An idealised assessment of Townsend's outer-layer similarity hypothesis for wall turbulence

University of New Hampshire Boundary Layer Workshop
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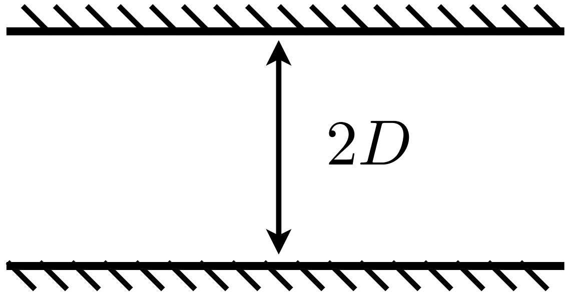
University of Melbourne

An idealised assessment of Townsend's outer-layer similarity hypothesis for wall turbulence

Reynolds number similarity (Townsend 1956, p.89):

In a fully turbulent flow, there exists a region including almost all of the flow, over which the direct action of viscosity on the mean flow is negligible, i.e. the Reynolds stresses are large compared with the mean viscous stresses. Within this region, the mean motion and the motion of the energy-containing components of the turbulence are determined by the boundary conditions of the flow alone, and are independent of the fluid viscosity, except so far as a change in the fluid viscosity may change the boundary conditions.

Townsend's hypothesis



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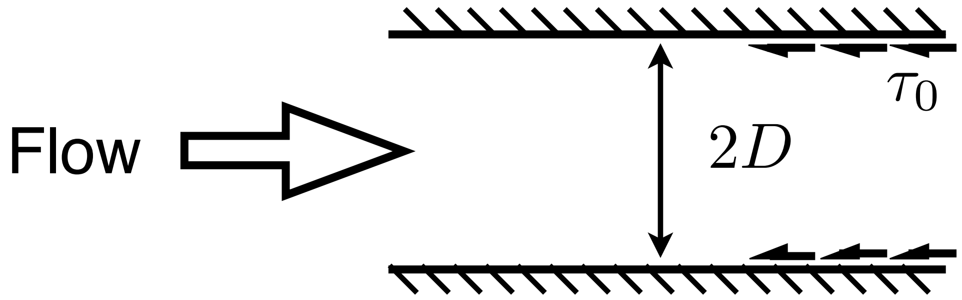

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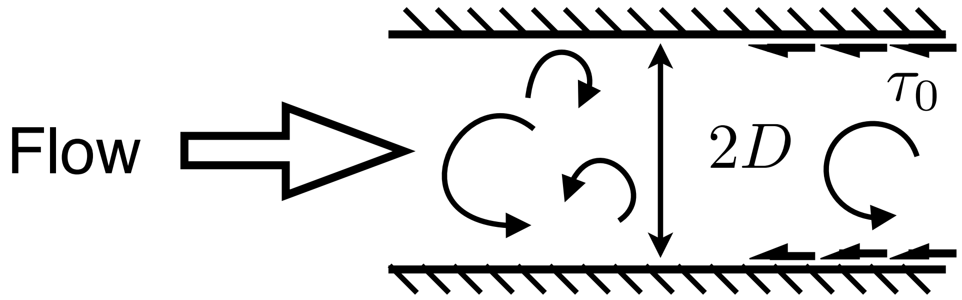
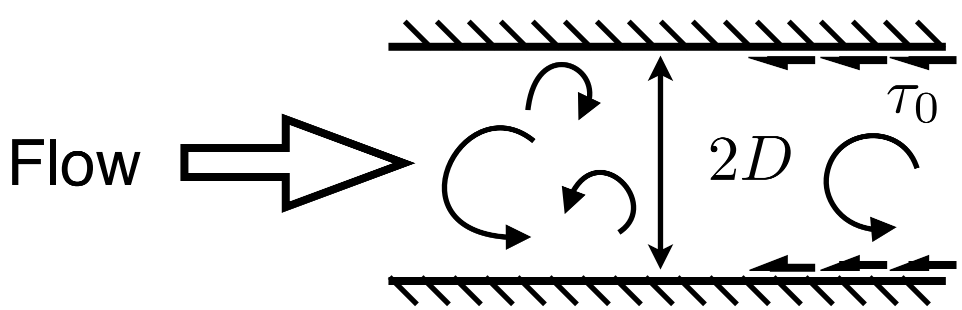
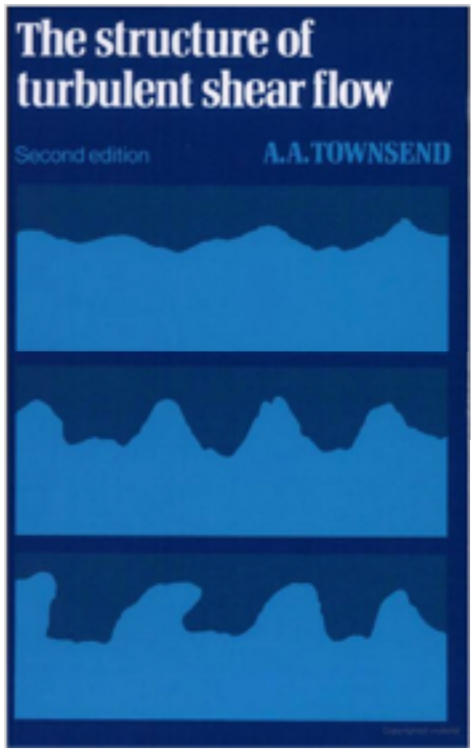


Townsend's hypothesis

the relative motion in the fully turbulent region depends only on the wall stresses and on the channel width.





(1976, Ch. 5)

Townsend's hypothesis

Outer layer

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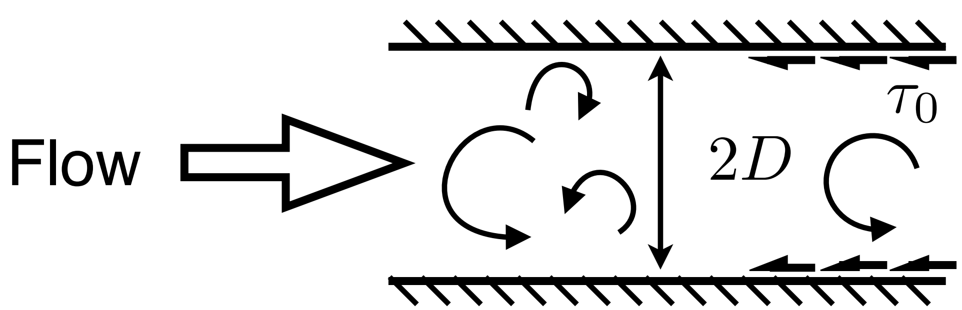

