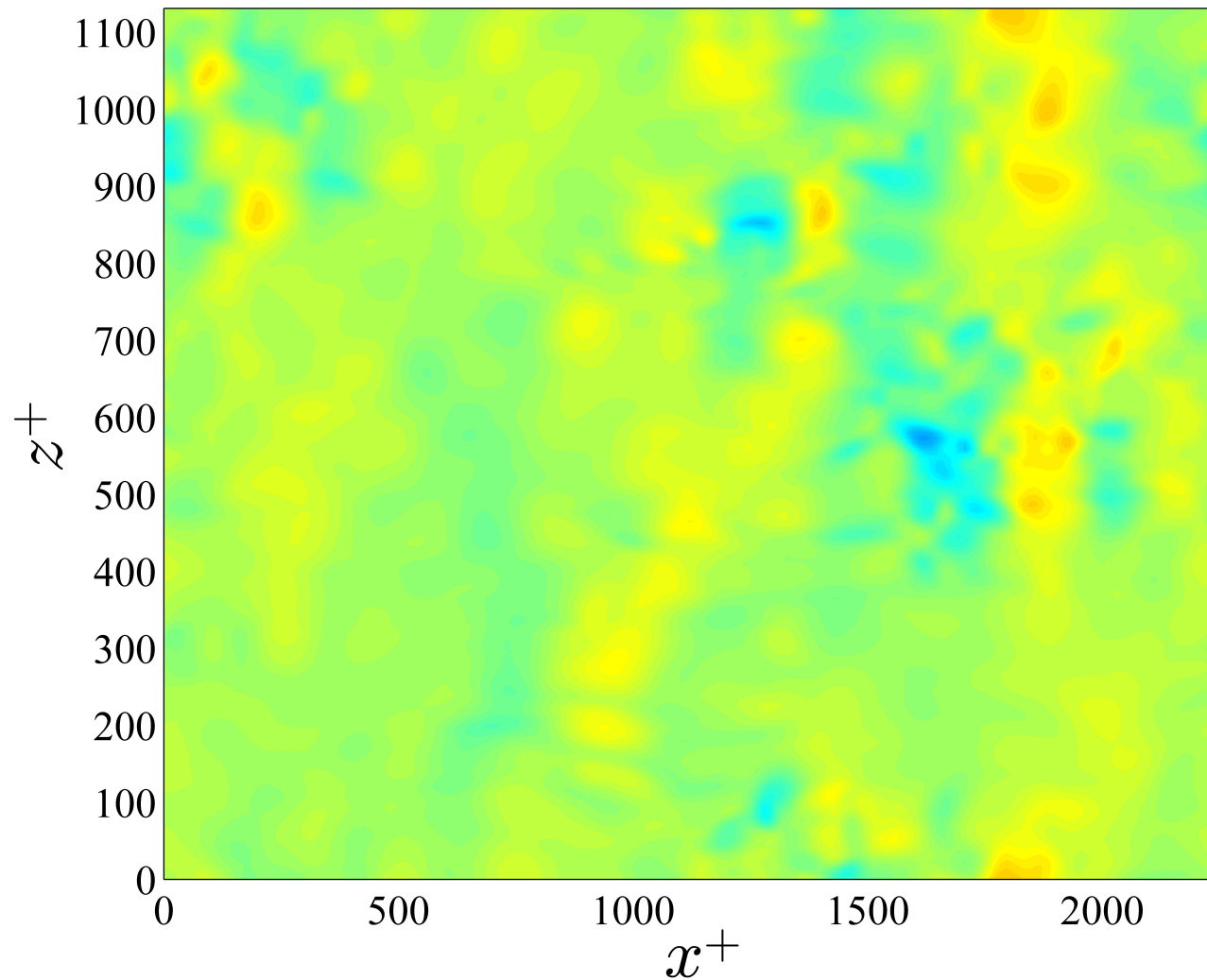


Controlled p : $y^+ = 20|_{\text{init}}$



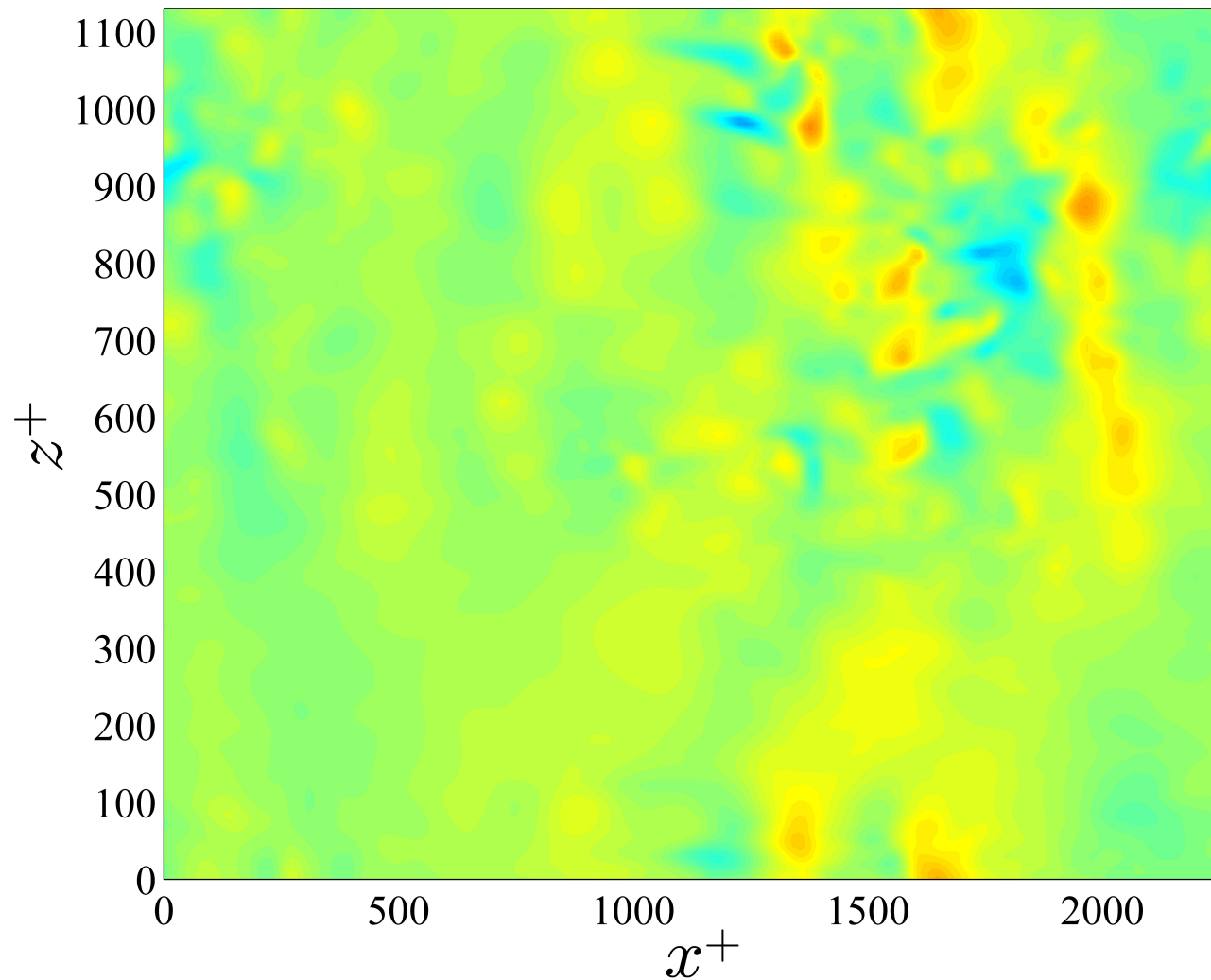
$tU/h = 10$

