Spatially localized solutions of plane Poiseuille and plane Couette flow

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High Reynolds Number Boundary Layer Turbulence Workshop
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Motivation

Ideology: coherent structures $=$ invariant solutions of Navier-Stokes

Many **invariant solutions** of Navier-Stokes computed 1990-present

- equilibria, traveling waves, periodic orbits, hetero & homoclinic orbits
- precise solutions of direct numerical simulations
- robust, efficient search algorithms & clever initial guesses

Physical features

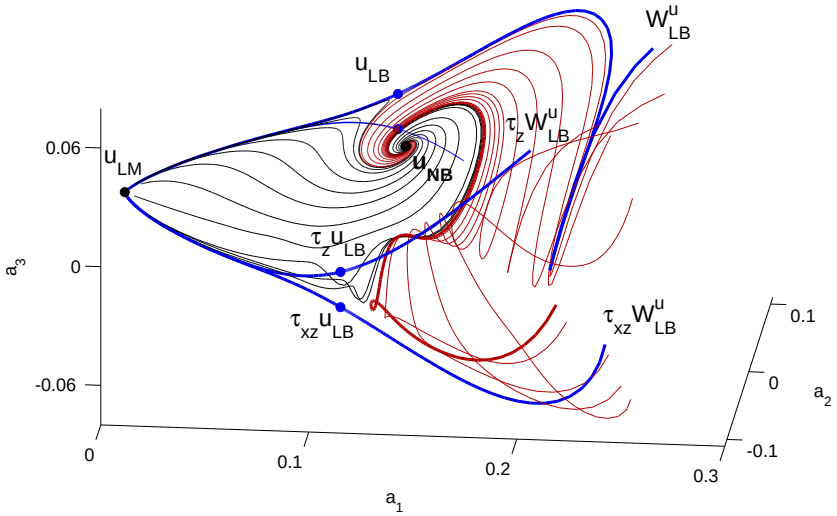
- pipes, plane Couette, channel flows
- fully 3D & nonlinear, large range Re, only a few unstable modes
- replicate large-scale flow structures, statistics, bursts

Dynamical systems theory motivation

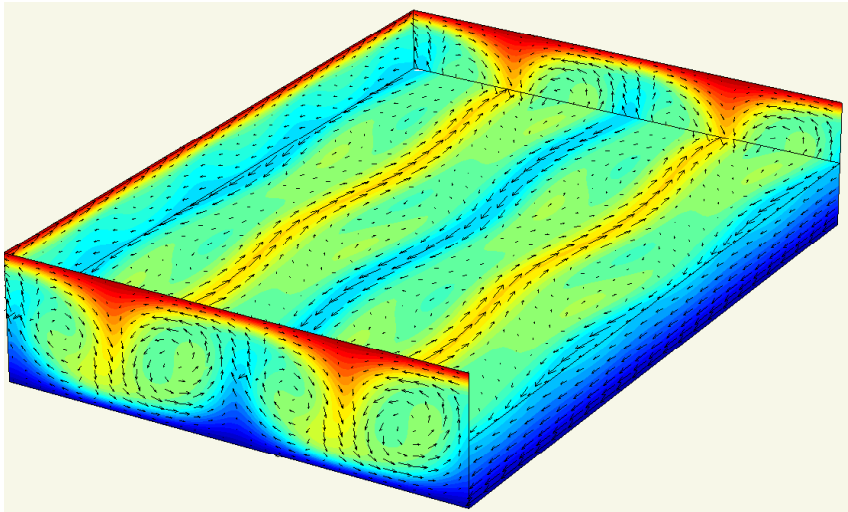
- invariant solutions organize state-space dynamics
- low-d instabilities produce low-d attracting set
- turbulent dynamics $=$ chaotic walk among low-d unstable solutions

Solutions enable precise nonlinear dynamical analysis of turbulence

Example: plane Couette flow, minimal flow unit, $Re = 400$



state-space portrait



periodic orbit, 10 unstable eigenvalues

In this example, invariant solutions

- capture roll-streak structure, bursting, self-sustaining process
- low-d instabilities produce low-d attracting set
- turbulent dynamics $=$ chaotic walk among low-d unstable solutions

Motivation

But low-Re minimal flow unit plane Couette is **highly constrained**.

- small periodic domains
- single flow structure spans entire domain
- "turbulent" flow is simple and repetitive

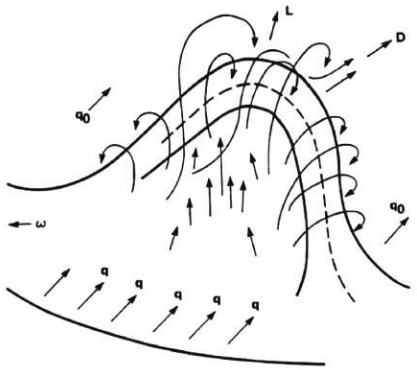
Goals of present work are to find

- **spatially localized** invariant solutions of extended flows
  - span and streamwise localization
  - concentration near walls
- invariant solutions corresponding to **boundary layer** structures