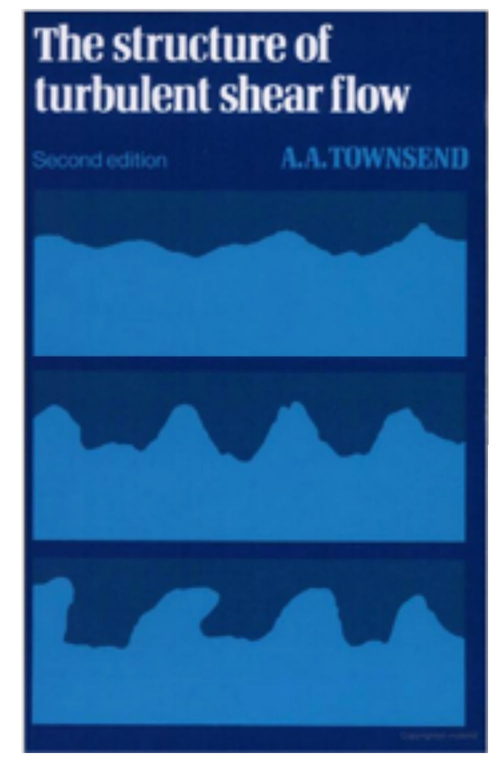

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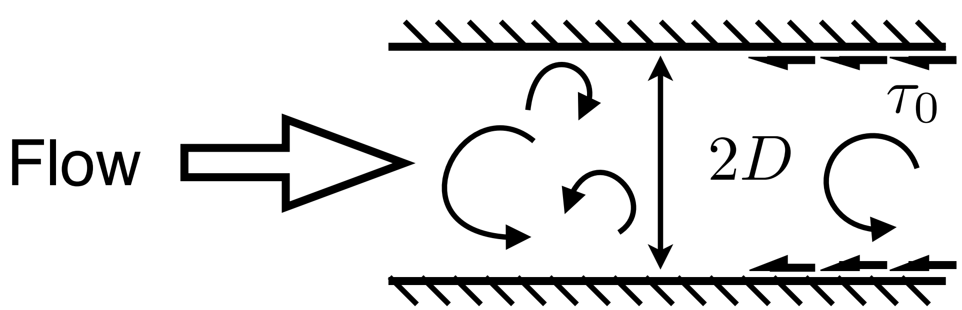
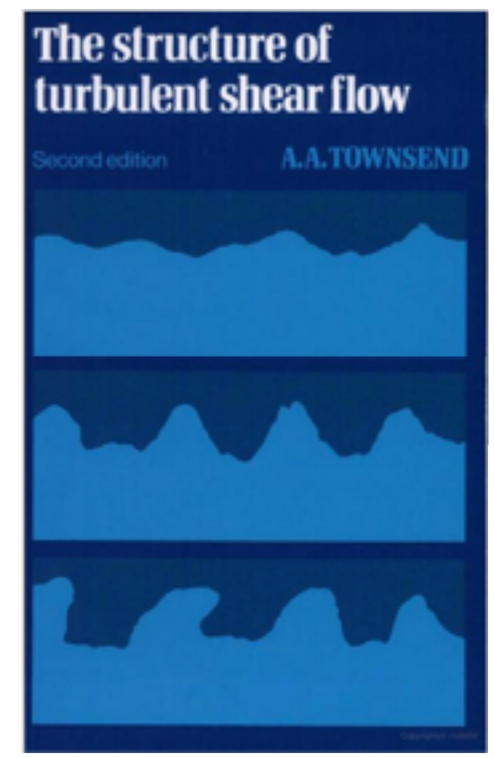
τ_0 (Kinematic stress)

D half-





(1976, Ch. 5)

Townsend's hypothesis

Outer layer

statistics of turbulent

the fully turbulent region depends only on the wall stresses and on the channel width.

the relative motion in the fully turbulent region depends only on the wall stresses and on the channel width.

τ_0

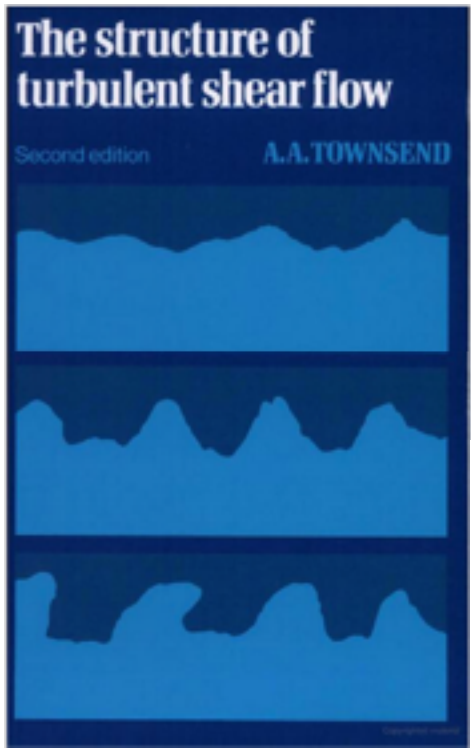
D

2nd-order

$$R_{ij}(z;\mathbf{r}) = \tau_0 R_{ij}{}^{*}[z/D;\mathbf{r}/D]. \qquad (5.3.5)$$

1st-order

$$\frac{U(D)-U}{\tau_0{}^{1/2}} = f(z/D), \qquad (5.3.6)$$



(1976, Ch. 5)

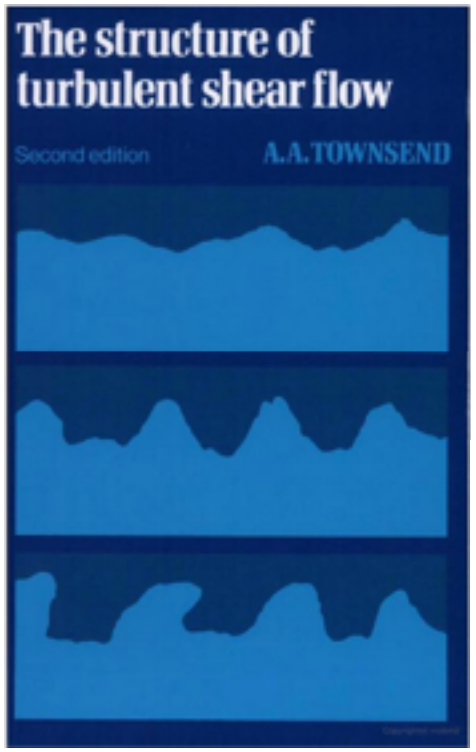
Townsend's hypothesis

statistics of turbulent
relative motion

Outer layer

τ_0

D



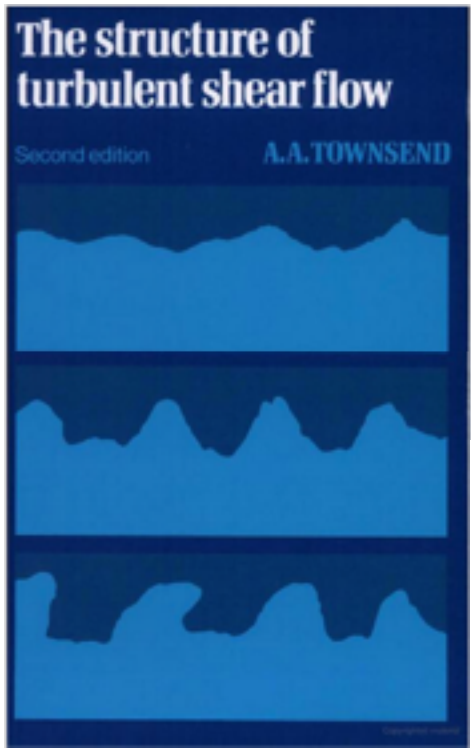
(1976, Ch. 5)

Townsend's hypothesis

Outer-layer

statistics of turbulent relative motion

depend only on D and τ_0.



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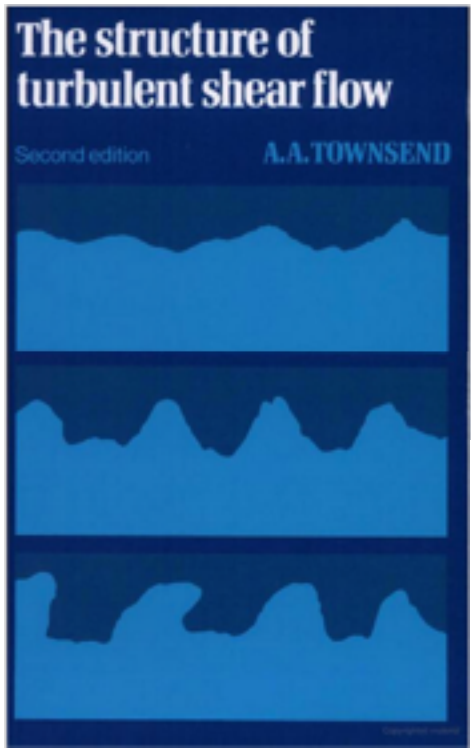

(1976, Ch. 5)

Townsend’s hypothesis

fully turbulent flow where the viscous
stresses are small compared with the Reynolds stresses, and this

/inertial

Outer-layer

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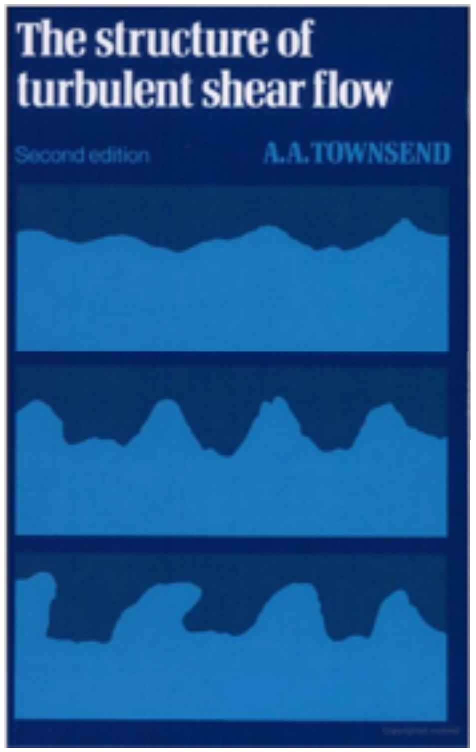

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Townsend’s hypothesis

Outer-layer (where Reynolds stresses dominate)

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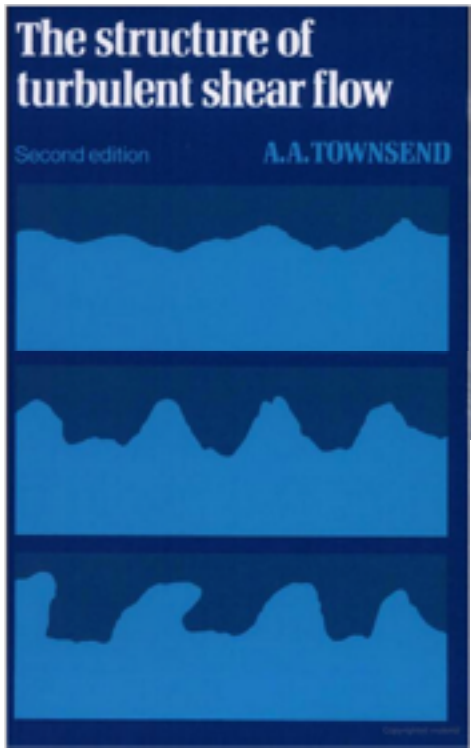