Controlled p : $y^+ = 20|_{\text{init}}$



$tU/h = 50$

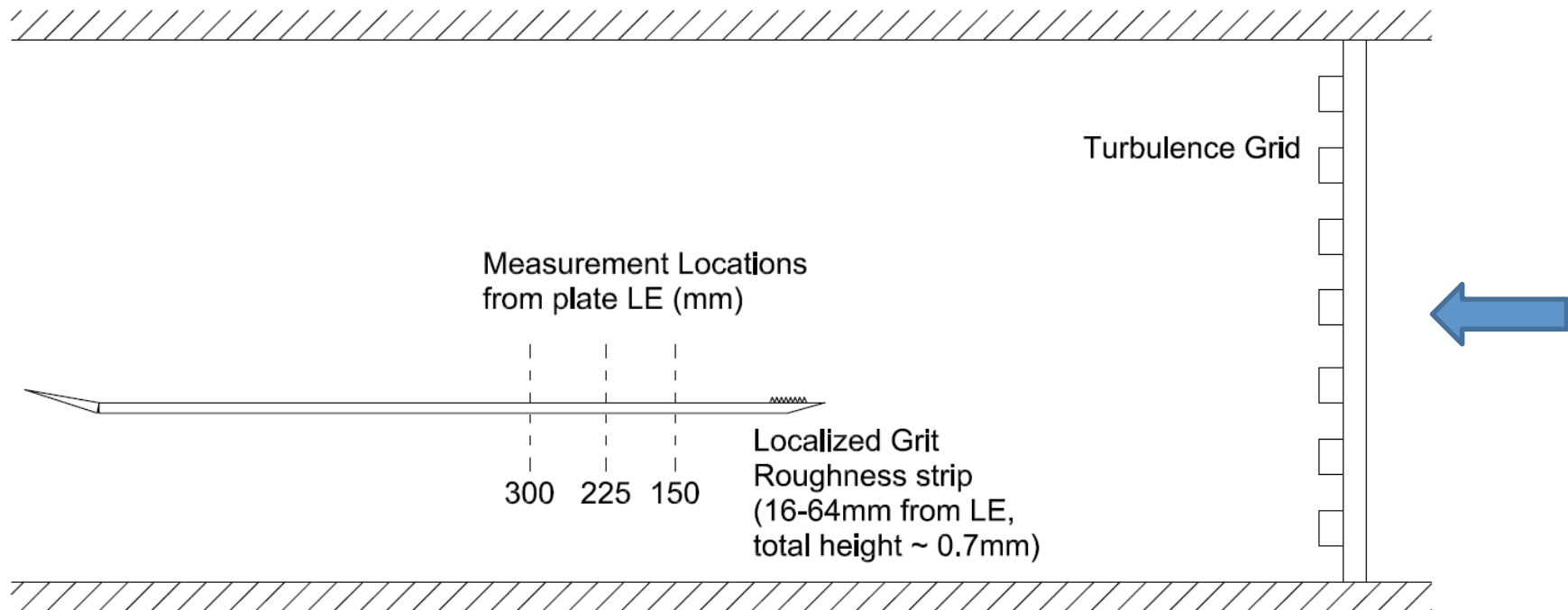
Controlled p : $y^+ = 20|_{\text{init}}$



