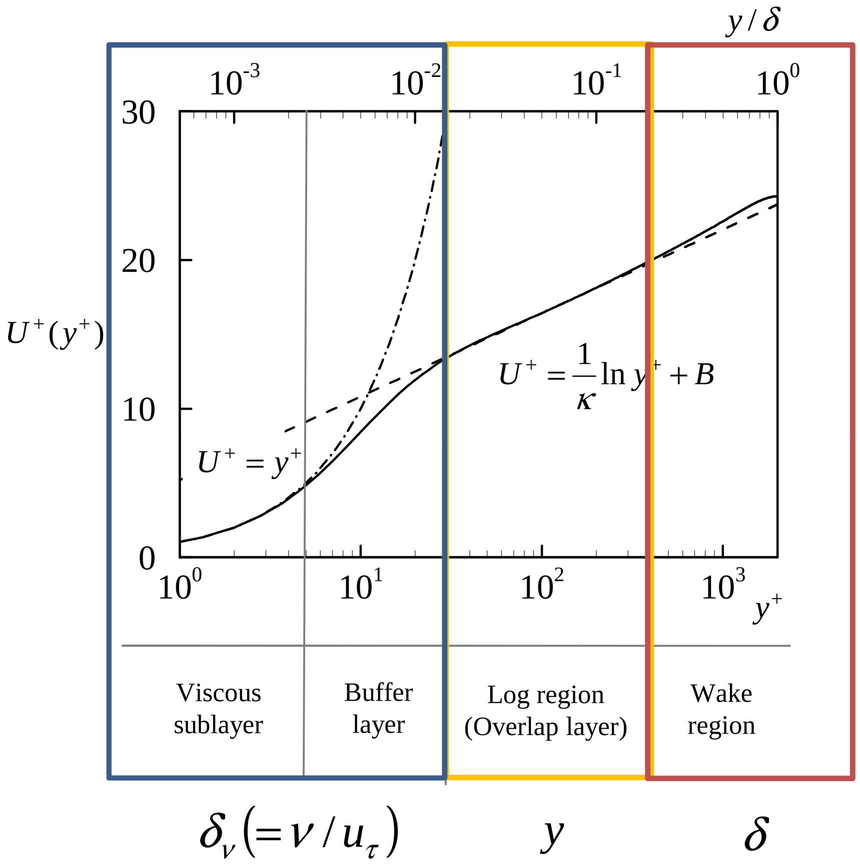
Statistical Structure of Self-Sustaining Attached Eddies in Turbulent Channel Flow

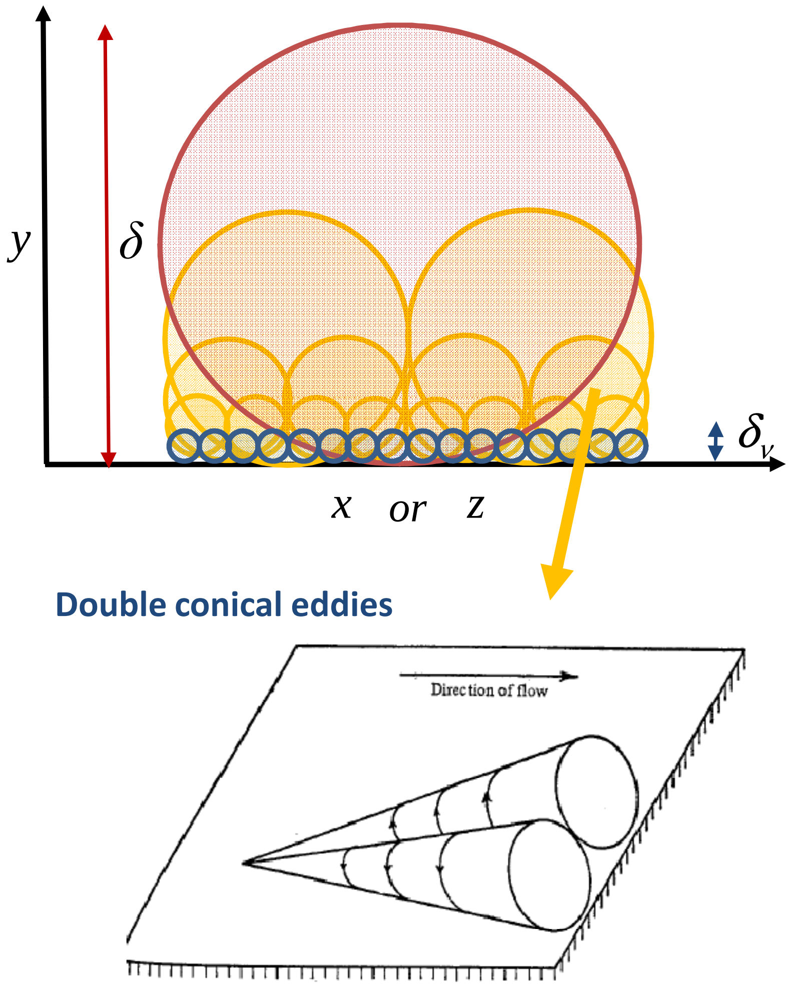
Yongyun Hwang[1,2]

[1] Department of Civil & Environmental Engineering
Imperial College London

[2] Department of Applied Mathematics & Theoretical Physics (DAMTP)
University of Cambridge

Attached Eddy Hypothesis – Townsend (1961)

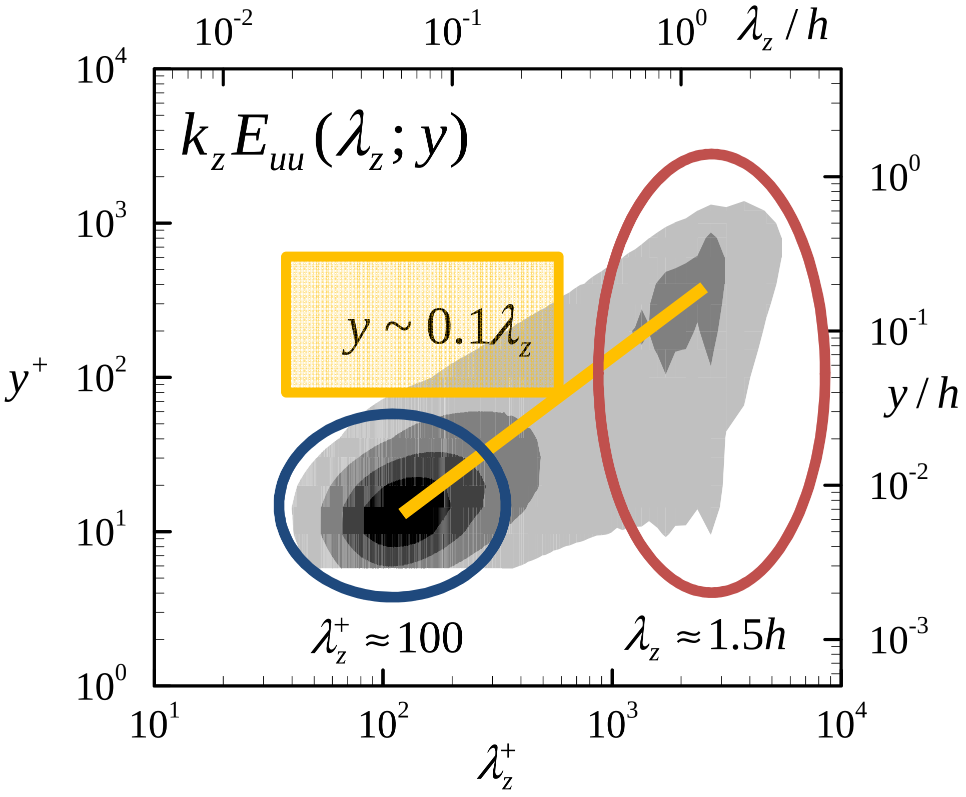




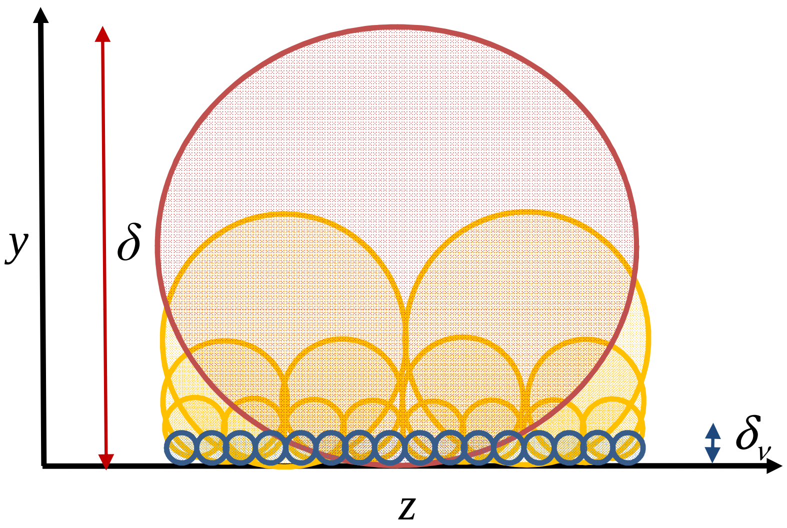
Double conical eddies

What Could be the Size of the Attached Eddy y_a ?