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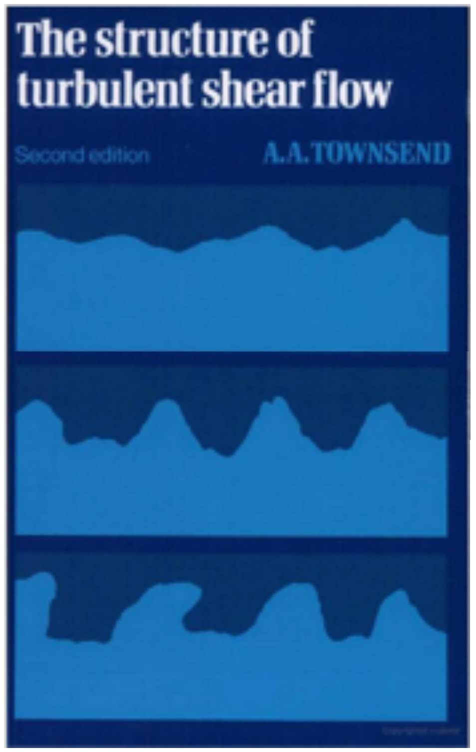
Turbulent flows bounded by rigid – or, at any rate, nearly immovable – walls



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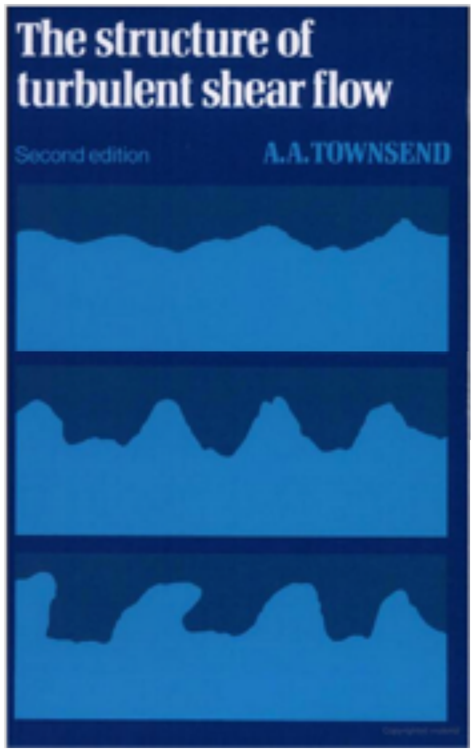

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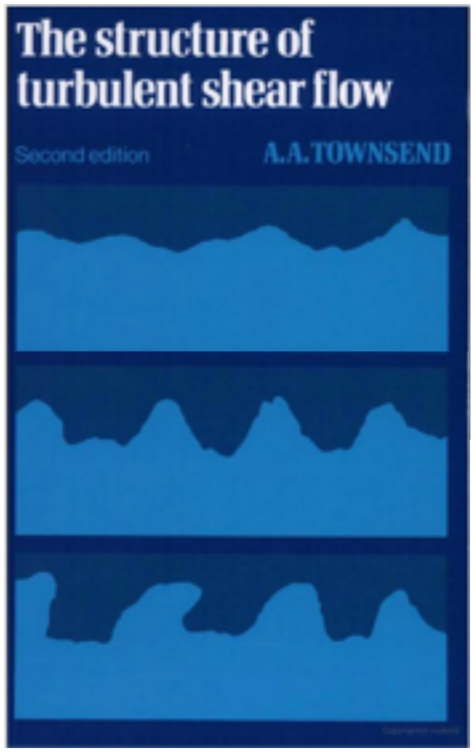

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Application of Townsend's hypothesis

Outer-layer (where Reynolds stresses dominate)

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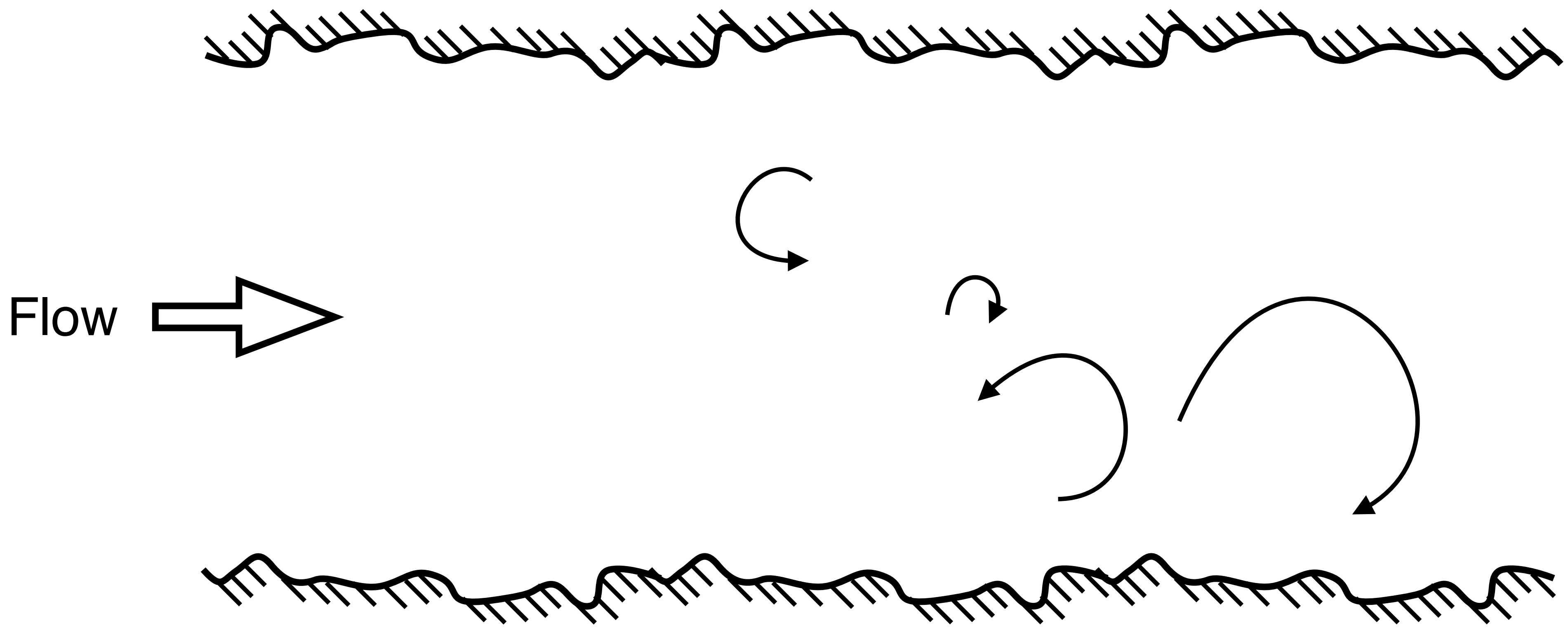