$tU/h = 100$

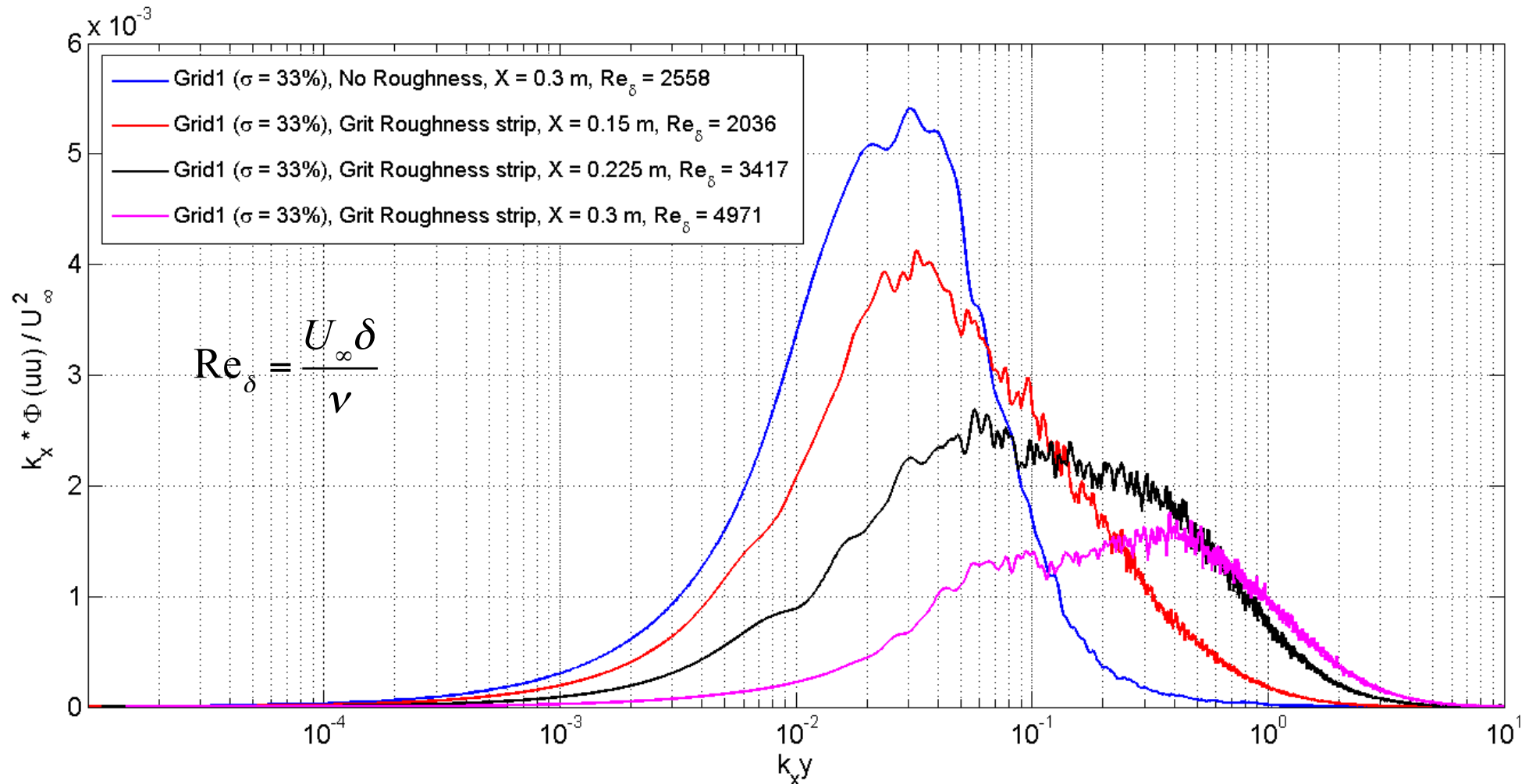
- *Experimental Design:*
 1. *Grid-generated turbulence*
 2. *Thin boundary layer on a flat surface to provide for rapid shearing*
 3. *Localised roughness strip to introduce broad-band disturbance*
- *Two configurations used:*