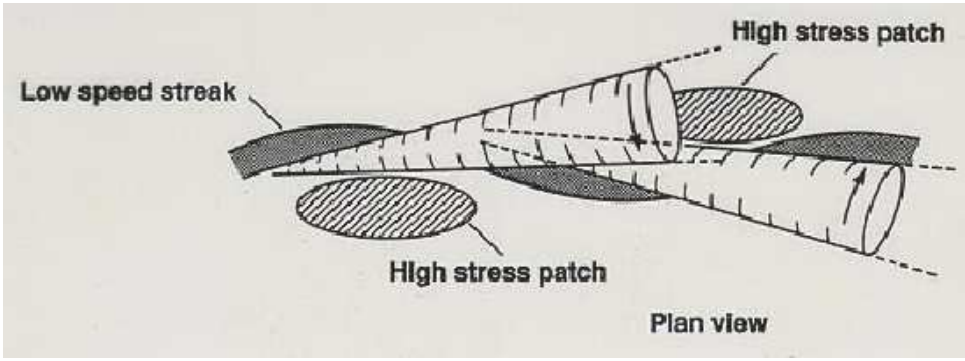
Related work: spatially localized . . .

- EQBs and TWs of plane Couette, Schneider et al. JFM 2010, PRL 2010
- edge state of developing boundary layer, Duguet et al. PRL 2012
- periodic orbits of asymp. suction boundary layer, Eckhardt talk, Friday

Boundary layer structures



Theodorsen (1952)



Stretch (1990)

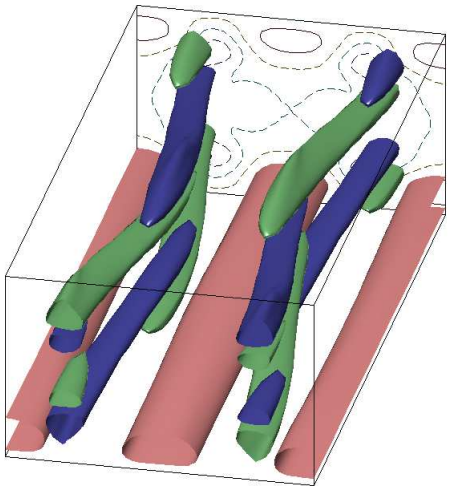
Features to replicate

- tilted rolls over low-speed streaks, flanked by high-speed streaks
- coherence is spatially localized, concentrated near walls
- streamwise sinuous and spanwise mirror symmetry

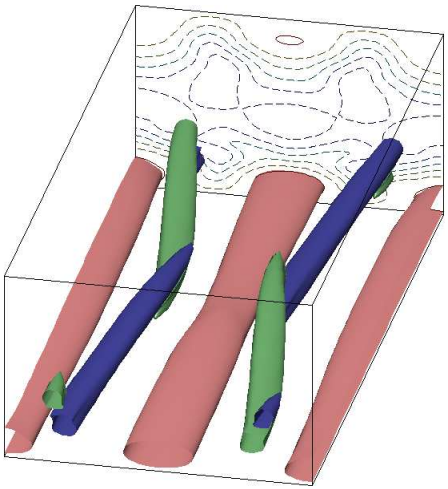
Exact localized traveling waves of channel flow

Gibson & Brand arXiv:1304.6323v1, submitted to JFM

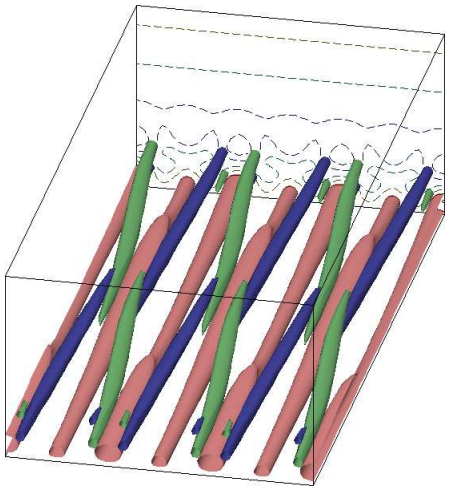
Exact traveling waves of channel flow: spatially periodic