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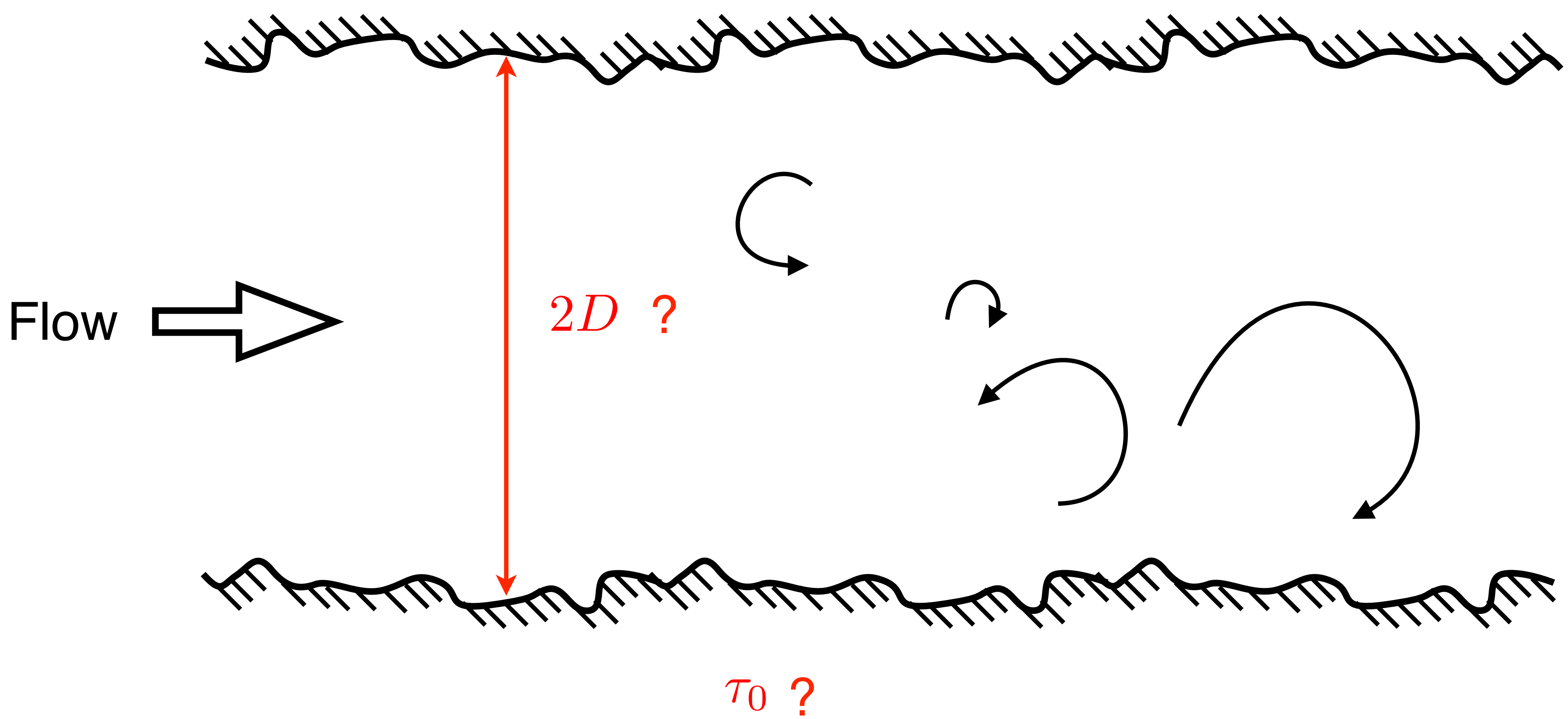

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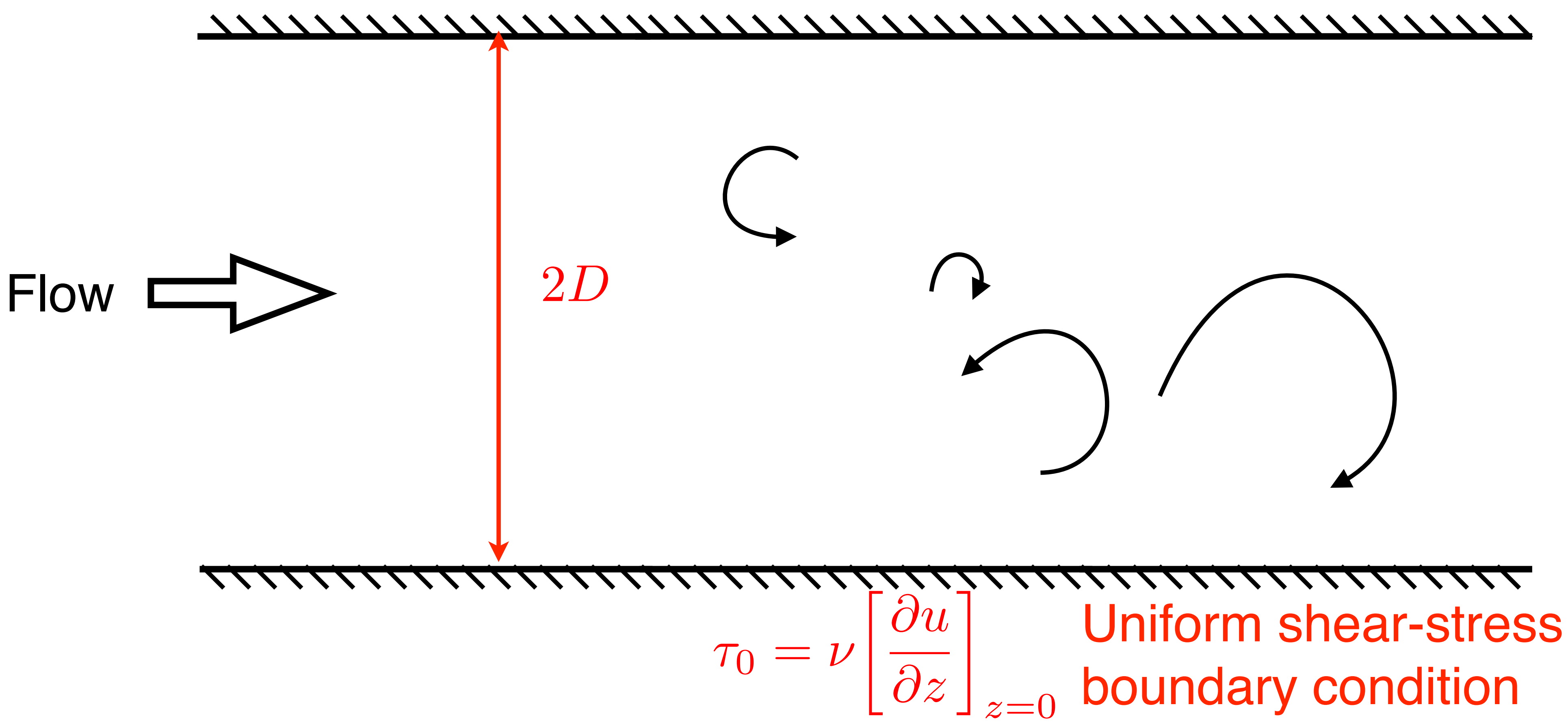
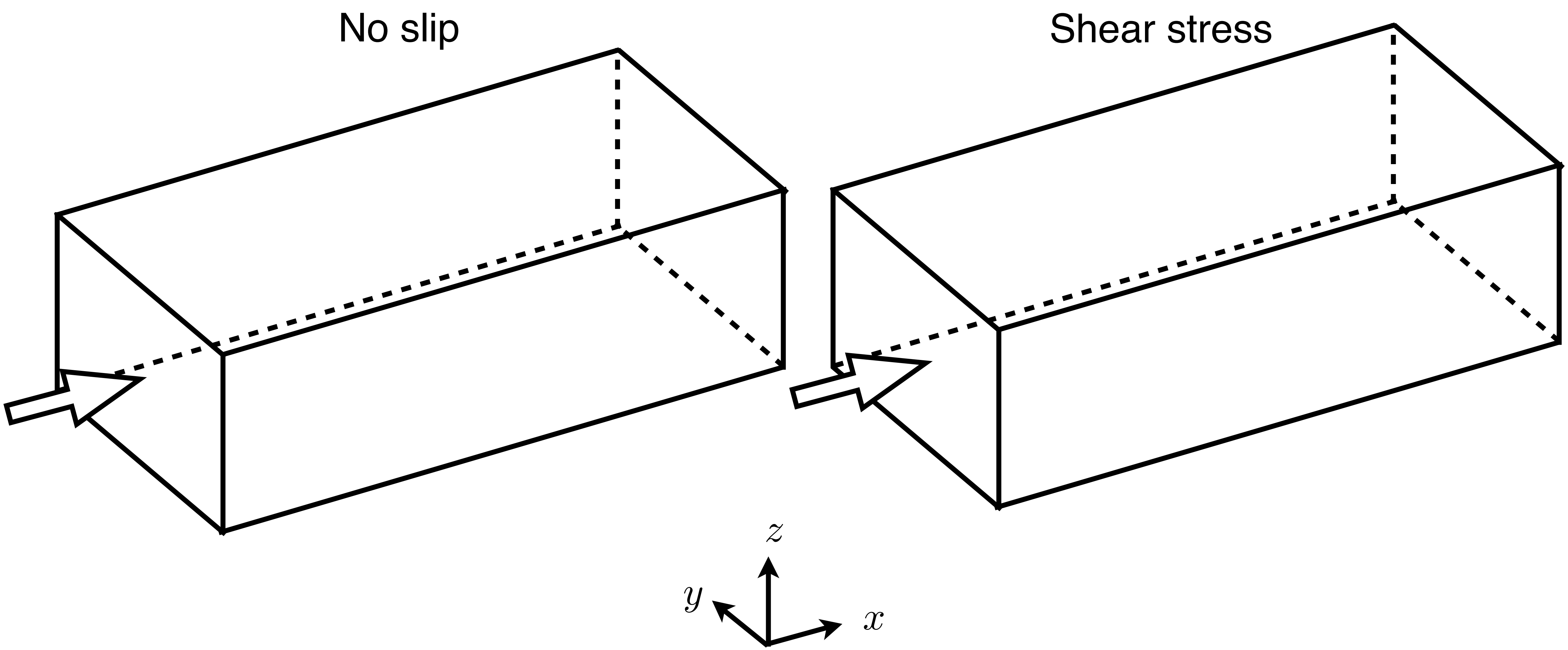