1) 18" x 18" tunnel (without a rolling road), on an Aluminium plate



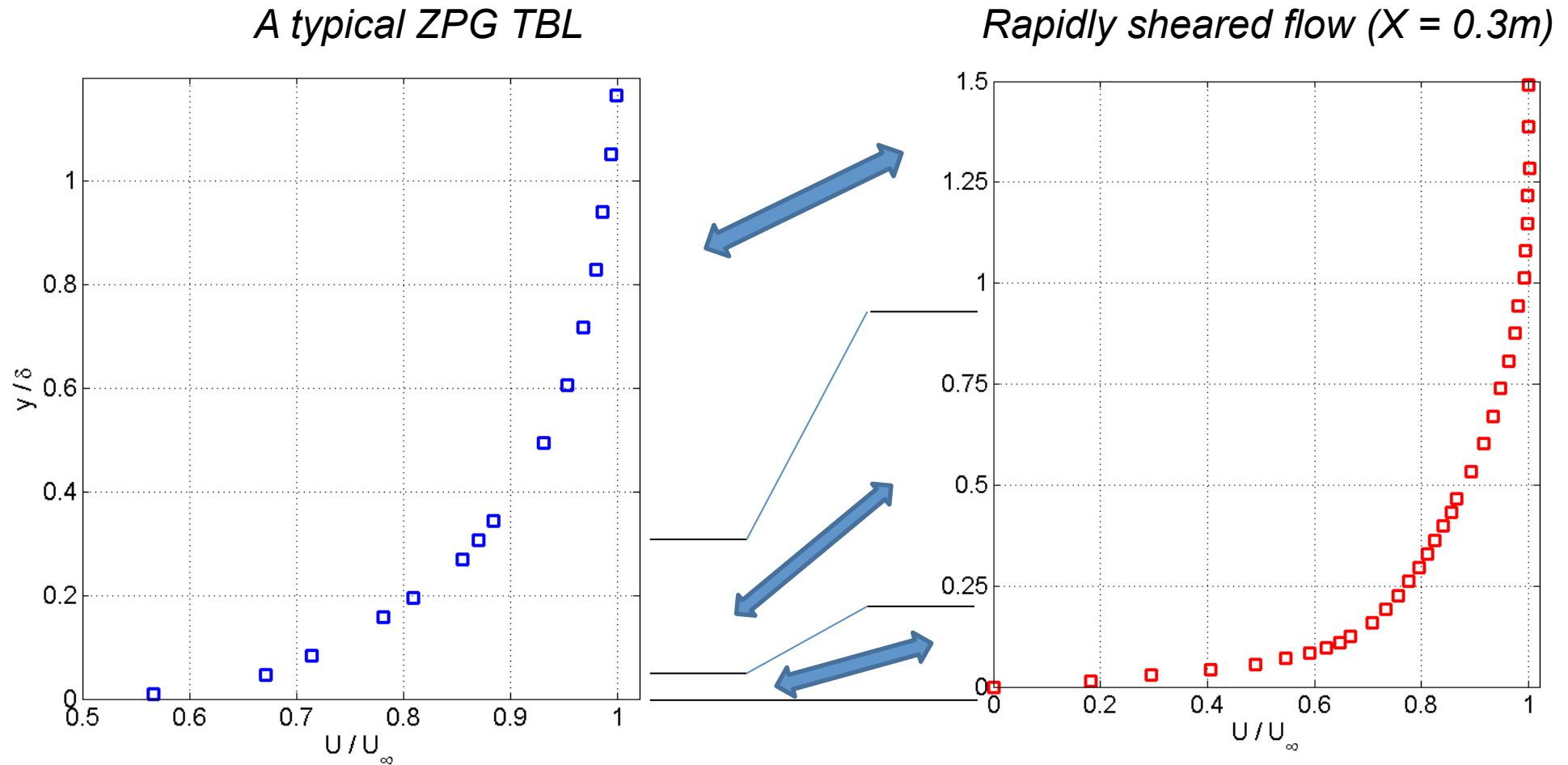
Preliminary Results

- *Single-wire HW measurements*
- *Spectrum calculated using Taylor's hypothesis*



- *BL evolves gradually towards a fully developed turbulent BL*
- *Localized grit roughness enhances high wave-number content in the spectrum (for a given*

Comparison between fully-developed TBL and a rapidly sheared flow



“synthetic” boundary layer that

- Approximately resembles in turbulence structure to a typical TBL*
- Expands the buffer layer of the TBL allowing for a closer inspection of structures*

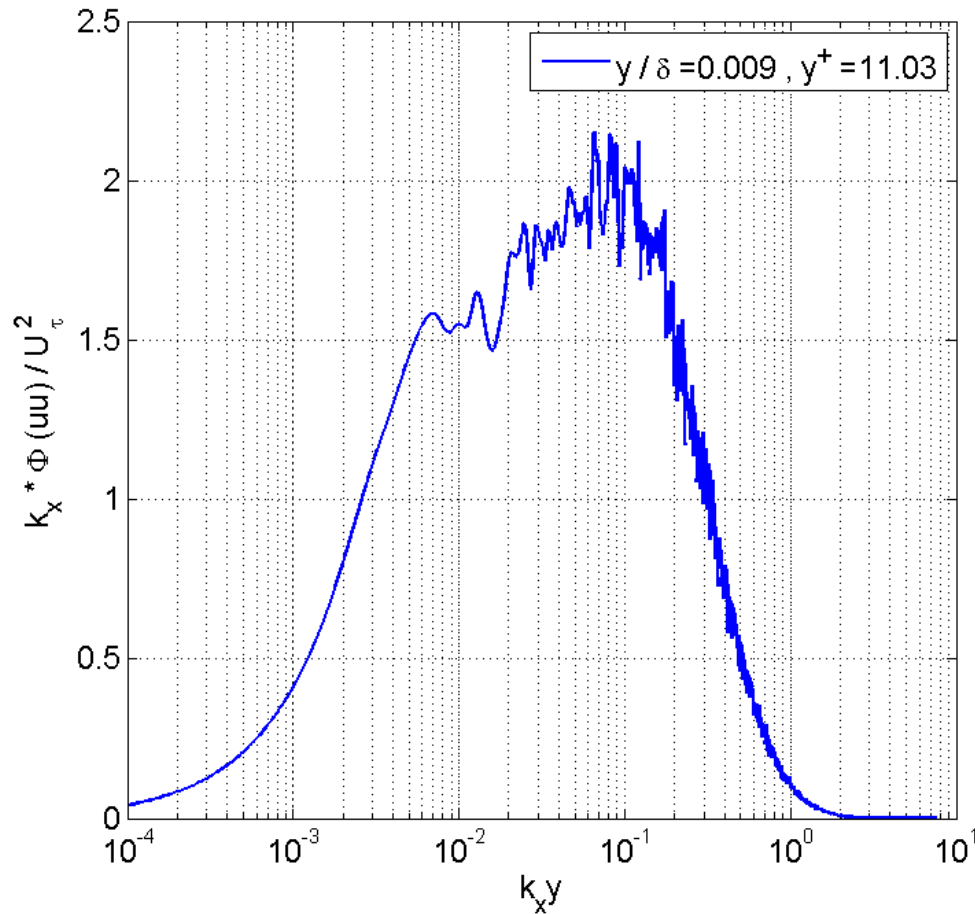
“Synthetic” Boundary Layer compared to a TBL

“Synthetic” Boundary Layer	Turbulent Boundary Layer
Grid-generated turbulence blocked by the presence of wall	Wake region close to the edge of the TBL
A thin wall layer (that rapidly shears the free-stream turbulence to generate long structures)	The buffer region of the TBL
A localized patch of roughness to enhance smaller scales	High wavenumber end of the turbulence spectrum