TW1



TW2

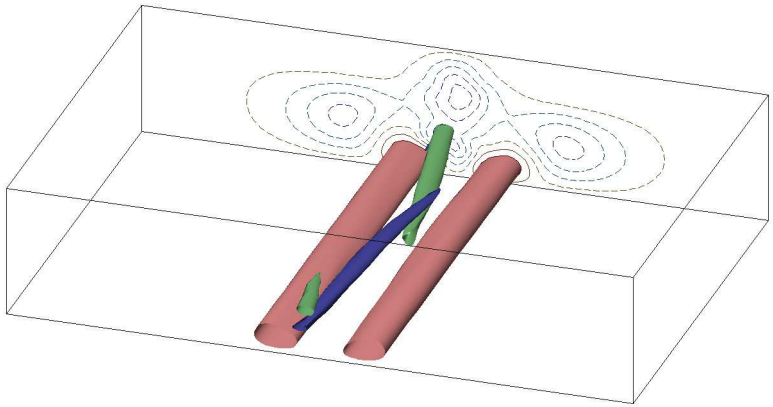


TW3

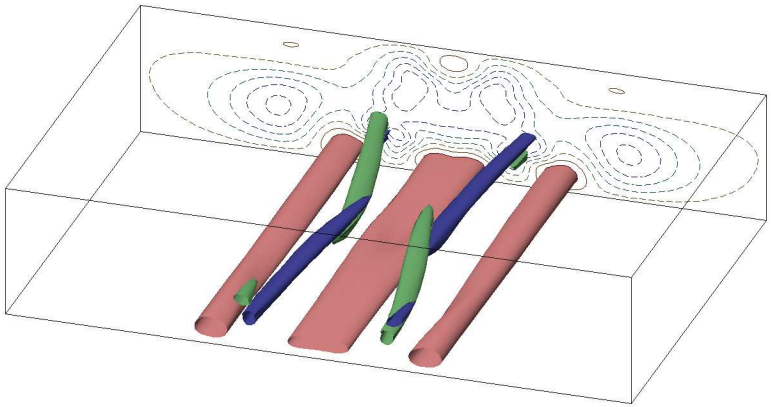
Visualization

- blue/green: $\circlearrowleft$ / $\circlearrowright$ signed swirling strength isosurfaces
- red: high-speed streamwise streak (low-speed streaks between)
- dashed/solid contours: $-/+$ streamwise velocity, relative to laminar

Exact traveling waves of channel flow: spatially localized



TW2-1



TW2-2

Features

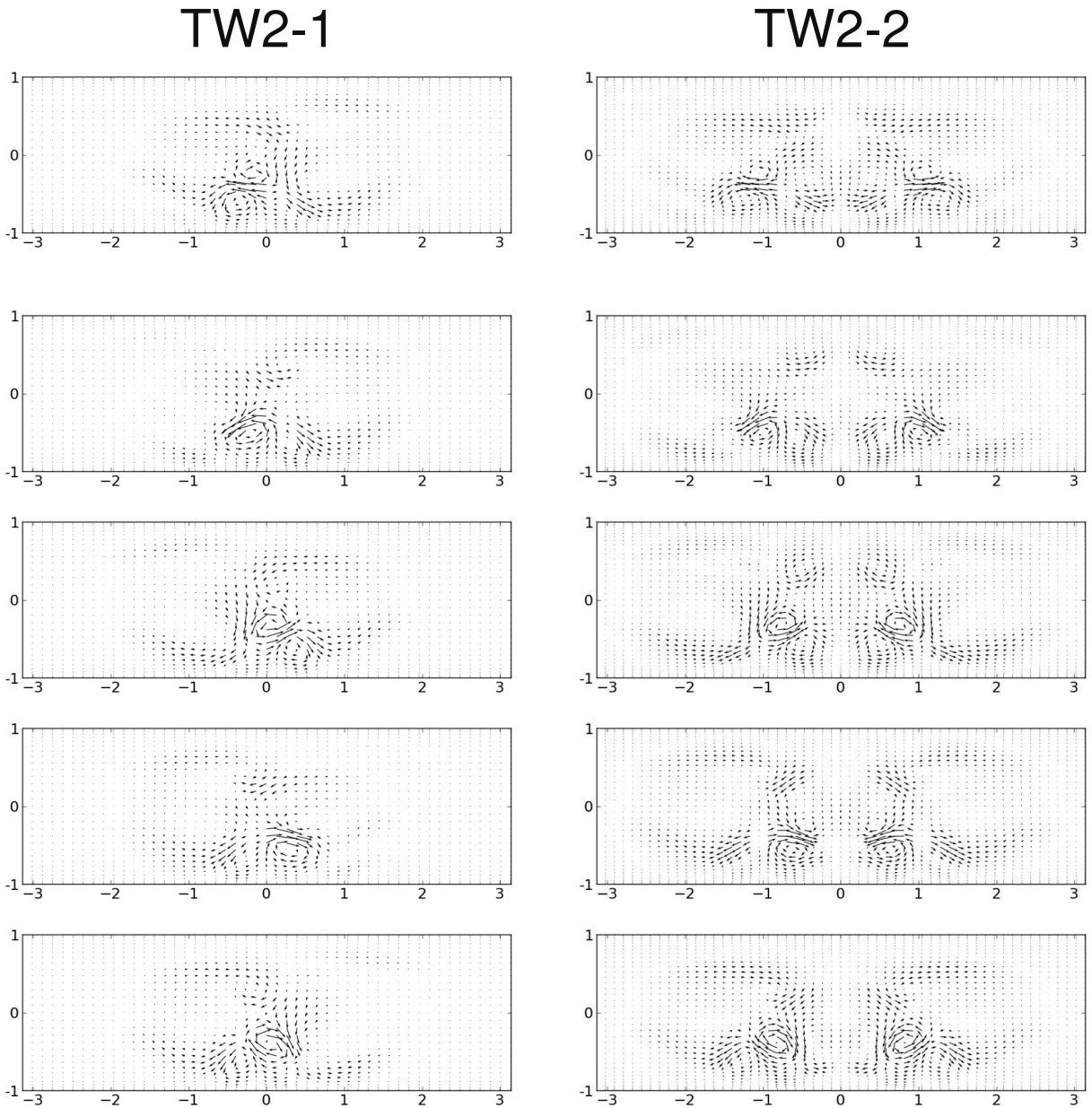
- concentrated, alternating, tilted, near-wall streamwise rolls
- centered over low-speed streaks, flanked by high-speed streaks
- large streamwise velocity deficit in core, relative to laminar