$$\tau_0 = \nu \left[\frac{\partial u}{\partial z} \right]_{z=0}$$

Idealised assessment: no-slip versus shear-stress boundary conditions



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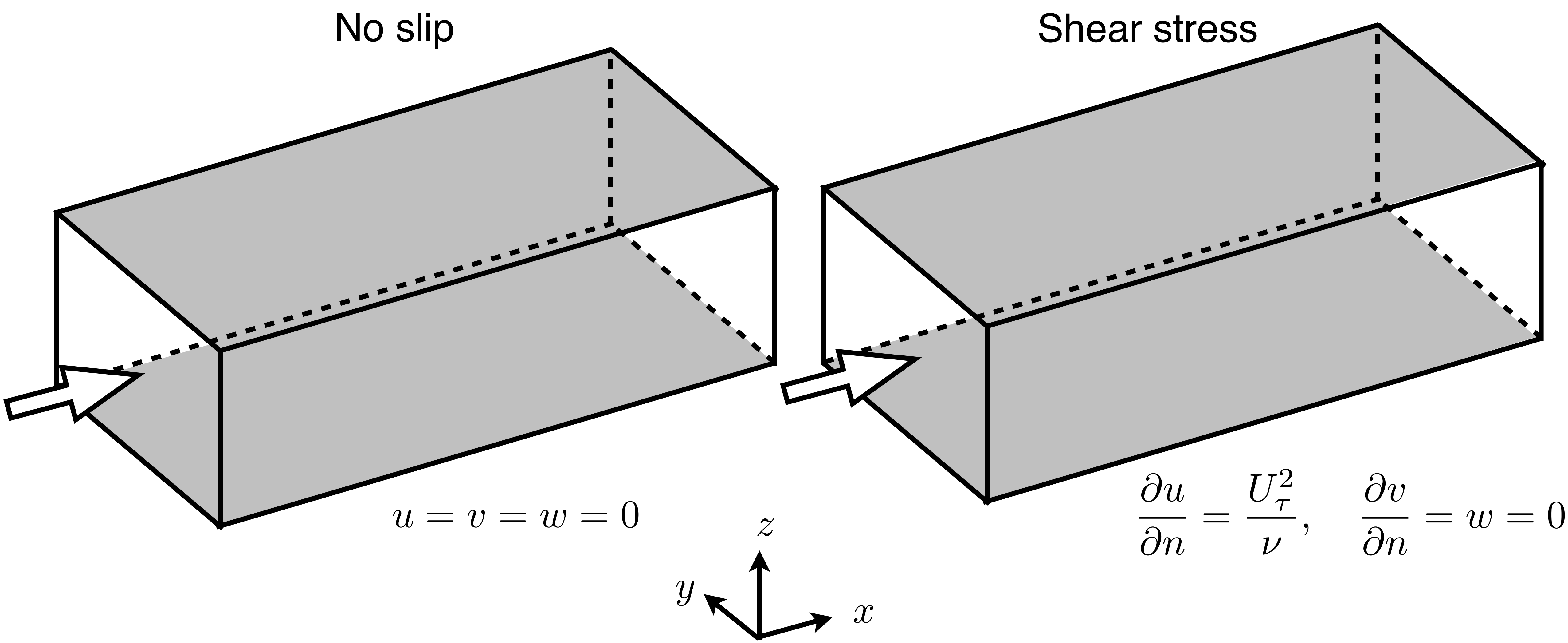

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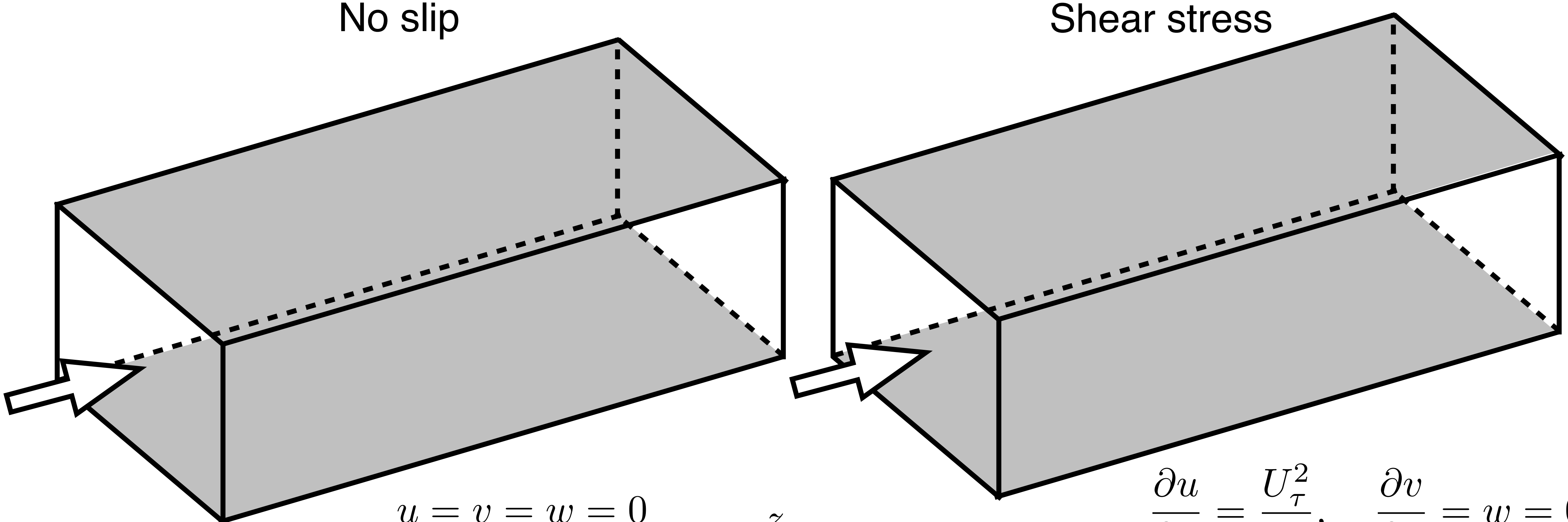
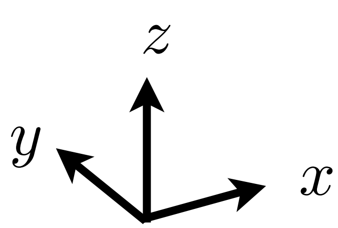


No slip:

$$u = v = w = 0$$

$$\frac{\partial u}{\partial n} \neq \frac{U_\tau^2}{\nu}, \qquad \frac{\partial v}{\partial n} \neq 0$$



Shear stress:

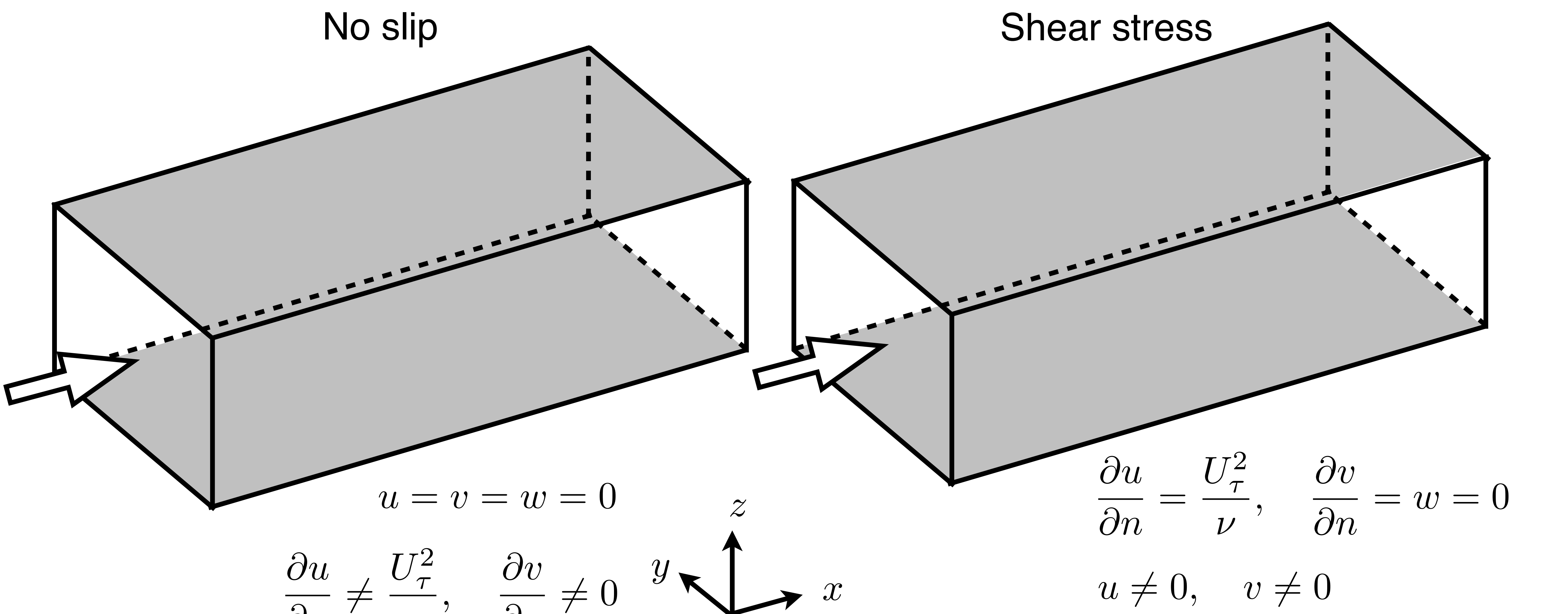
$$\frac{\partial u}{\partial n} = \frac{U_\tau^2}{\nu}, \qquad \frac{\partial v}{\partial n} = w = 0$$