Comparison between fully-developed TBL and a rapidly sheared flow

Buffer layer

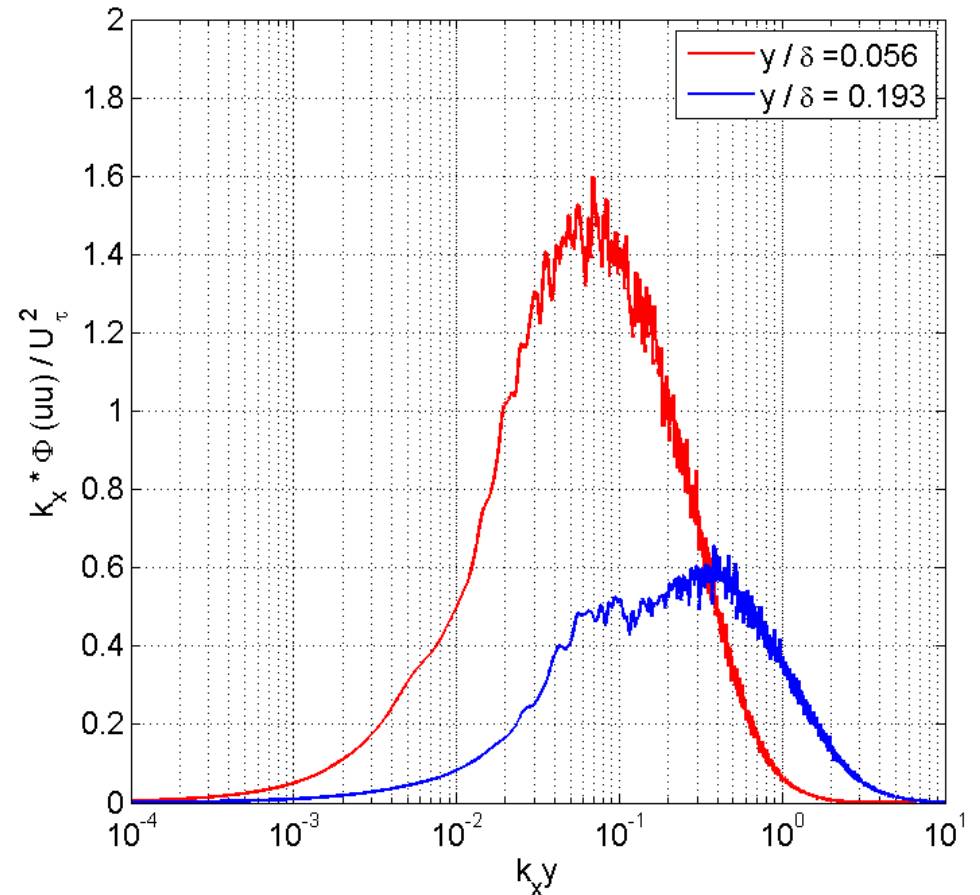
ZPG TBL (with freestream $Tu = 3\%$)



10' x 5'

from Clauser Chart

*Rapidly sheared flow ($X = 0.3m$)
Freestream $Tu = 2.2\%$*



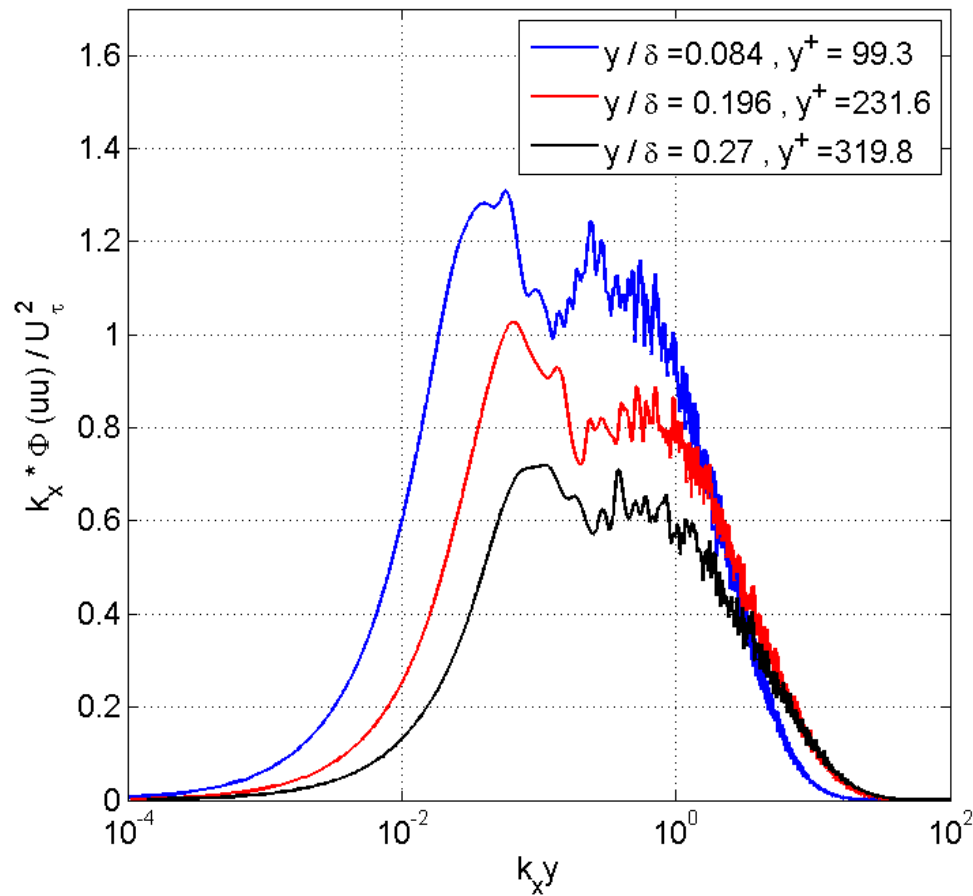
18" x 18"