Computed from periodic TW2 solution by

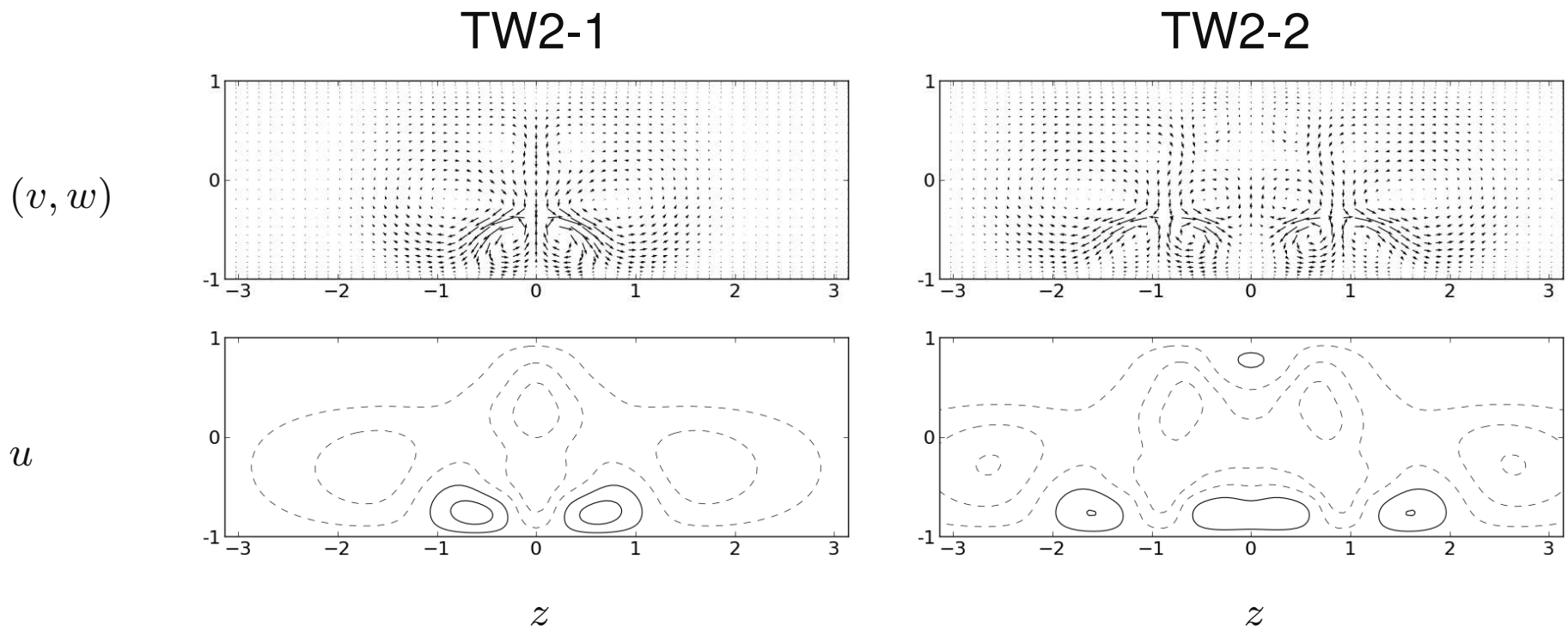
- applying $\tanh$-based windowing function to TW2 in different phases
- refining windowed initial guess with Newtown-Krylov-hookstep

TW2-1, TW2-2: cross-stream velocity slices

increasing x



TW2-1, TW2-2: streamwise-averaged velocity

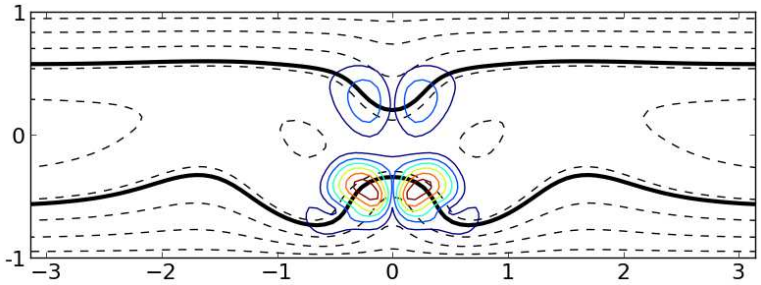


dashed/solid contours: $-/+$ streamwise velocity relative to laminar

- localized counter-rotating mean vortices
- high/low-speed streaks via lift-up
- large velocity deficit region in core