$$u \neq 0, \quad v \neq 0$$

Flow is Galilean invariant:

$$u_1 = U_t + u$$

Idealised assessment: no-slip versus shear-stress boundary conditions



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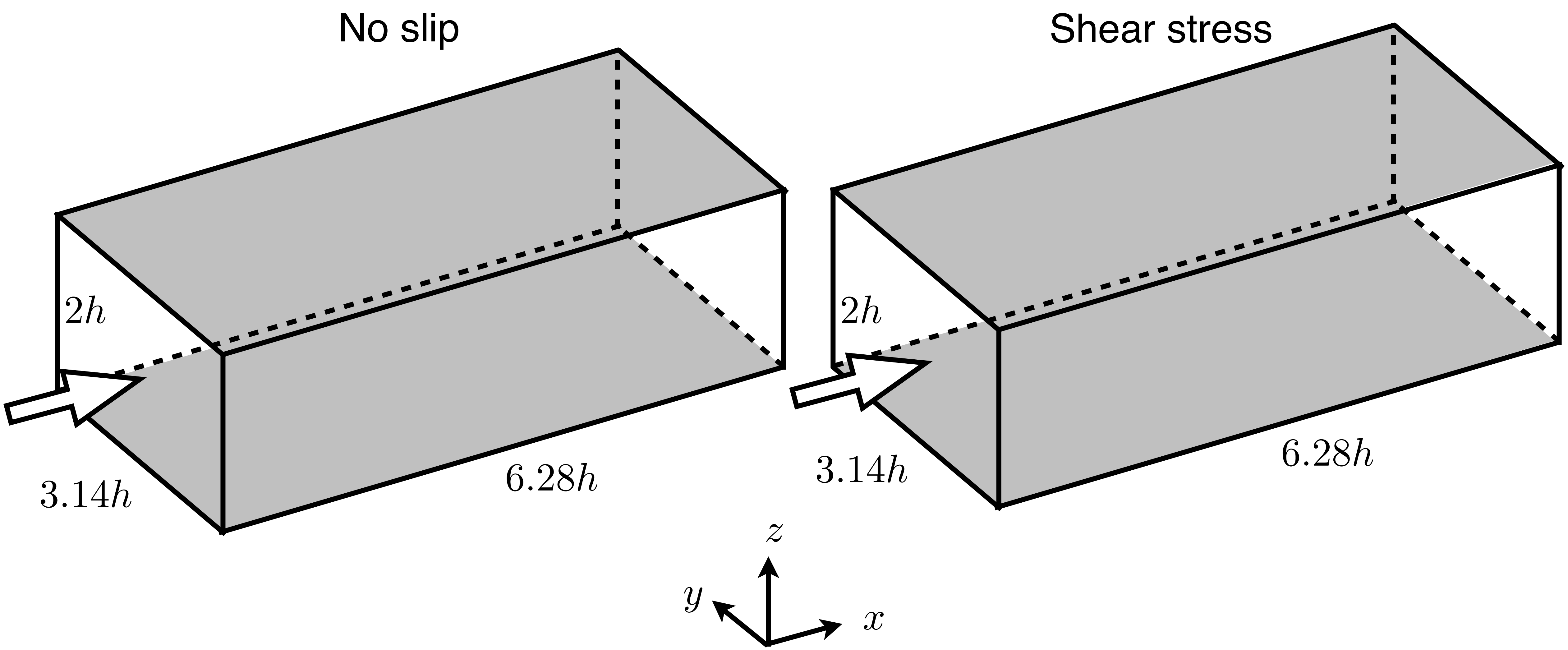
$$u \neq 0, \quad v \neq 0$$

$$\frac{\mathrm{d}U}{\mathrm{d}z} = \frac{U_\tau^2}{\nu}, \quad \frac{\mathrm{d}V}{\mathrm{d}z} = 0$$

Idealised assessment: no-slip versus shear-stress boundary conditions



Simulation setup identical to Moser *et al.* (1999):

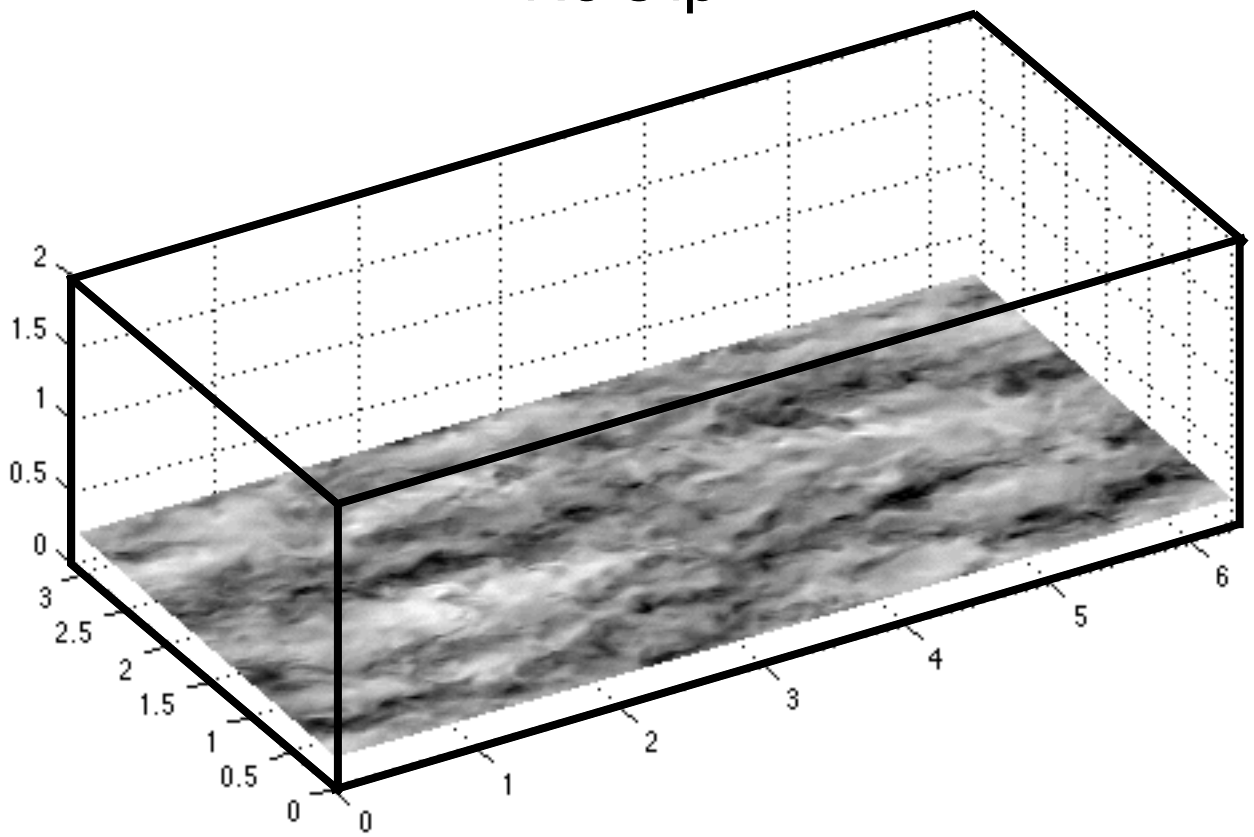
$D = h$

Re_τ	Δx^+	Δy^+	Δz_c^+
590	9.7	4.8	7.2

All statistics checked with Moser *et al.* (1999).