- Spanwise spectra in turbulent channel flow at $\mathrm{Re}_\tau = 2000$



Hoyas & Jimenez (2006, Phys. Fluids)



$\lambda_z \sim y$

- **Spanwise width** roughly characterizes **the size of the attached eddies**

The Near-Wall Motions at $\lambda_z^+ \leq 100$

Hwang, 2013, *J. Fluid Mech*. **727** p264

Removal of motions wider than $\lambda_z^+ \approx 100$

1. **Spanwise minimal** and **streamwise long** computational domain

$$\boxed{L_z^+ \approx 100}$$

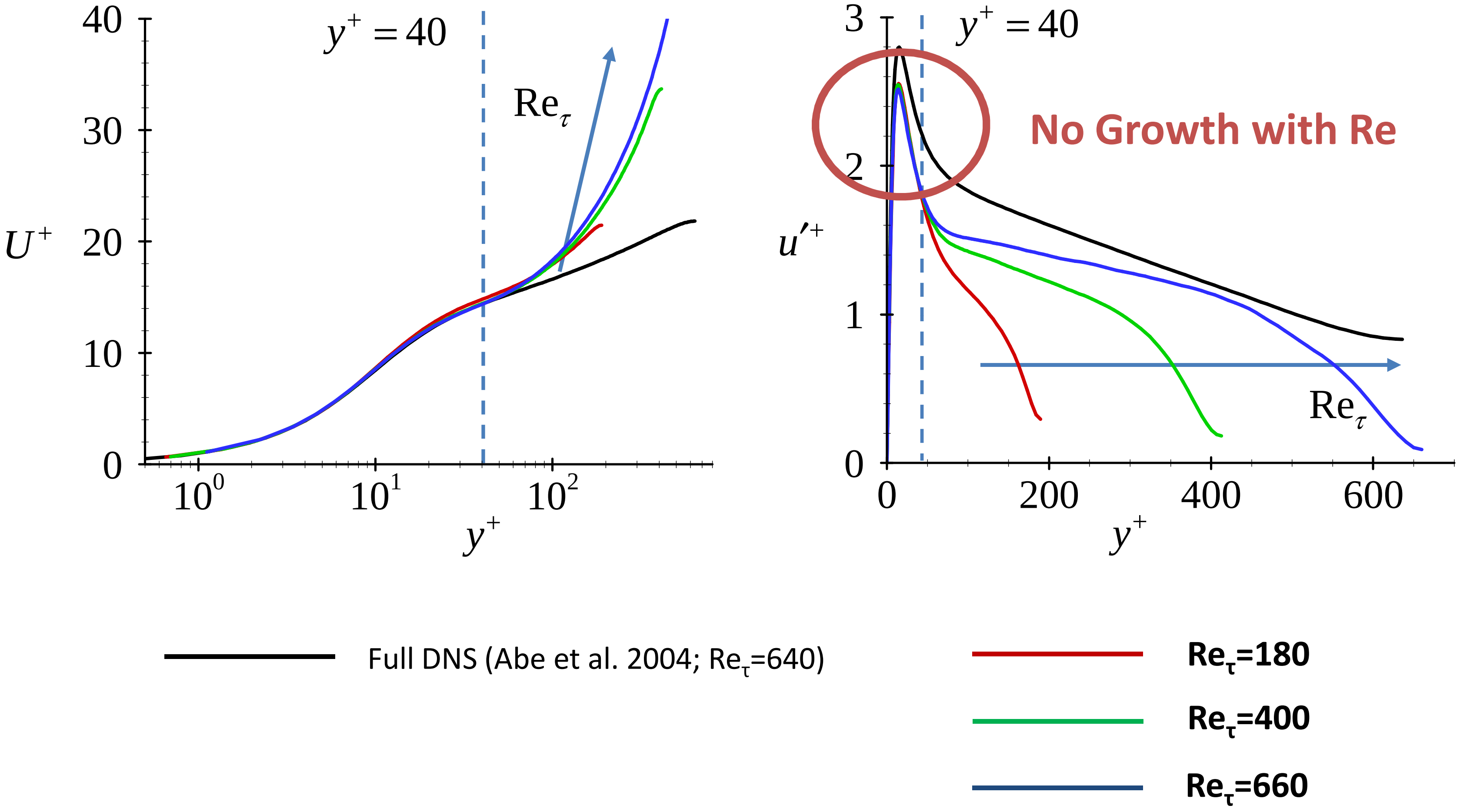
$$\boxed{L_x^+ \approx 3000 \sim 4000}$$