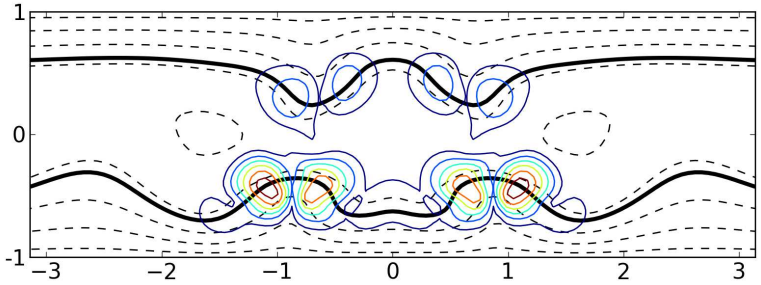
TW2-1, TW2-2: critical layers

TW2-1



$Re = 2300$

TW2-2



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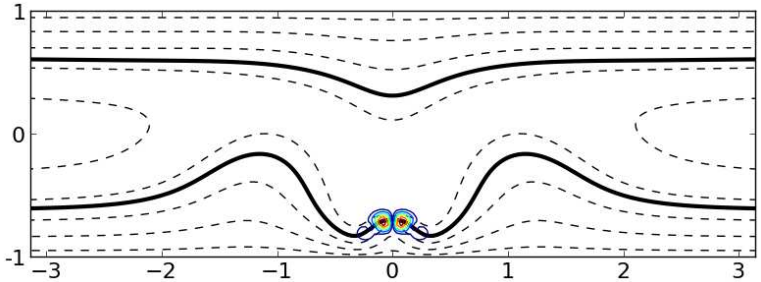
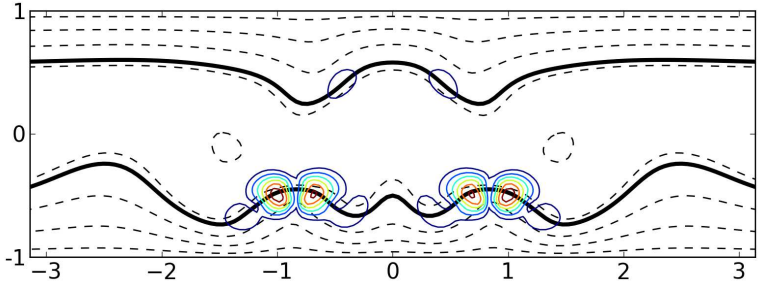


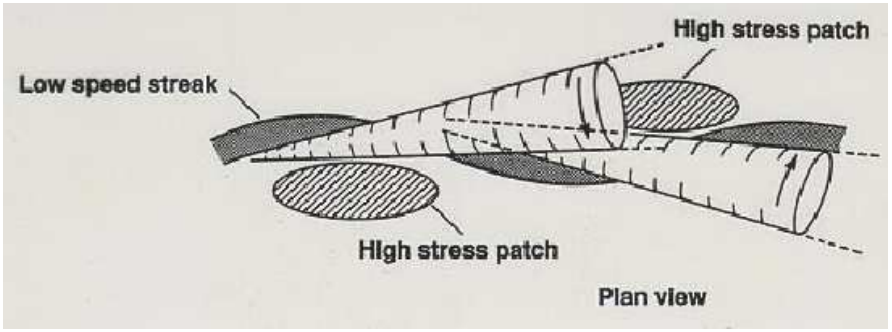
$Re = 20000$



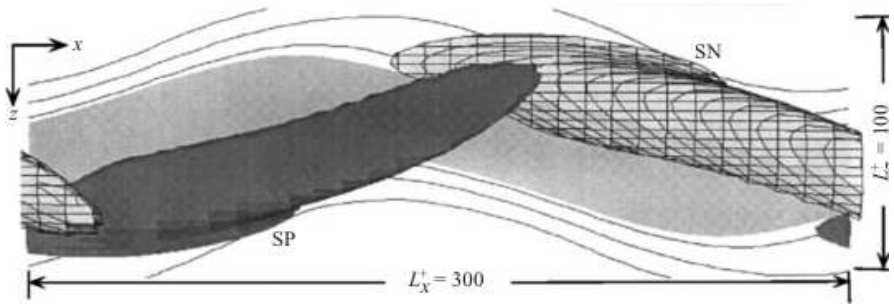
$Re = 5500$

dotted: total streamwise velocity
thick: critical layer $\langle u_{\text{tot}} \rangle_x(y, z) = c_x$
colored: magnitude of fundamental streamwise Fourier mode