Streamwise velocity visualisations

No slip



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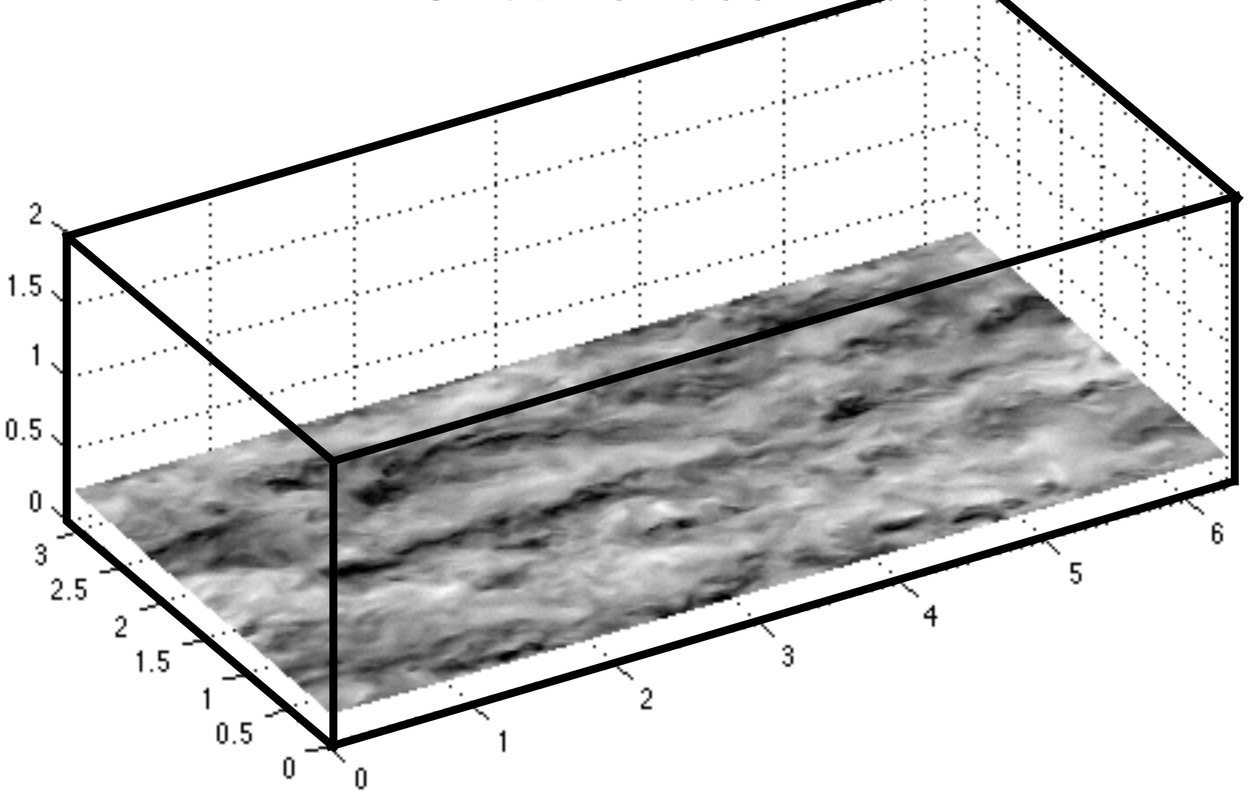
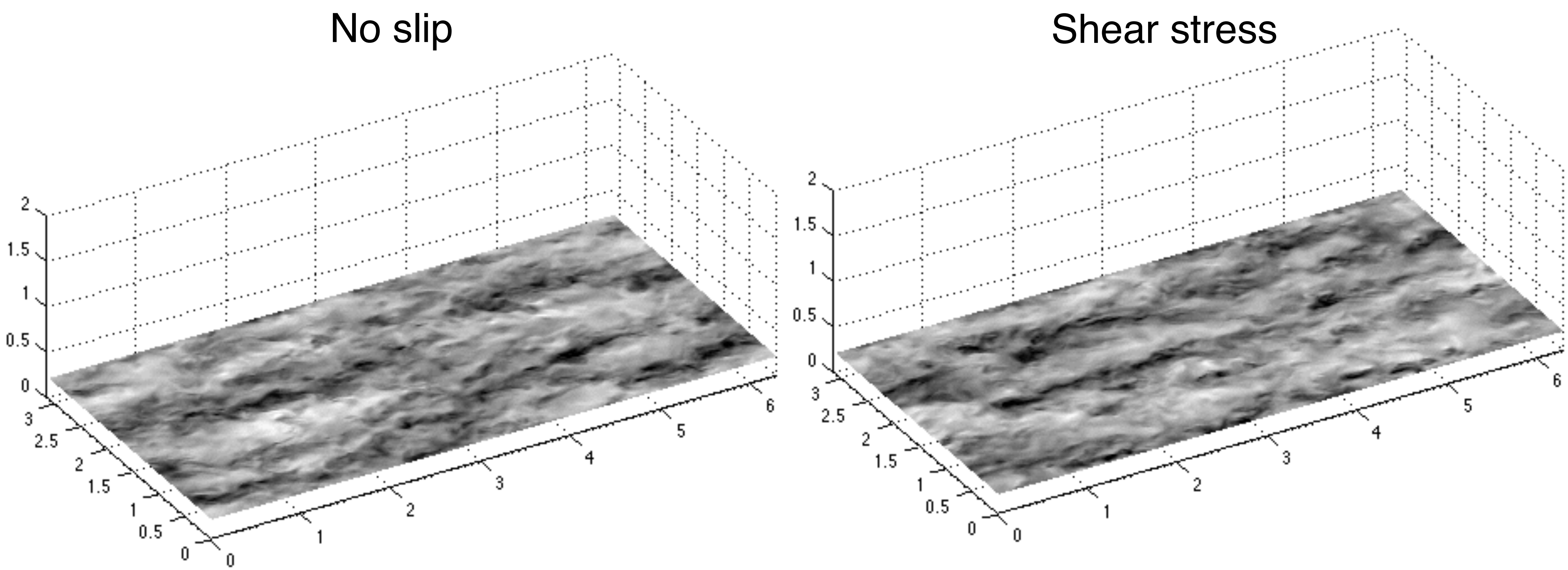


$$z = 123\nu/U_\tau = 0.21h$$

Streamwise velocity visualisations



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Streamwise velocity visualisations