2. **Explicit filtering** of **spanwise uniform (2D) eddies**

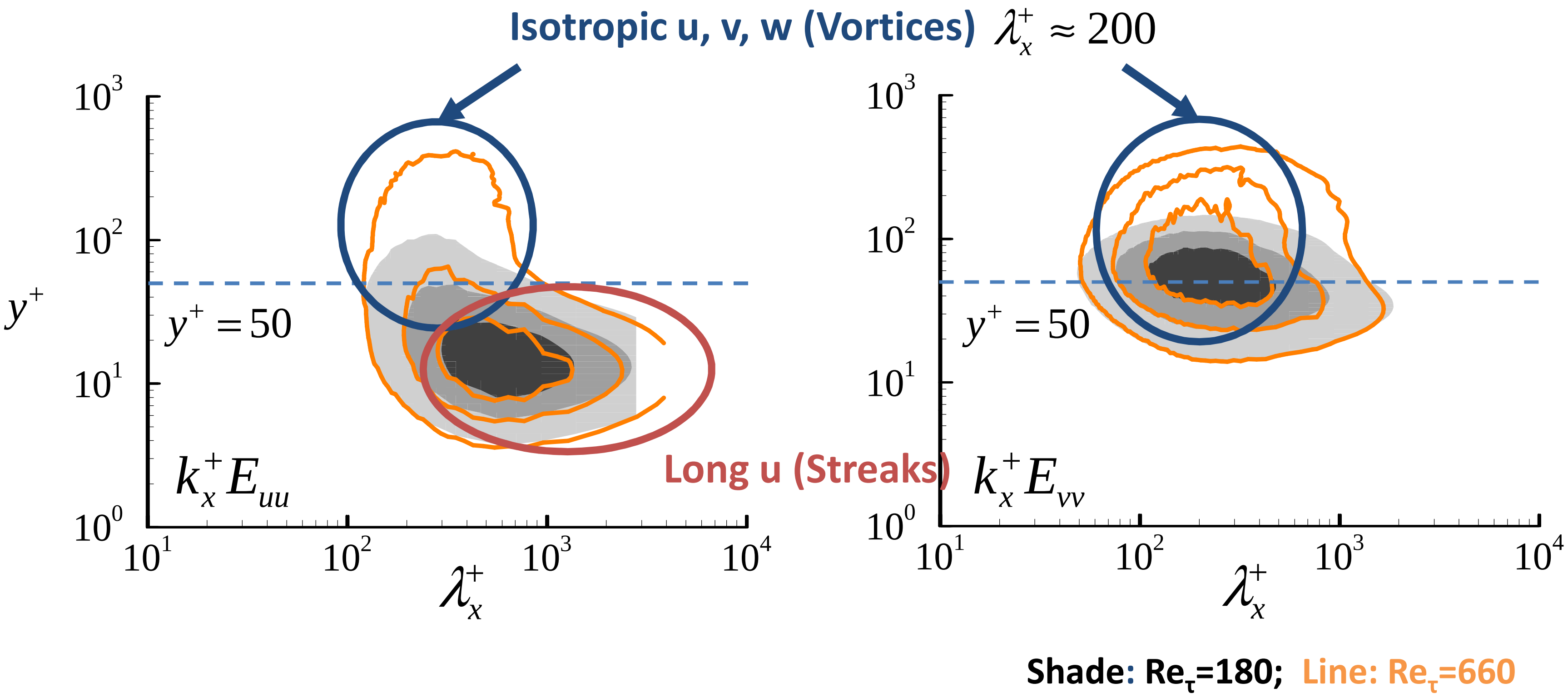
$$A\phi^{n+1} = RHS(\phi^n)$$

Remove spanwise uniform components

Mean-Velocity Profile and Turbulence Intensity



One-dimensional Spectra



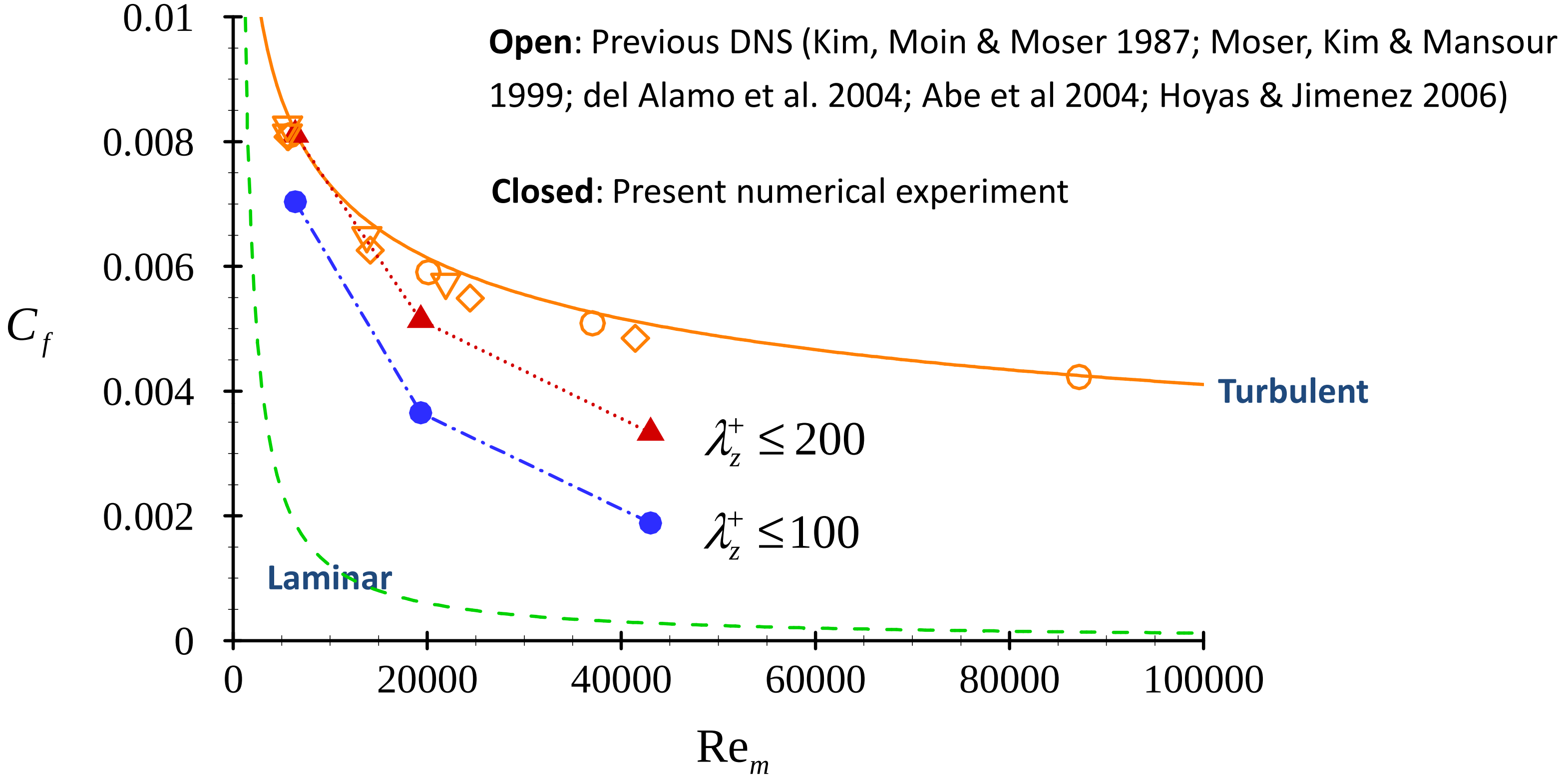
Isotropic u, v, w (Vortices) $\lambda_x^+ \approx 200$

Long u (Streaks)

Shade: Re_τ=180; Line: Re_τ=660

- All the **velocity fluctuations scale very well with the Reynolds number**
- **Bimodal Behavior** – **Vortices** & **Streaks**

The Skin Friction by the Near-Wall Motions May be Very Small



Open: Previous DNS (Kim, Moin & Moser 1987; Moser, Kim & Mansour 1999; del Alamo et al. 2004; Abe et al 2004; Hoyas & Jimenez 2006)

Closed: Present numerical experiment

- **Need to control the scales larger than the near-wall motions**

Summary I

- In the near-wall region

$\lambda_z^+ \approx 100$ →

Long u	$y^+ \approx 15$	$\lambda_x^+ \approx 1000$
Tall & short u,v,w	$y^+ \approx 50$	$\lambda_x^+ \approx 200$

The Outer Motions at $\lambda_z \approx 1 \sim 2h$

Hwang & Cossu**,** 2010, *Phys. Rev. Lett*. **105** 044505

Hwang, 2013, In preparation

Removal of the motions at $\lambda_z \leq 1 \sim 2h$

- **Over-damped large eddy simulations (LES)**

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_i}\frac{\partial \overline{u_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \nu\frac{\partial \overline{u_i}}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}^R}{\partial x_j} \quad \text{where} \quad \overline{u_i} = \int G(\mathbf{r},\mathbf{x}) u_i(\mathbf{x}-\mathbf{r},t)d\mathbf{r}$$

Static Smagorinsky model

$$\tau_{ij} - 1/3\delta_{ij}\tau_{kk} = 2\nu_T \overline{S_{ij}} \quad \text{with}$$

$$\nu_T = D\left(C_s\overline{\Delta}\right)^2\overline{S}$$

Filter width

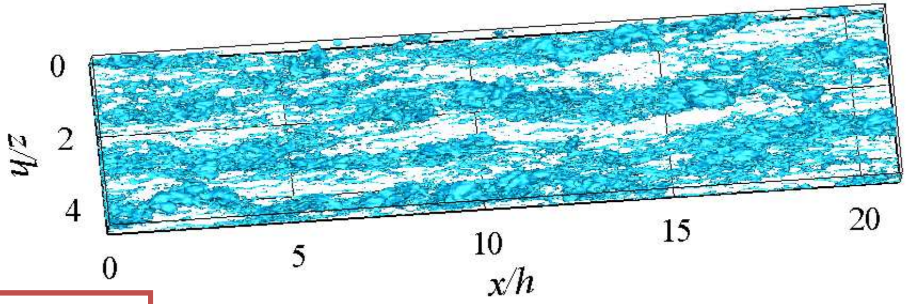
l :mixing length

$$l = C_s\overline{\Delta}$$

Increase Cs !!!!

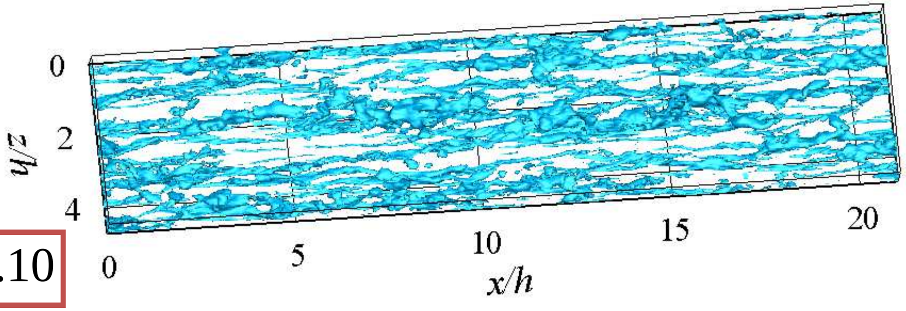
Self-Sustaining Outer Motions at $\mathrm{Re}_\tau \approx 1000$

- Iso-surface of instantaneous streamwise velocity fluctuation (u^+=-2)