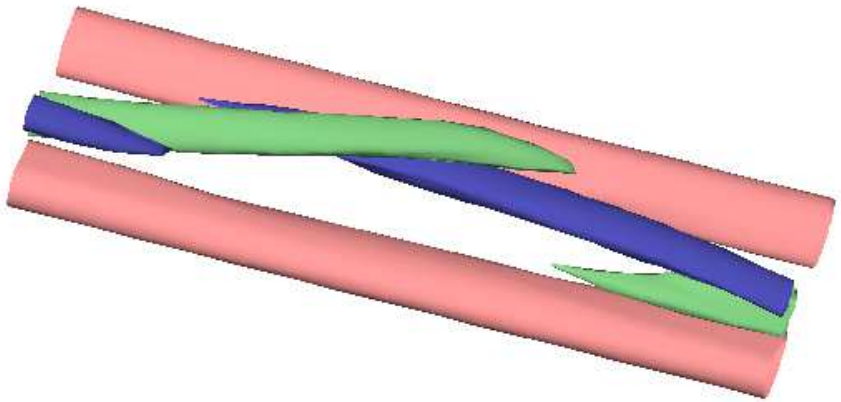
TW2-1: comparison to sinuous boundary-layer structures



Stretch (1990)
educed from DNS data



Schoppa & Hussain (2002)
transient growth mode

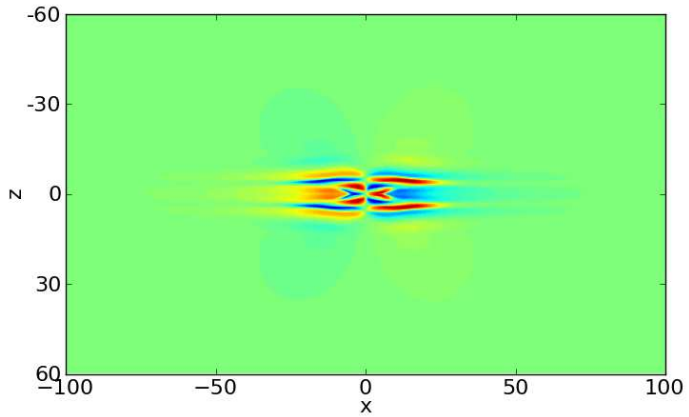


TW2-1: exact traveling wave
same orientation of swirling, wall-unit length scales

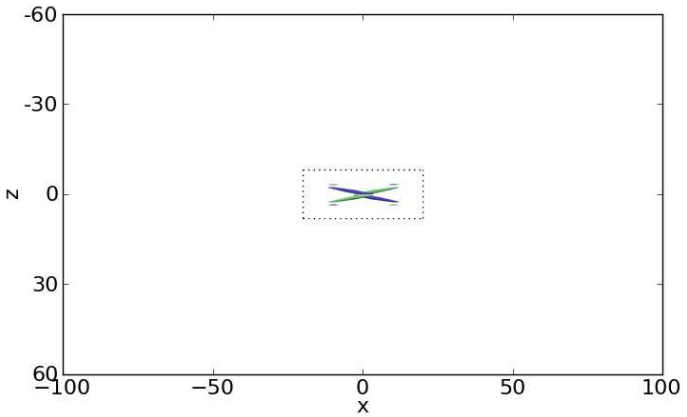
Doubly-local equilibrium of plane Couette flow

Brand & Gibson, in preparation

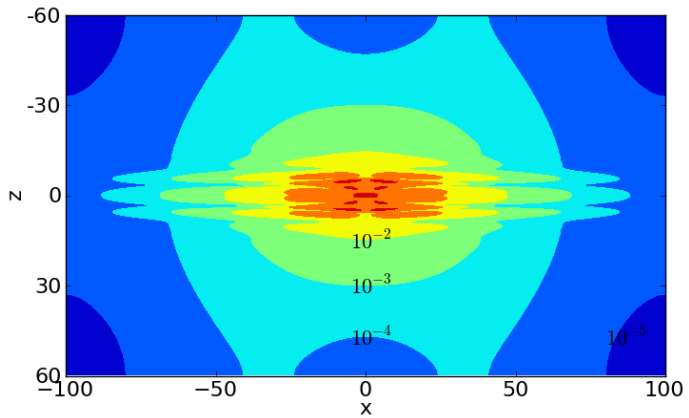
Doubly-local equilibrium of PCF: global flow