*Grid Turbulence with localized roughness
from wall gradient (within $\pm 3\%$)*

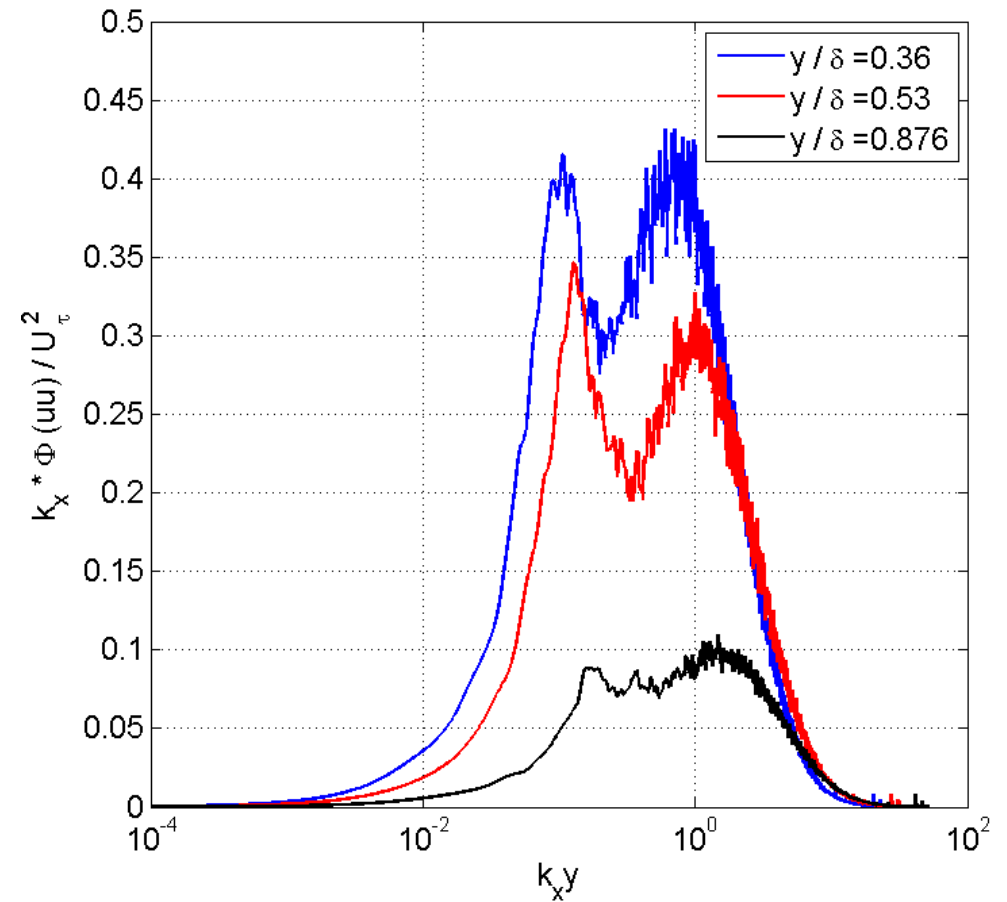
Comparison between fully-developed TBL and a rapidly sheared fl

Log layer

ZPG TBL



Rapidly sheared flow ($X = 0.3m$)