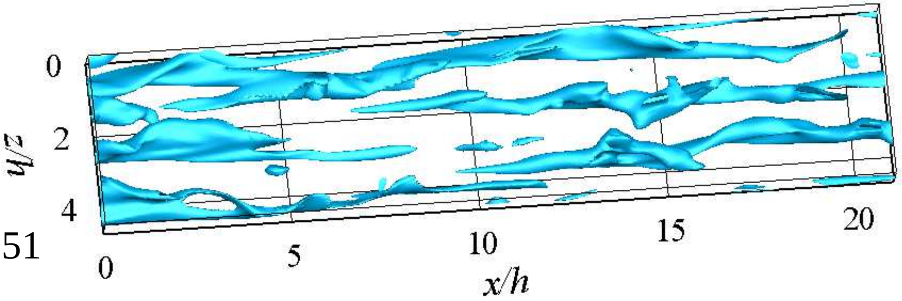
$C_s = 0.05$

$\mathrm{Re}_\tau = 955$



$C_s = 0.10$

$\mathrm{Re}_\tau = 887$

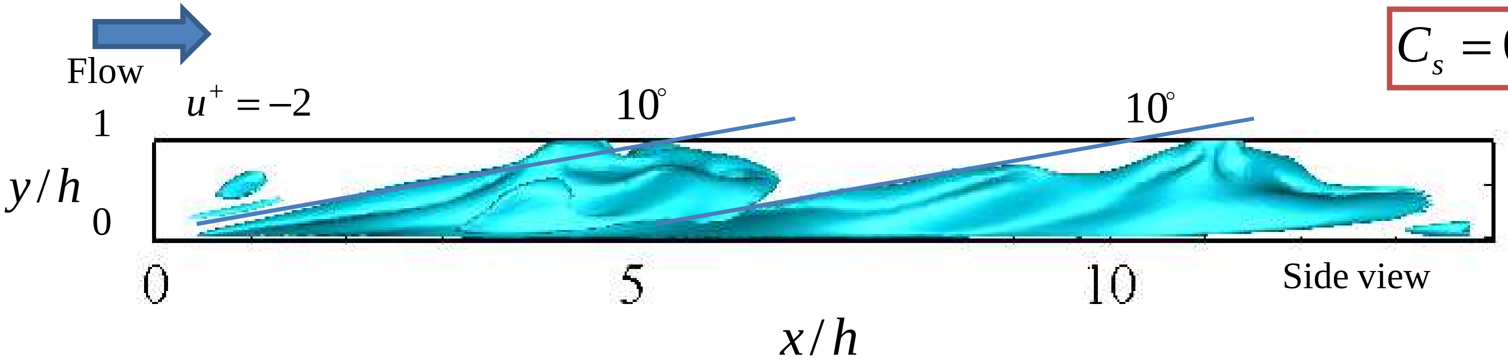


$C_s = 0.30$

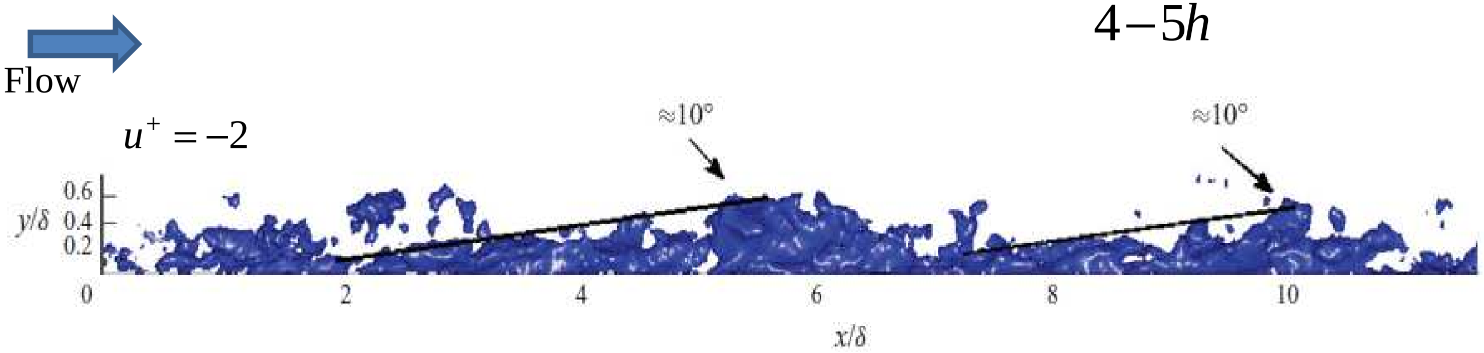
$\mathrm{Re}_\tau = 1051$

The Over-Damped LES Looks Not So Bad

$\boxed{C_s = 0.30}$



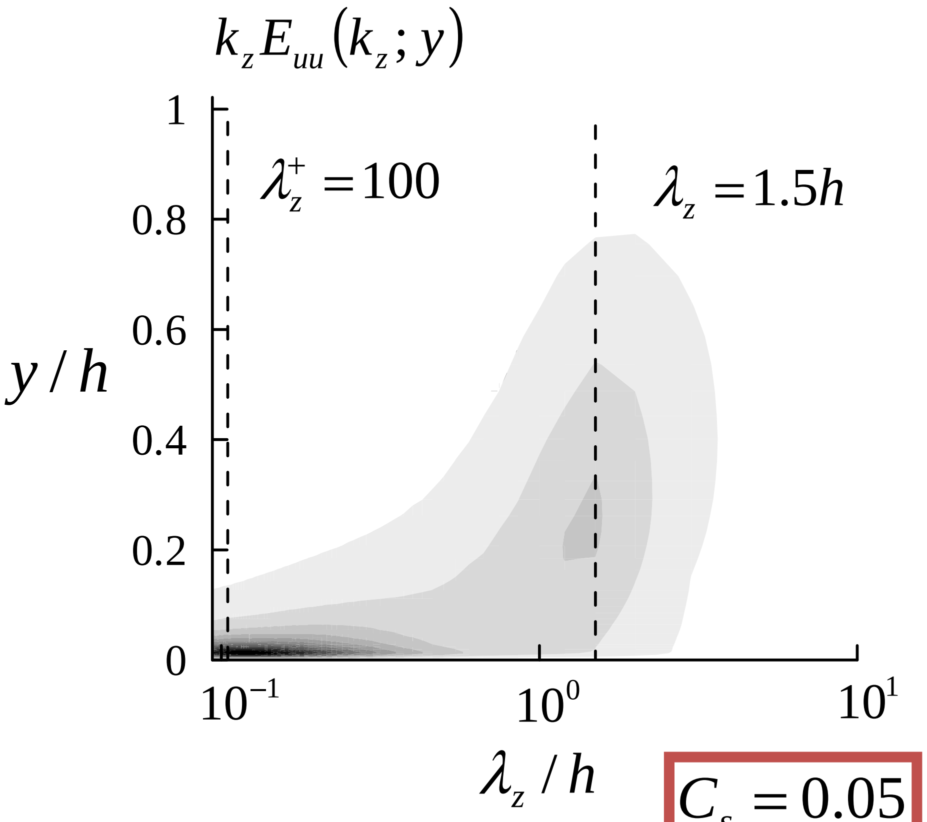
$4-5h$



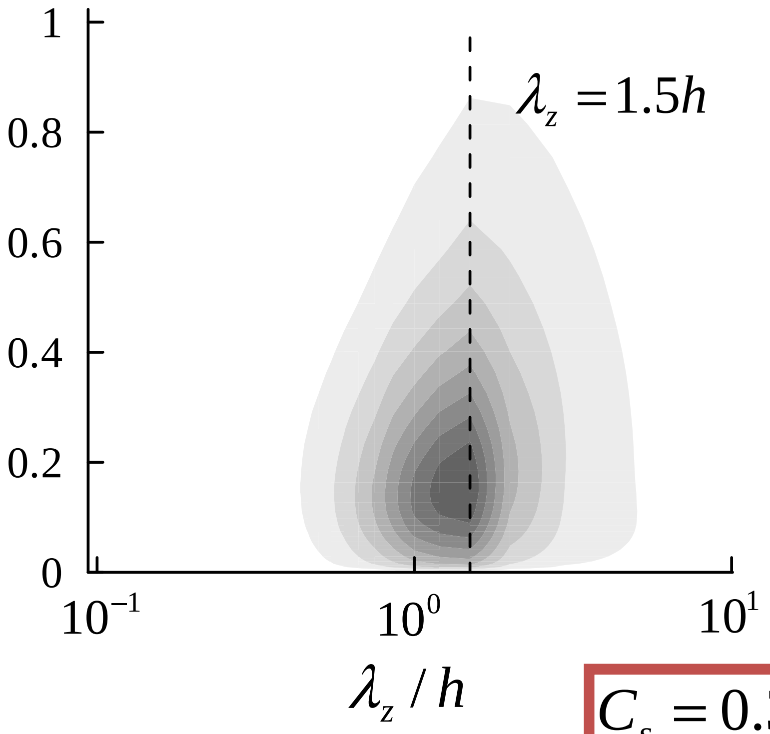
PIV measurement

By Dennis & Nickels (2011)

One-dimensional Spanwise Spectra

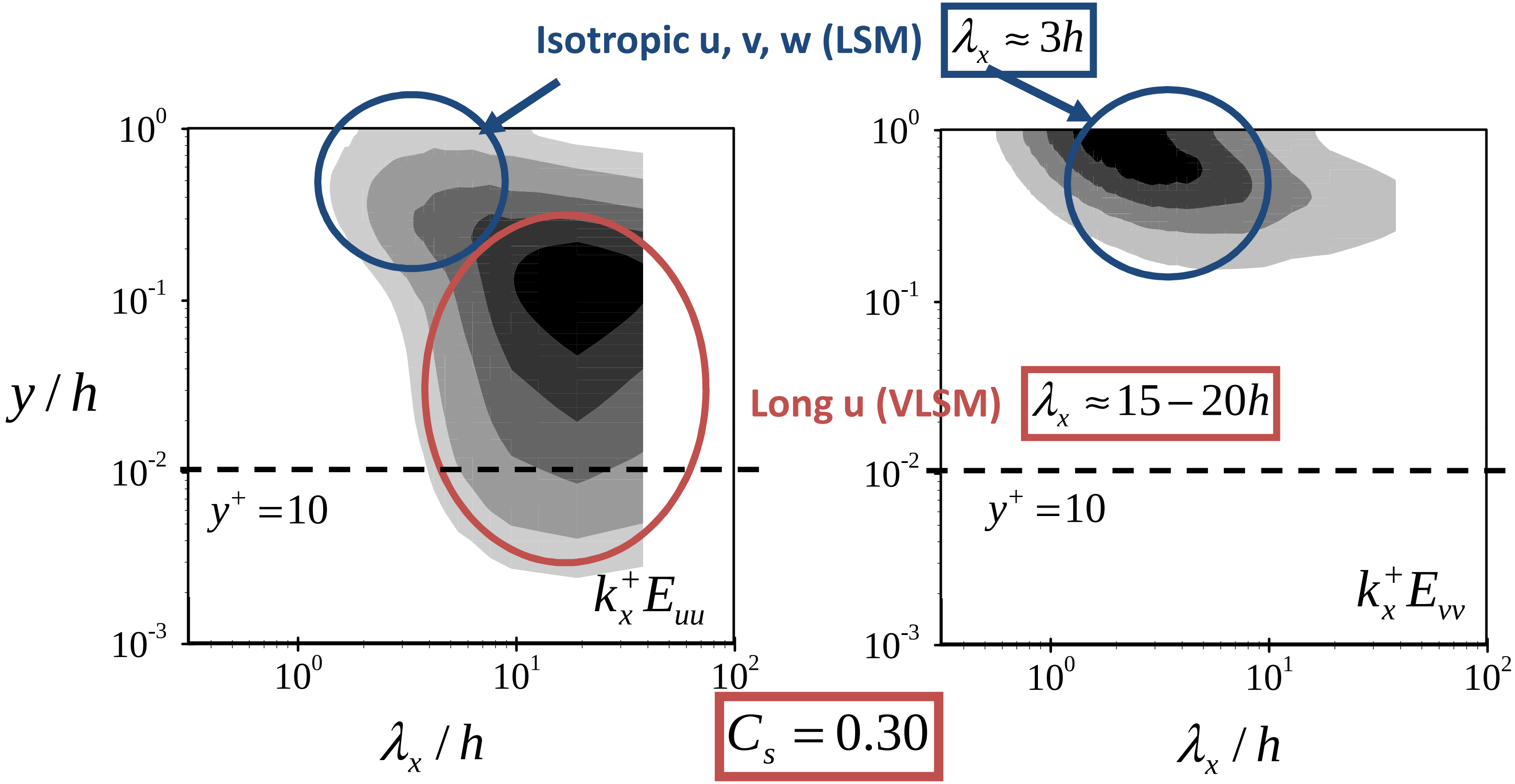


$C_s = 0.05$



$C_s = 0.30$

One-dimensional Streamwise Spectra of $\lambda_z \approx 1.5h$



Isotropic u, v, w (LSM) $\lambda_x \approx 3h$

Long u (VLSM) $\lambda_x \approx 15-20h$

$C_s = 0.30$

- **Bimodal Behavior** – **LSM (u,v,w)** & **VLSM (long u)**
- **Highly Penetrating VLSM** into the near-wall region