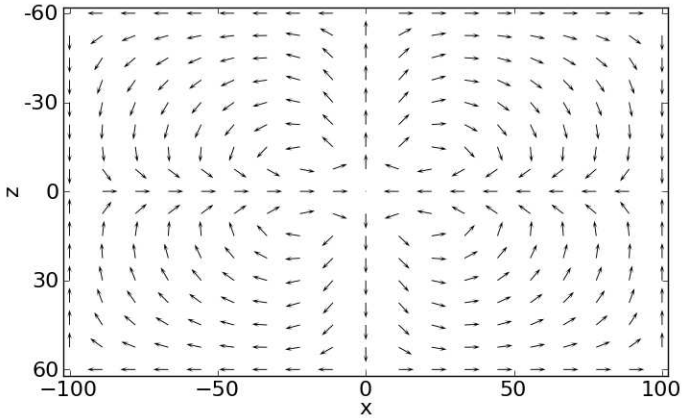
midplane streamwise velocity



swirling strength



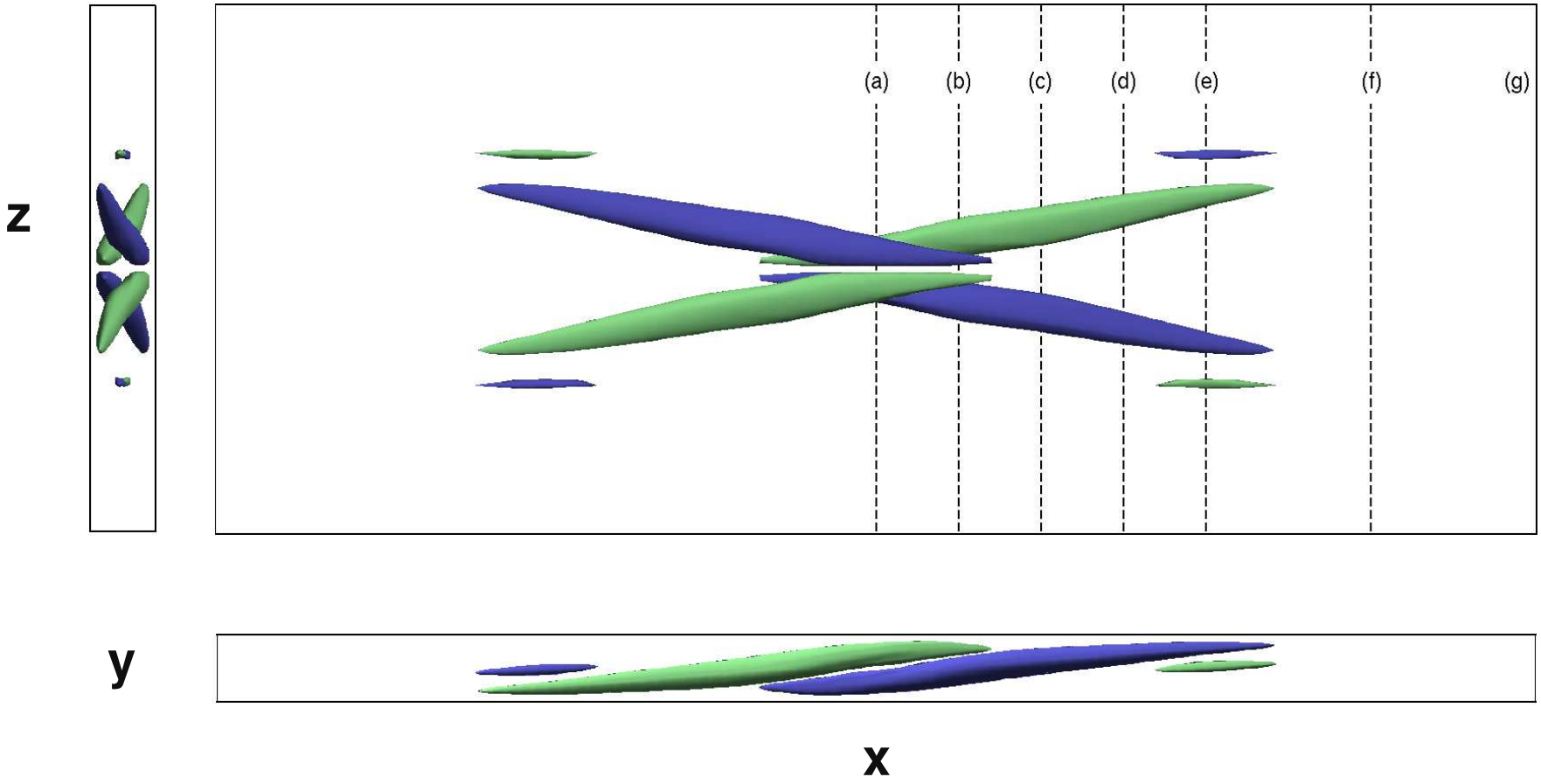
y-avg energy



normalized y-avg (u,w) flow

small nonlaminar spot decaying exponentially to laminar flow

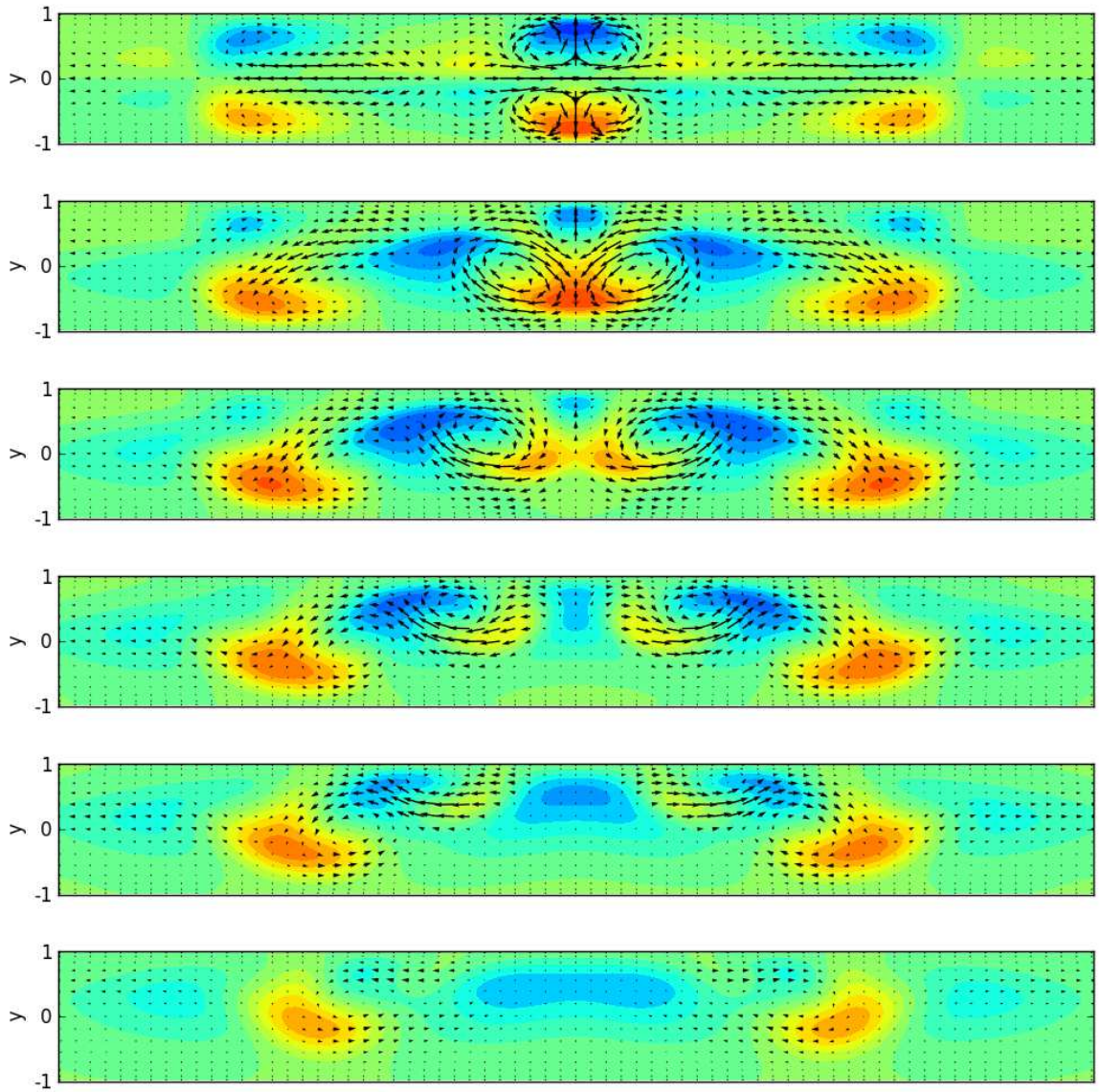
Doubly-local equilibrium of PCF: detail



isolated pair of symmetrically-opposed lambda vortices

blue/green: ↺ / ↻ signed swirling strength

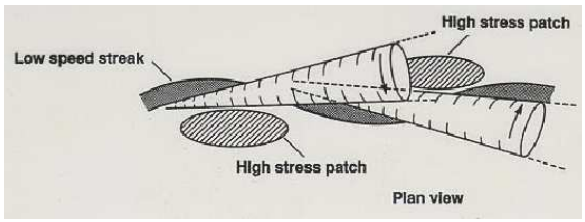
Doubly-local equilibrium of PCF: cross-stream slices



arrows: cross-stream (v,w) flow, color: streamwise u (red/blue = +/-)

Conclusions

- exact invariant solutions exist for flows beyond confined domains
- spanwise localized, near-wall traveling waves of plane Poiseuille
- doubly-localized equilibrium of plane Couette
- analysis of exponential decay rates of tails (in papers)