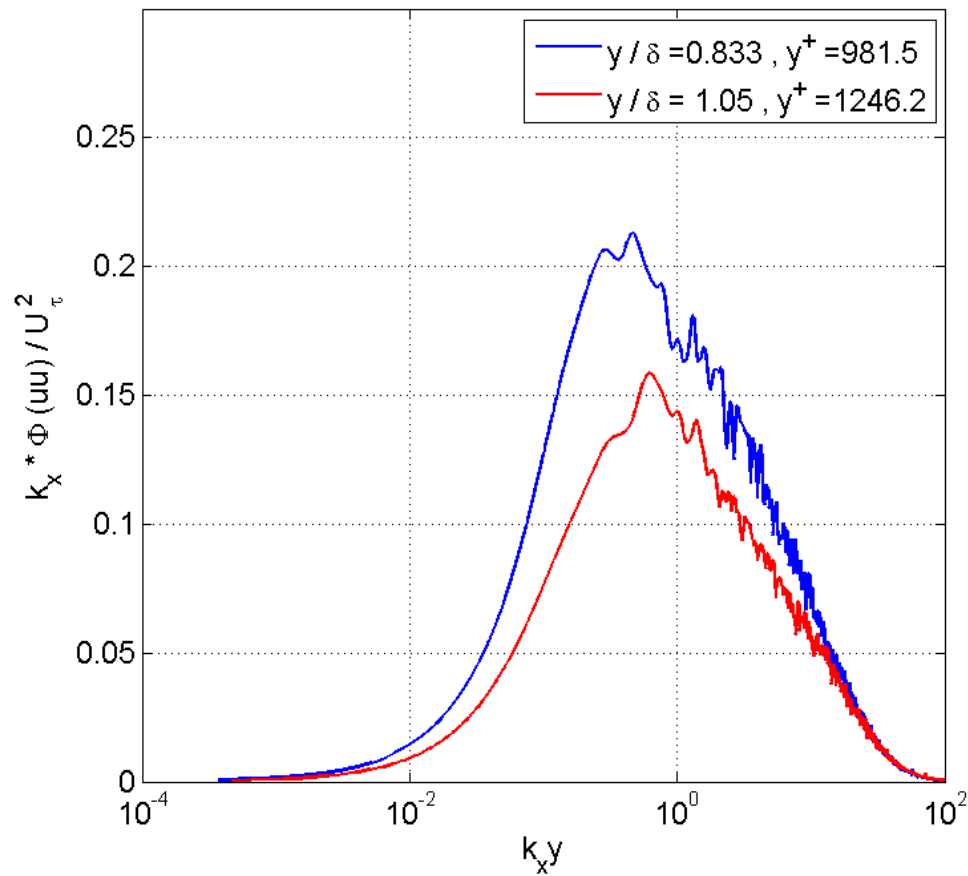


Streamwise length scales of the same order in both flows in appropriate regions

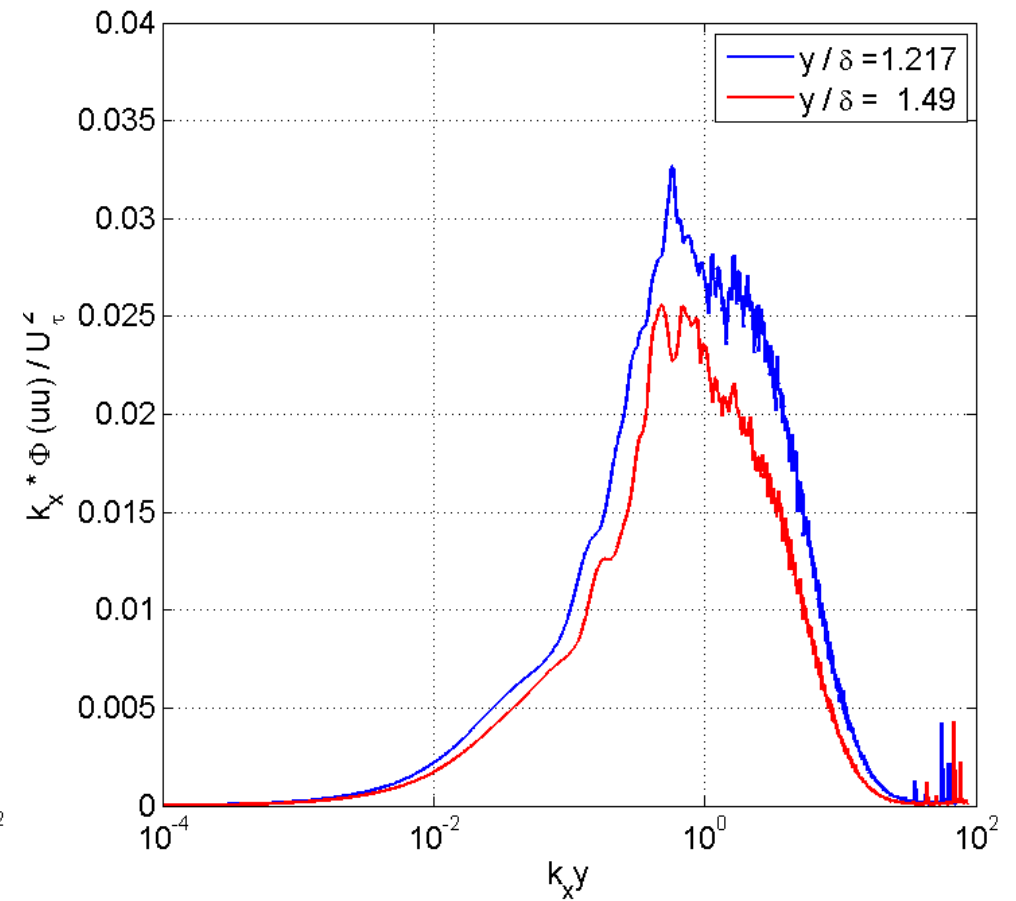
Comparison between fully-developed TBL and a rapidly sheared flow

Outer layer

ZPG TBL



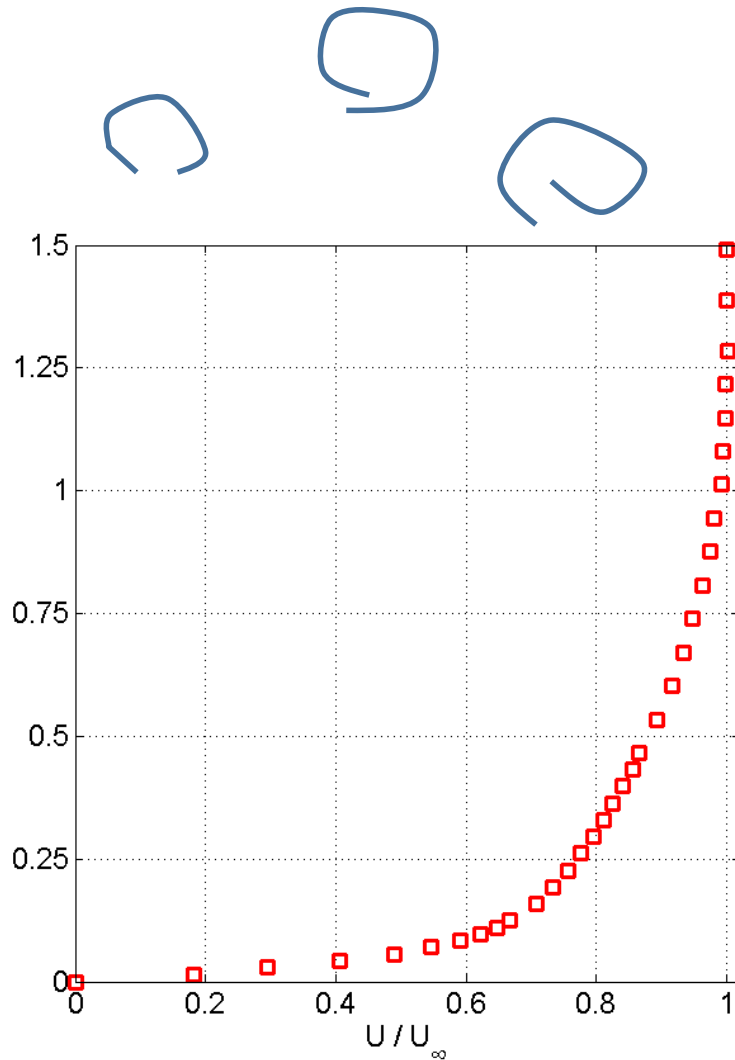
Rapidly sheared flow ($X = 0.3m$)



Streamwise length scales of the same order in both flows in appropriate regions

Time-scale Disparity

τ = Integral time scale of Freestream Turbulence
= Shear interaction time scale =



T

Time-scale disparity suggests that the ideas of the Rapid Distortion Theory might be applicable to this “synthetic” flow