Summary II

- In the near-wall region

$\lambda_z^+ \approx 100$	**Long u**	$y^+ \approx 15$	$\lambda_x^+ \approx 1000$
	Tall & short u,v,w	$y^+ \approx 50$	$\lambda_x^+ \approx 200$

- In the outer region

$\lambda_z \approx 1.5h$	**Long u**	$y \approx 0.2h$	$\lambda_x \approx 15 \sim 20\,h$
	Tall & short u,v,w	$y \approx 0.6 \sim 1h$	$\lambda_x \approx 2 \sim 4h$

The Log-layer Motions at $\lambda_z \sim y$

Hwang & Cossu**,** 2011, *Phys. Fluids*. **23** 061702

Hwang, 2013, In preparation

Isolating the Motions at a Given λ_z $(100\delta_v < \lambda_z < 1.5h)$

1. **Motions wider than a given** λ_z

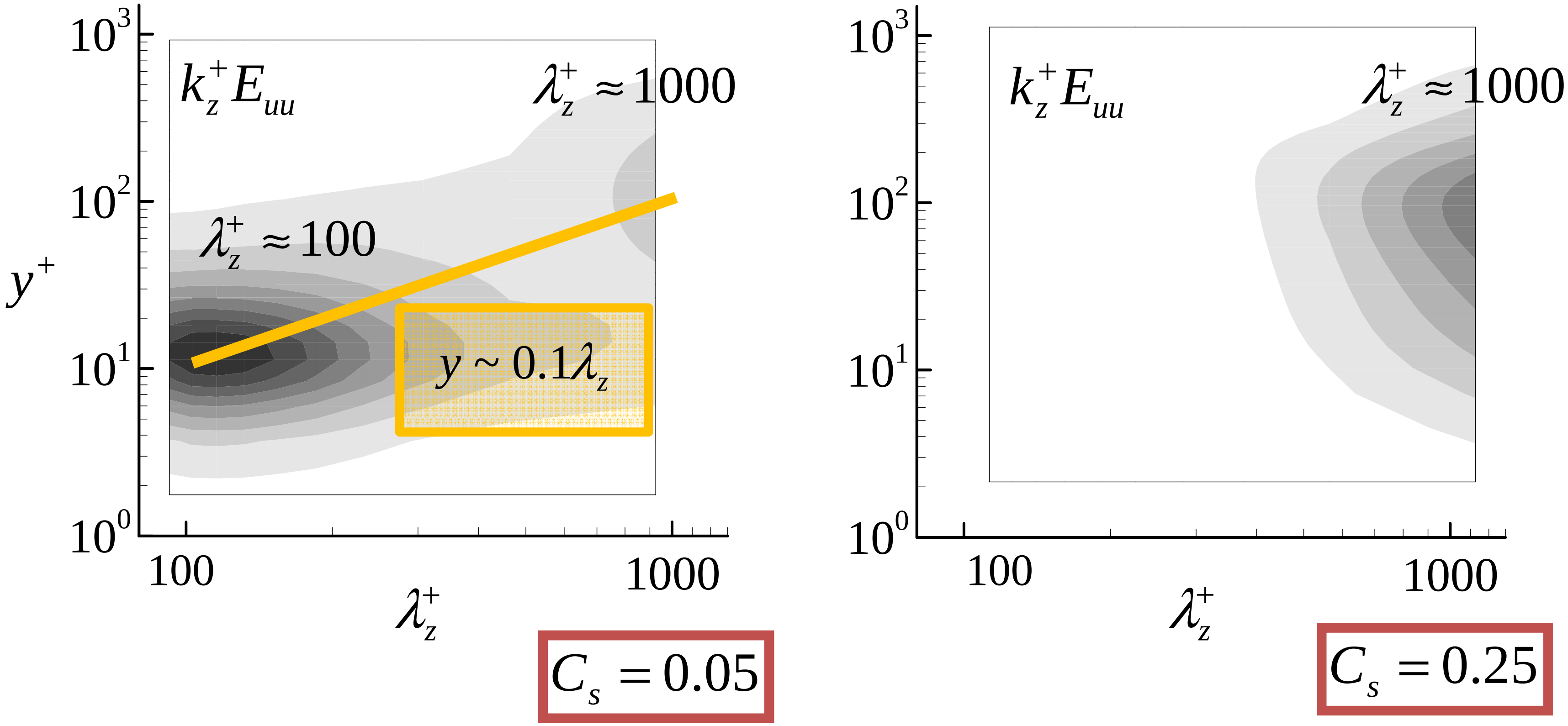
➡ **Filtering of these motions with a computational box**

$$\boxed{L_z \approx \lambda_z} \qquad \boxed{L_x \approx 37h}$$

2. **Motions narrower than a given** λ_z

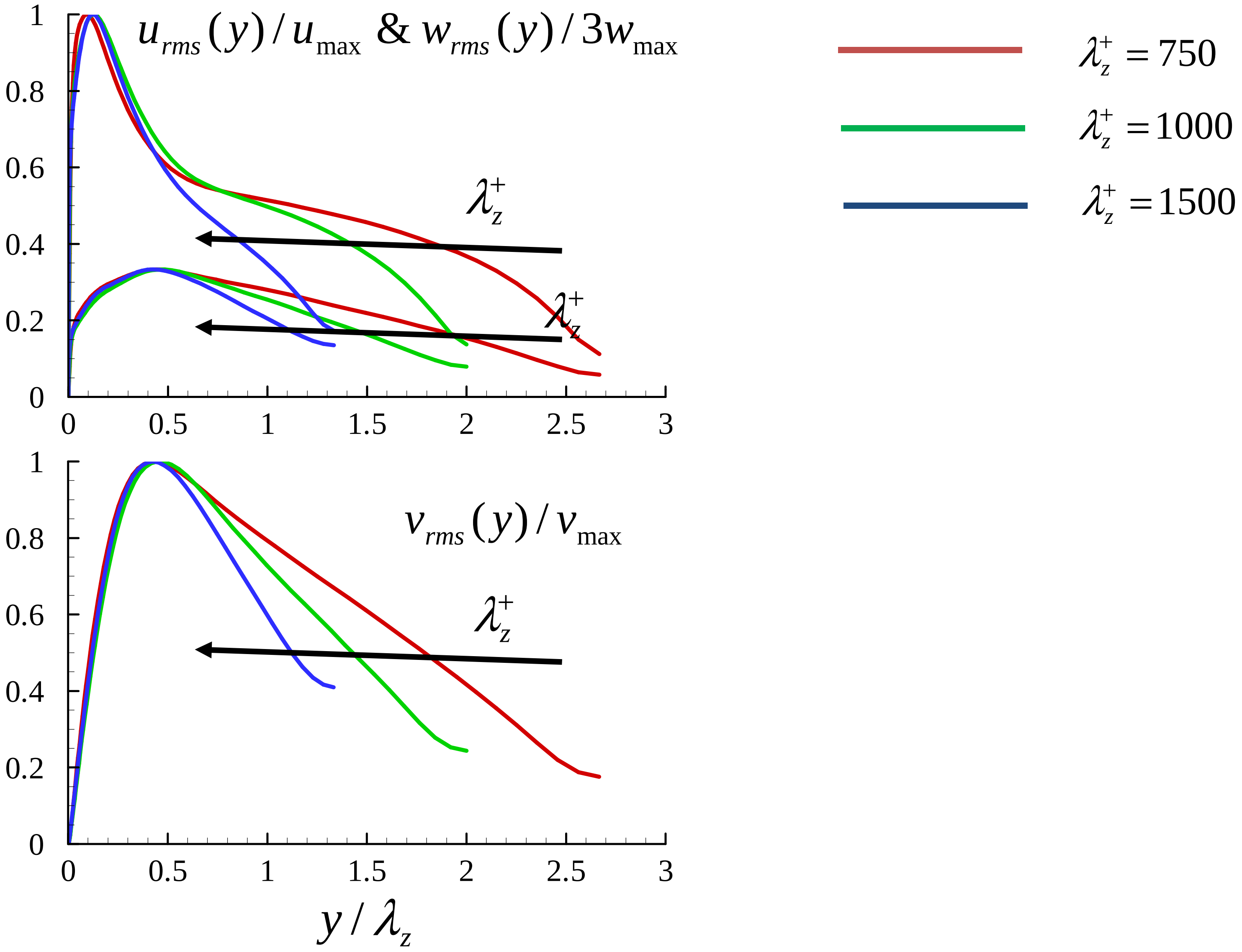
➡ **Over-damped LES**

Example: Isolating the motions at $\lambda_z^+ \approx 1000 \ (\mathrm{Re}_\tau \approx 2000)$