- suggestive similarity of TW2-1 to boundary-layer structure



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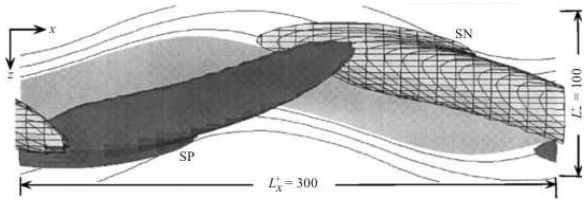

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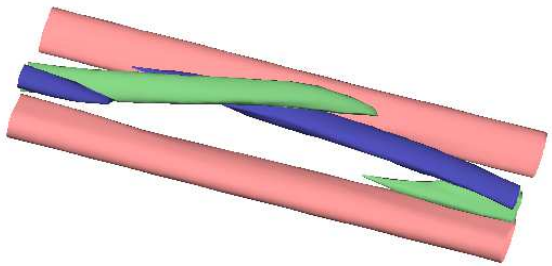

TW2-1

exact traveling wave

Questions

- Is there hope for understanding turbulence in extended flows as sets of dynamically coupled structures?
- What's the significance of embedding a localized structure in a background of laminar flow, versus a background of turbulent flow?
- What are theoretical limitations of computing invariant solutions, as spatial structure and dynamic complexity increase with Re?

Acknowledgments

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- Tobias Schneider (Max Planck Institut Göttingen)

Please send me your talks!

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or

via USB stick