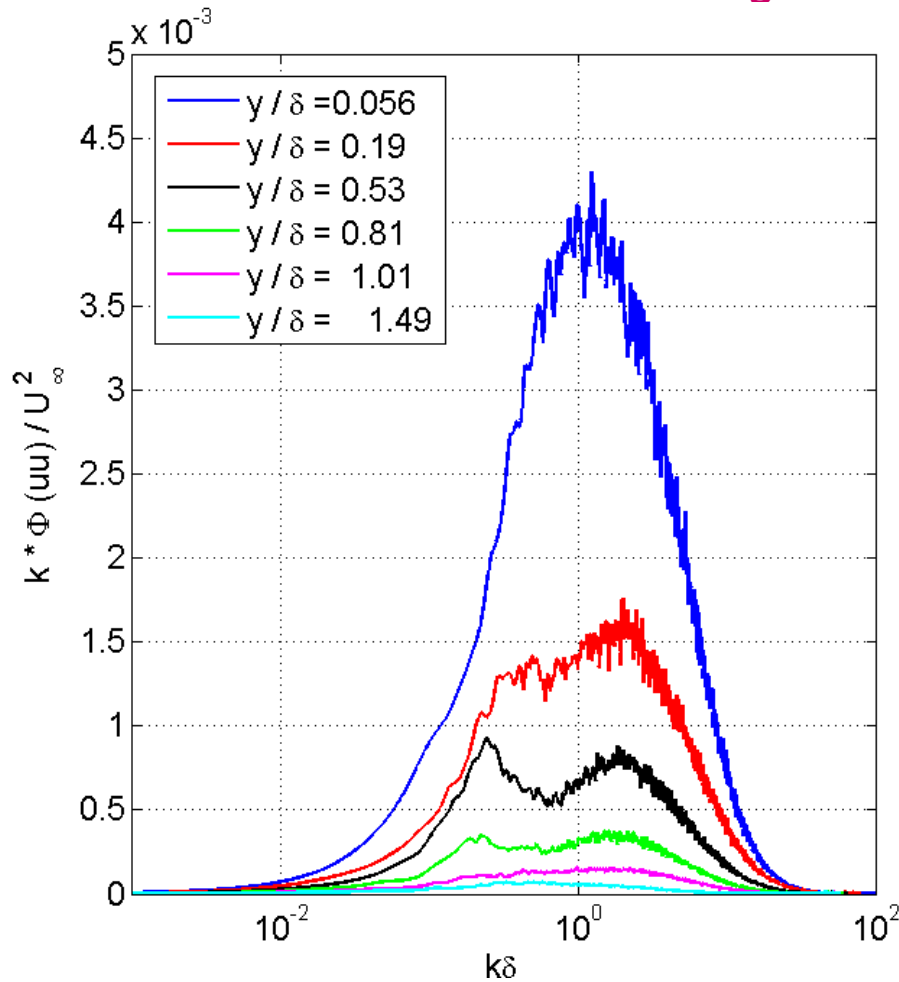
18" x 18"

$X=0.3\text{ m}$

Grid Turbulence with localized roughness

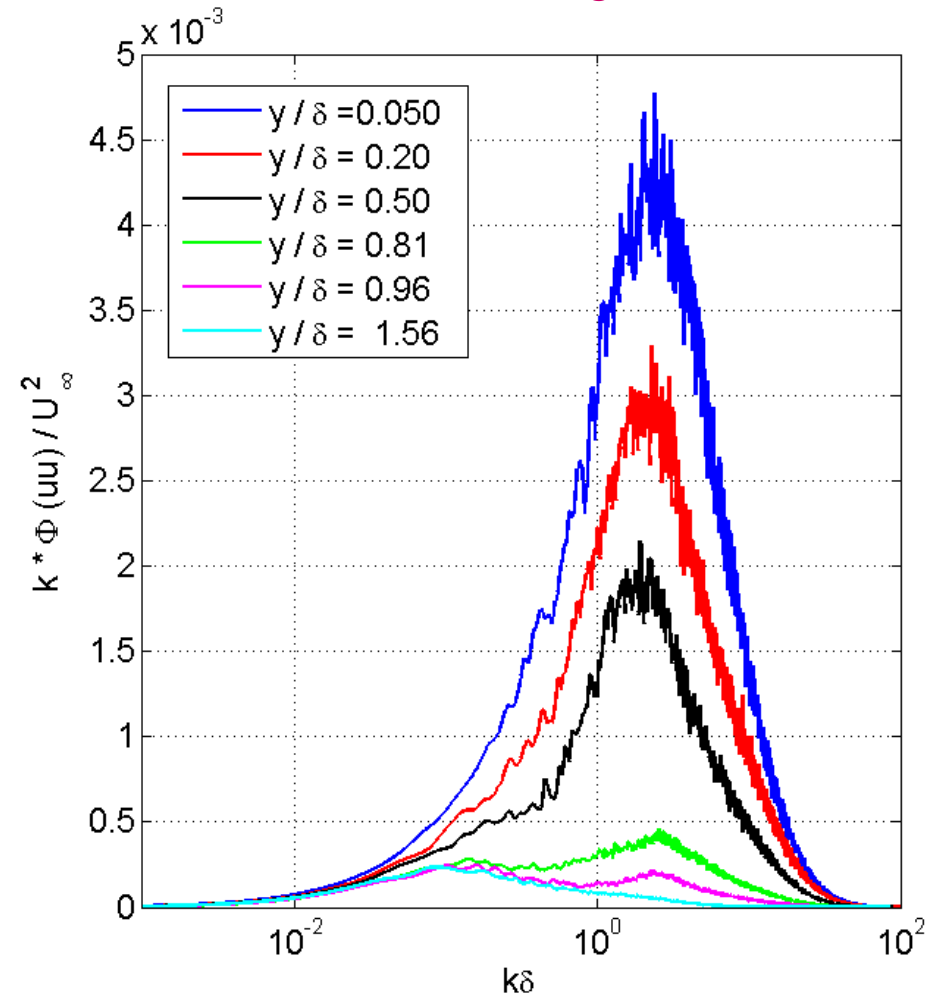
Effect of upstream boundary condition (UBC) on spectra

*UBCs: Grid turbulence
Localized Wall Roughness*



18" x 18" tunnel

*UBCs: Shearless Boundary Layer
No Wall Roughness*



10' x 5' tunnel