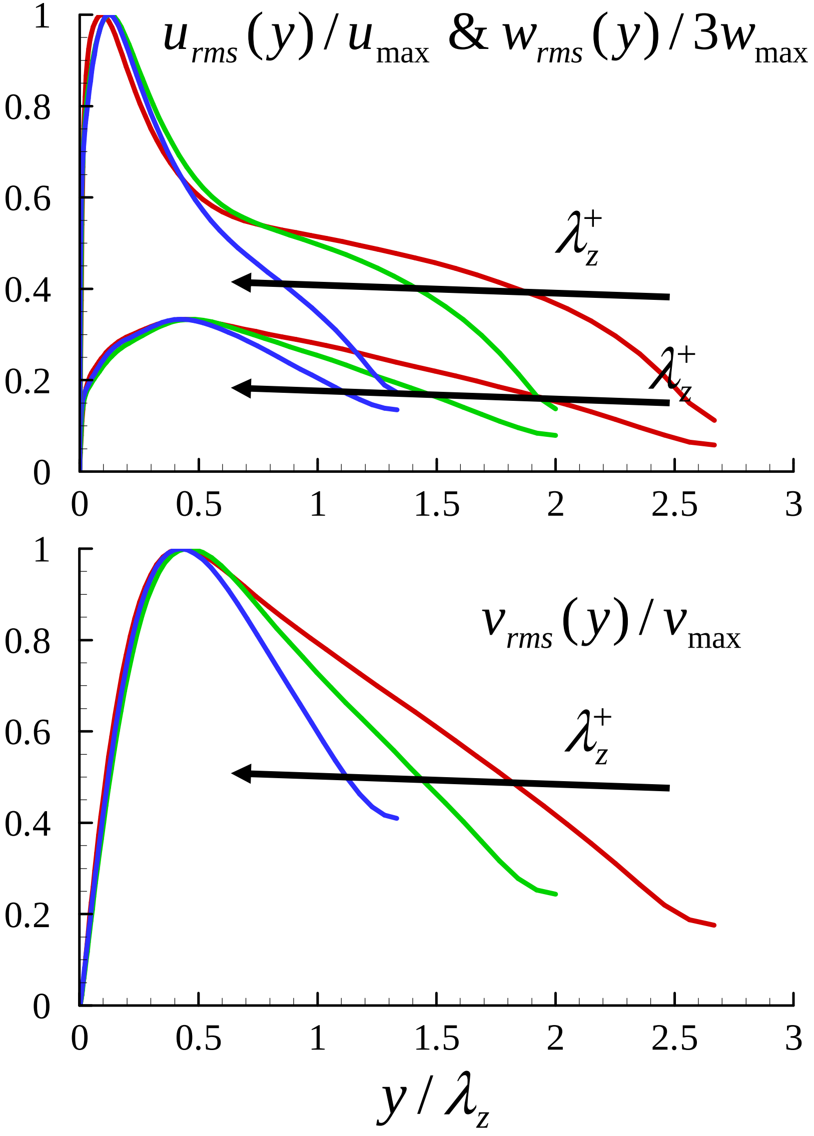


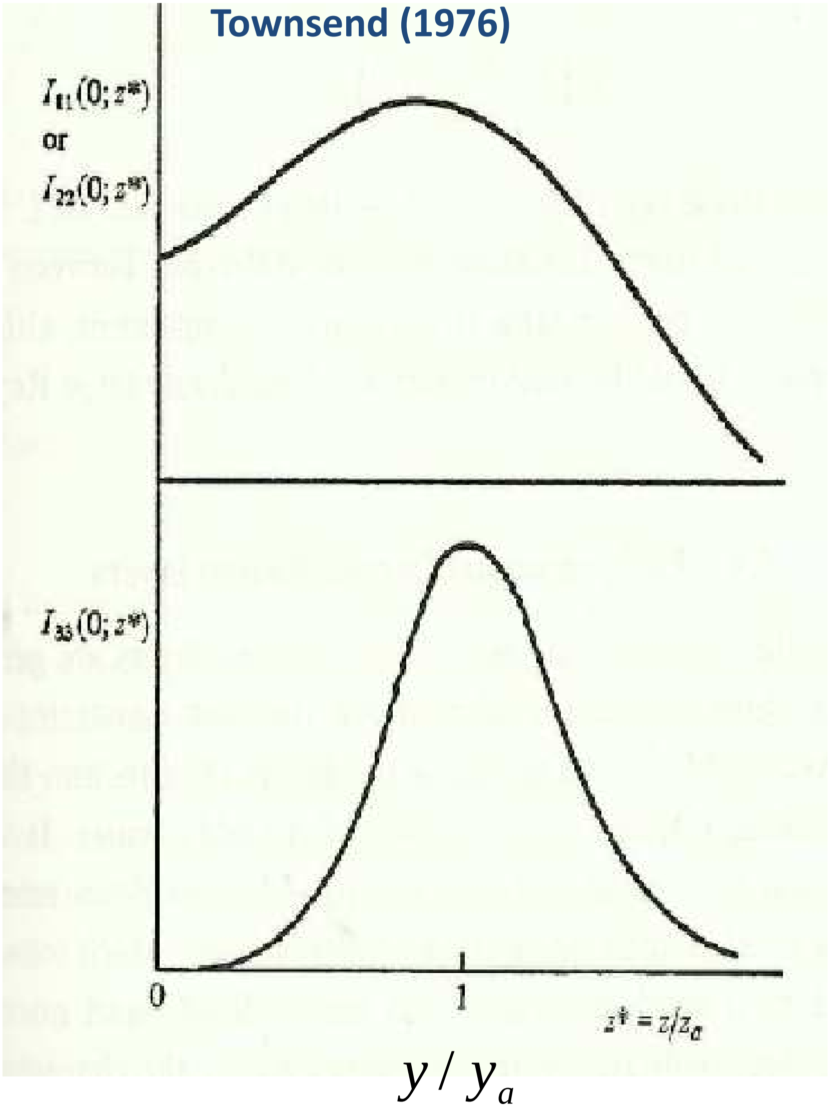
- $\lambda_z^+ > 1000$ → **Filtering**
- $\lambda_z^+ < 1000$ → **Over-damped LES**

Statistics of the Eddies at a Given λ_z $\left(\mathrm{Re}_\tau \approx 2000\right)$

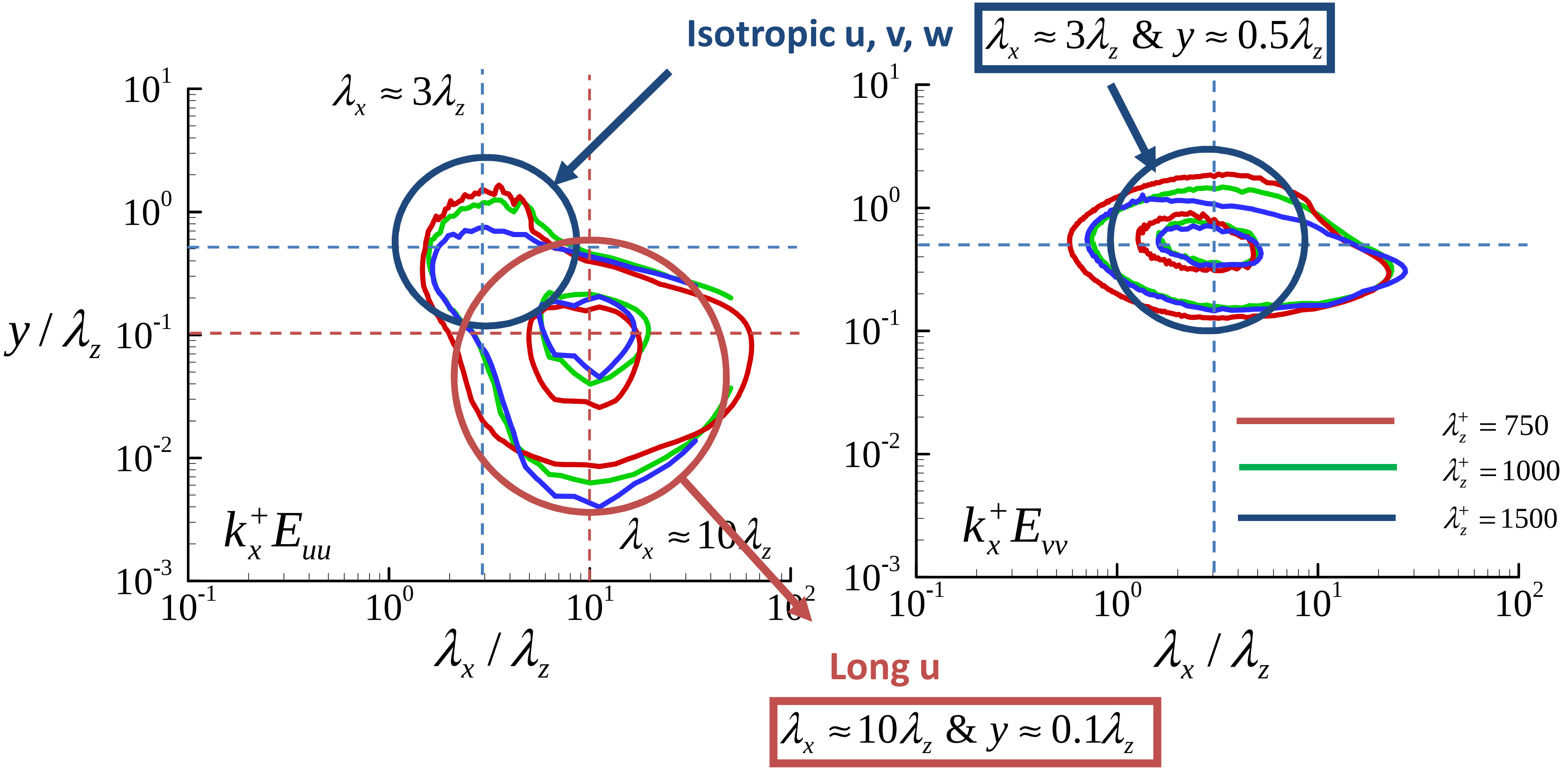


They are the Attached Eddies Townsend Has Seen




Townsend (1976)

One-dimensional Spectra of the Attached Eddies



Isotropic u, v, w $\boxed{\lambda_x \approx 3\lambda_z \ \& \ y \approx 0.5\lambda_z}$

Long u $\boxed{\lambda_x \approx 10\lambda_z \ \& \ y \approx 0.1\lambda_z}$

- **Bimodal Behavior** – **Isotropic u,v,w** & **Long u**
- **Highly Penetrating Long u** into the near-wall region

Summary III

- In the near-wall region

$\lambda_z^+ \approx 100$ →

Long u	$y^+ \approx 15$	$\lambda_x^+ \approx 1000$
Tall & short u,v,w	$y^+ \approx 50$	$\lambda_x^+ \approx 200$

- In the outer region

$\lambda_z \approx 1.5h$ →

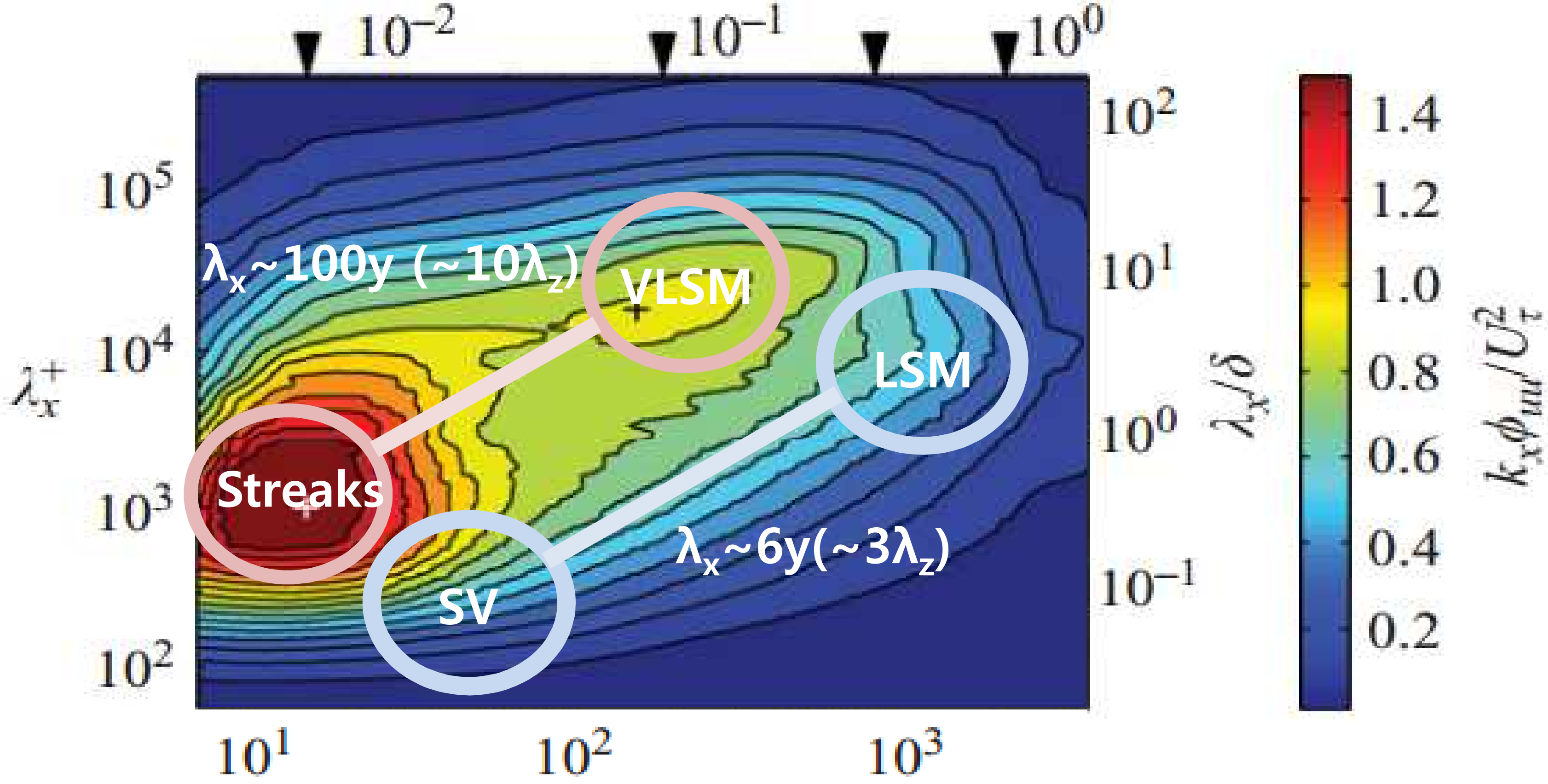
Long u	$y \approx 0.15\,h$	$\lambda_x \approx 15 \sim 20\,h$
Tall & short u,v,w	$y \approx 0.6 \sim 1h$	$\lambda_x \approx 3 \sim 4\,h$

- In the log layer

$\lambda_z \sim y$ →

Long u	$y \approx 0.1\lambda_z$	$\lambda_x \approx 10\,\lambda_z$
Tall & short u,v,w	$y \approx 0.5\lambda_z$	$\lambda_x \approx 3\lambda_z$

Interpretation of One-Dimensional Spectra



Monty et al. (2009, J. Fluid Mech.)

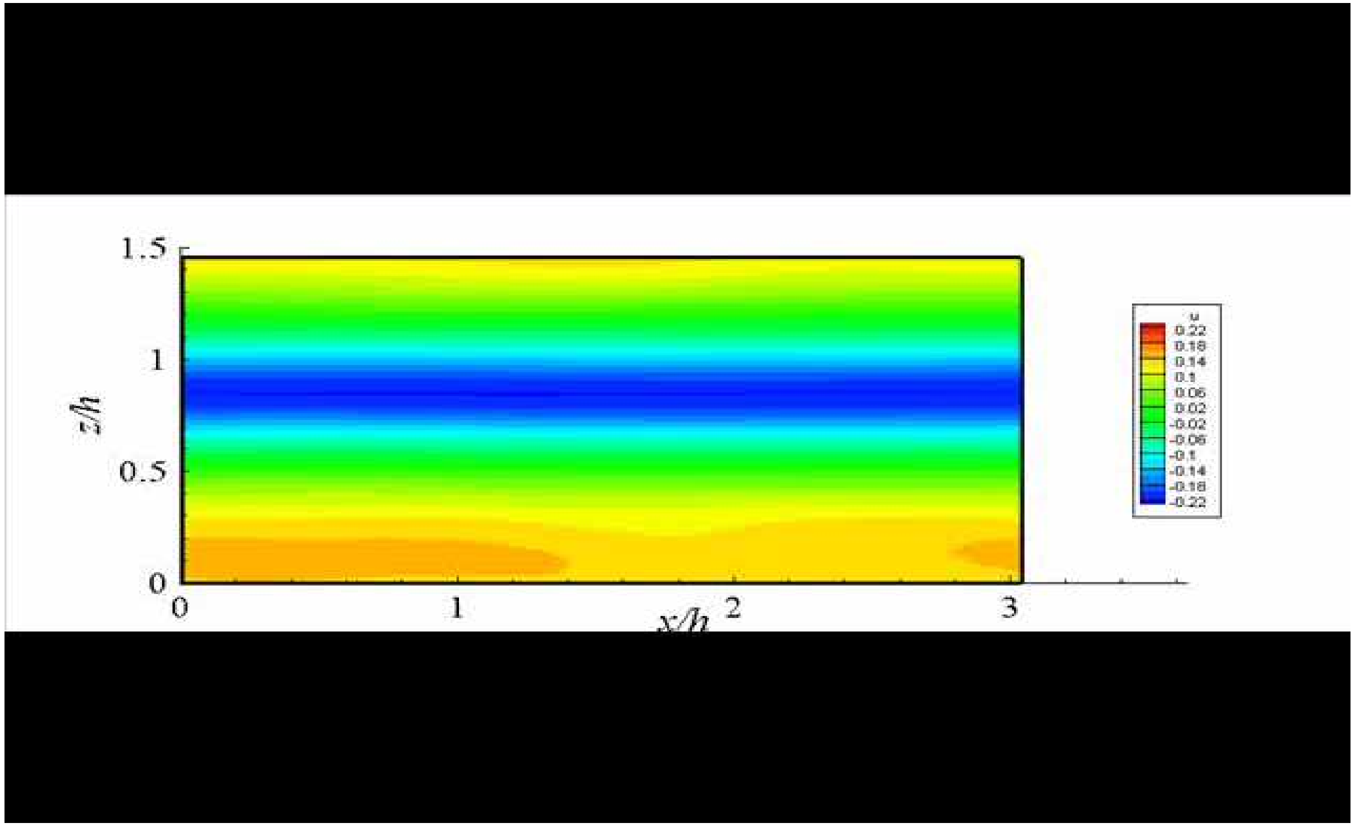
- **Bimodal** over **the entire wall-normal direction**

Isotropic u,v,w (vortices) : Near-wall SV – Log-layer Vortices – LSM

Long u (streaks) : Near-wall Streaks – Log-layer Streaks – VLSM

Minimal Box Dynamics

$$L_x / L_z \approx 2$$



at $y / h = 0.2$

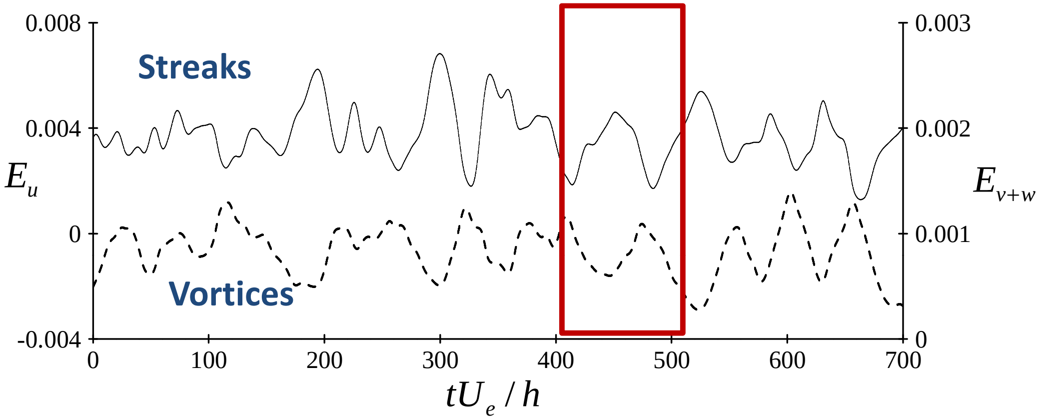
Self-Sustaining Process of the Attached Eddies

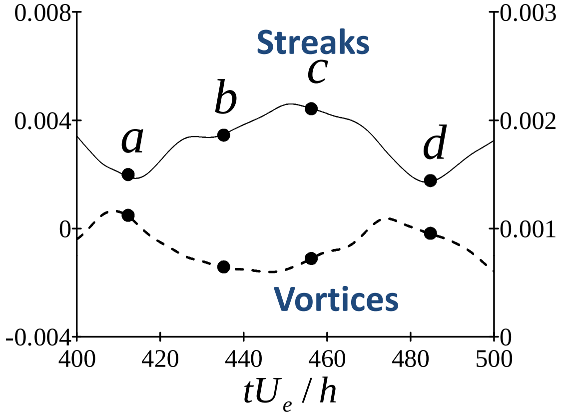
Streaks

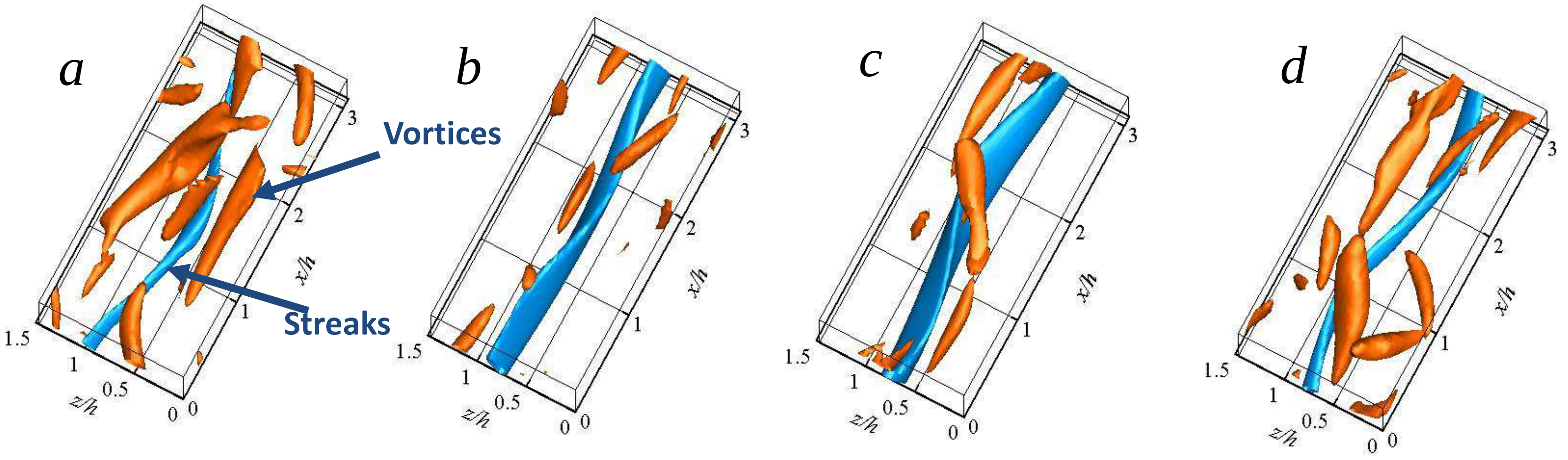
$$E_u \equiv 2/V\int_0^h |u'|^2\, dV$$

Vortices

$$E_{v+w} \equiv 2/V\int_0^h |v'|^2 + |w'|^2\, dV$$

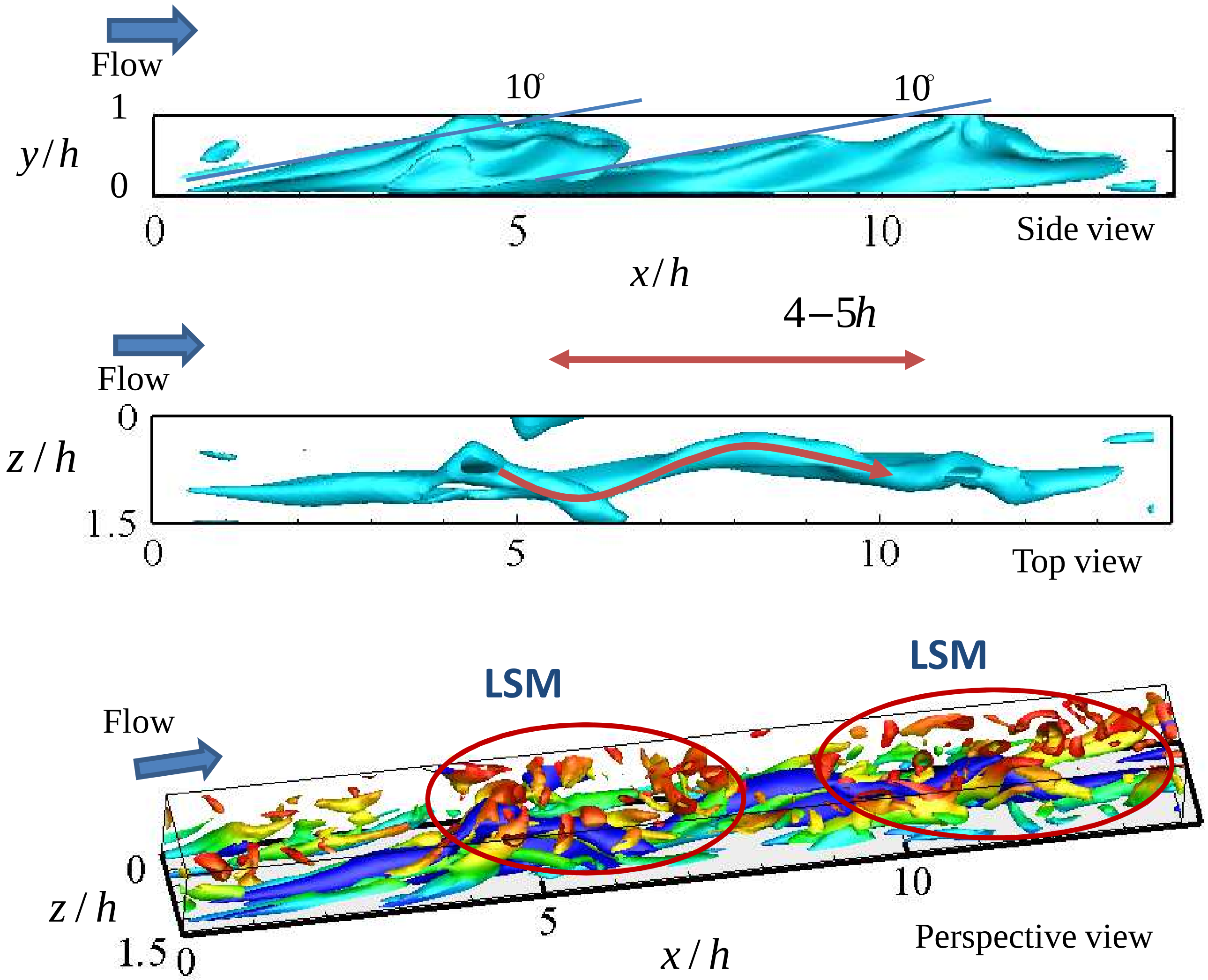






Thank you

LSMs are aligned along a VLSM



Statistics with respect to Smagorinsky const.

- One-dimensional statistics (Cs=0.05, 0.1, 0.2, 0.3)