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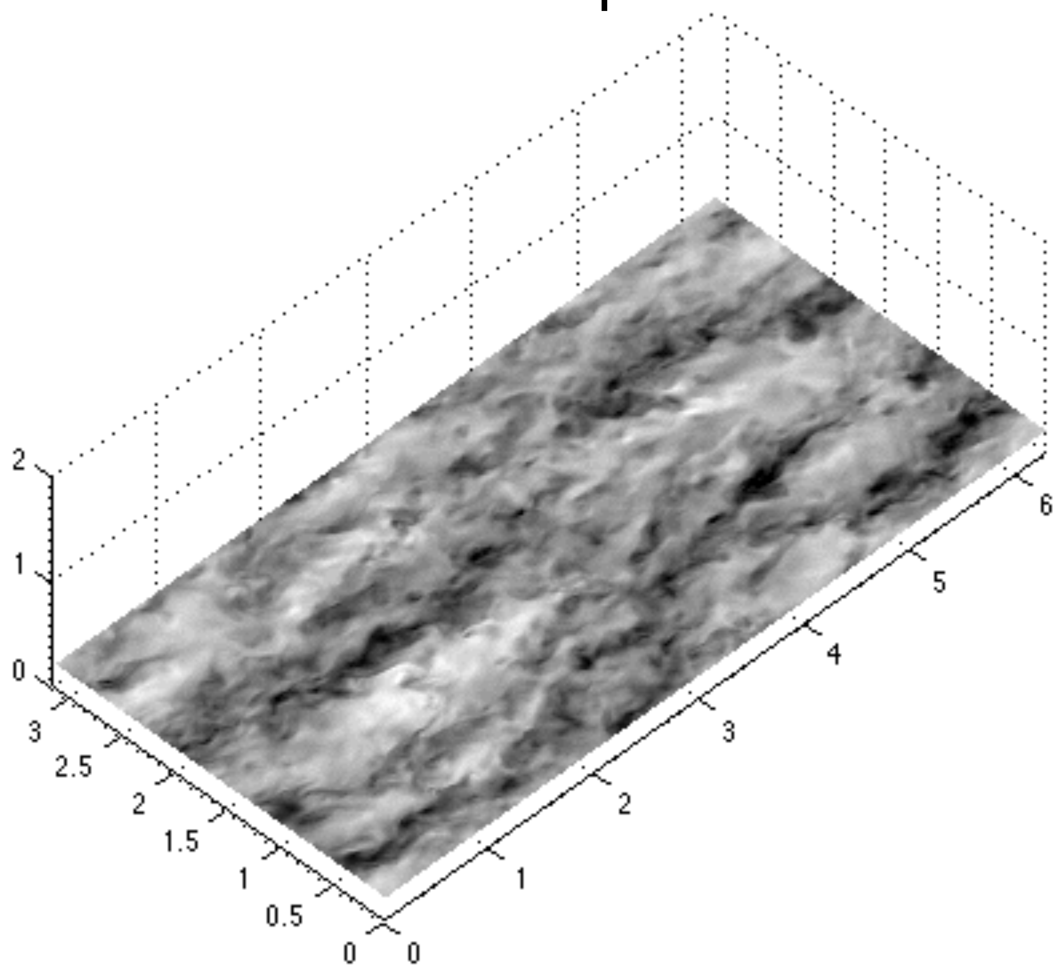
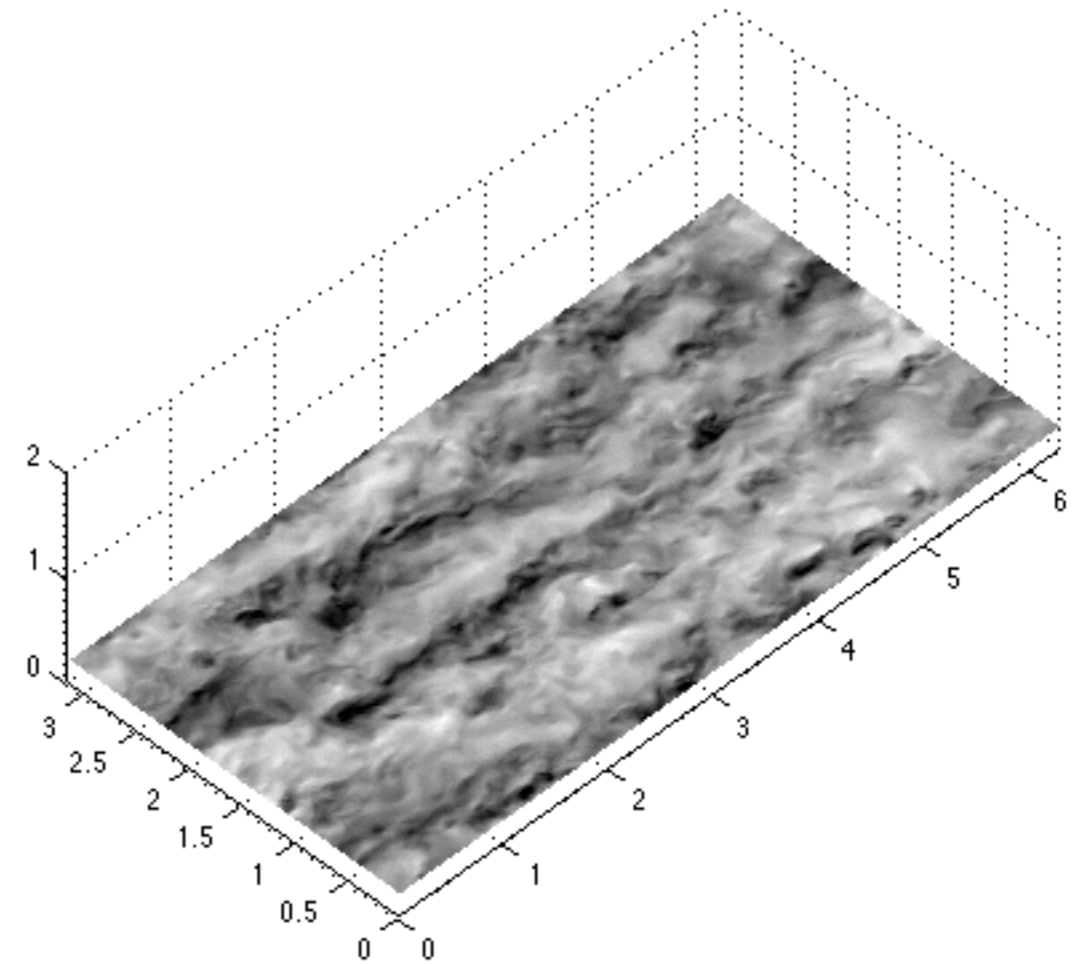
No slip



Shear stress



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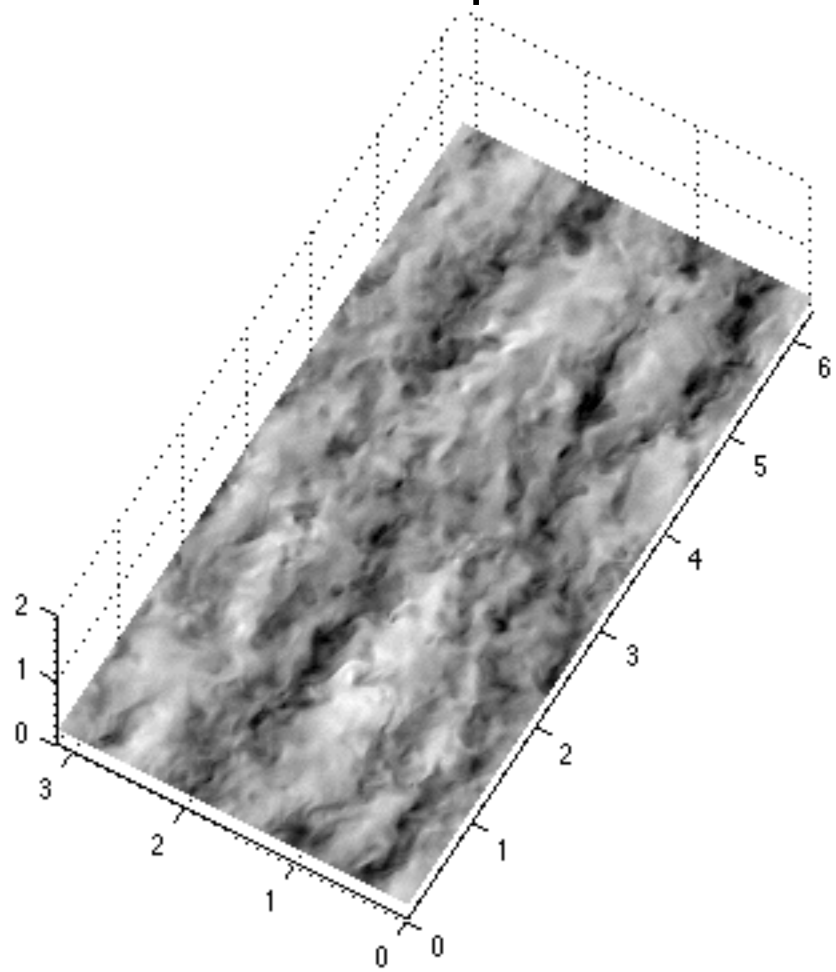

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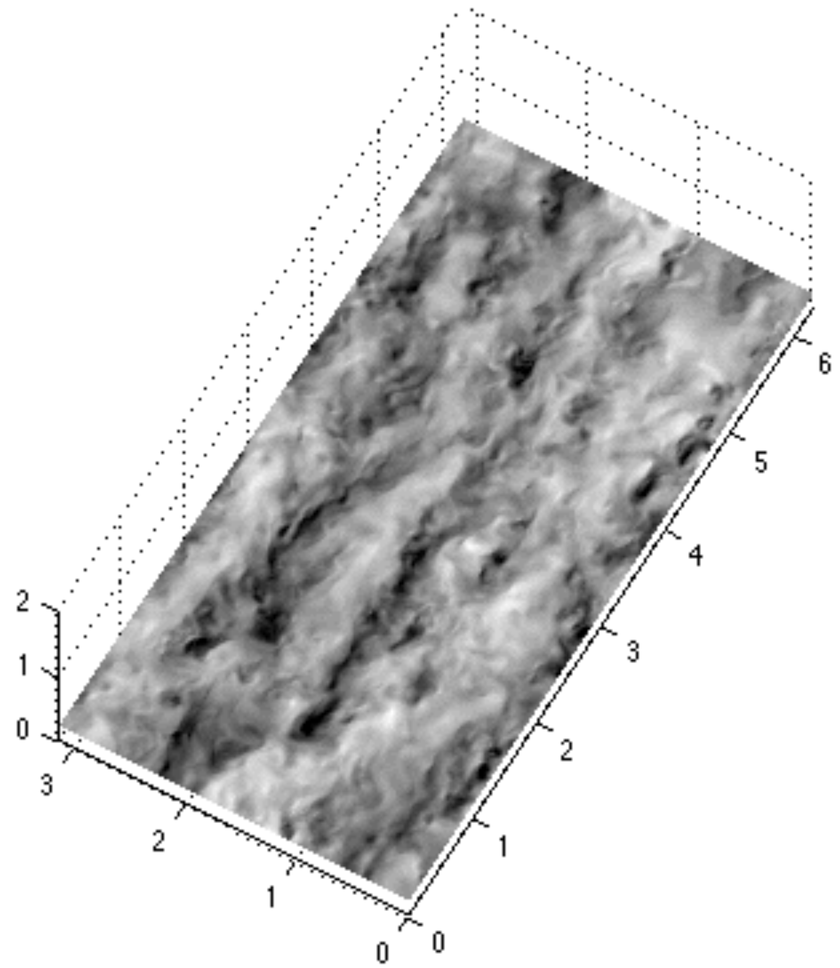

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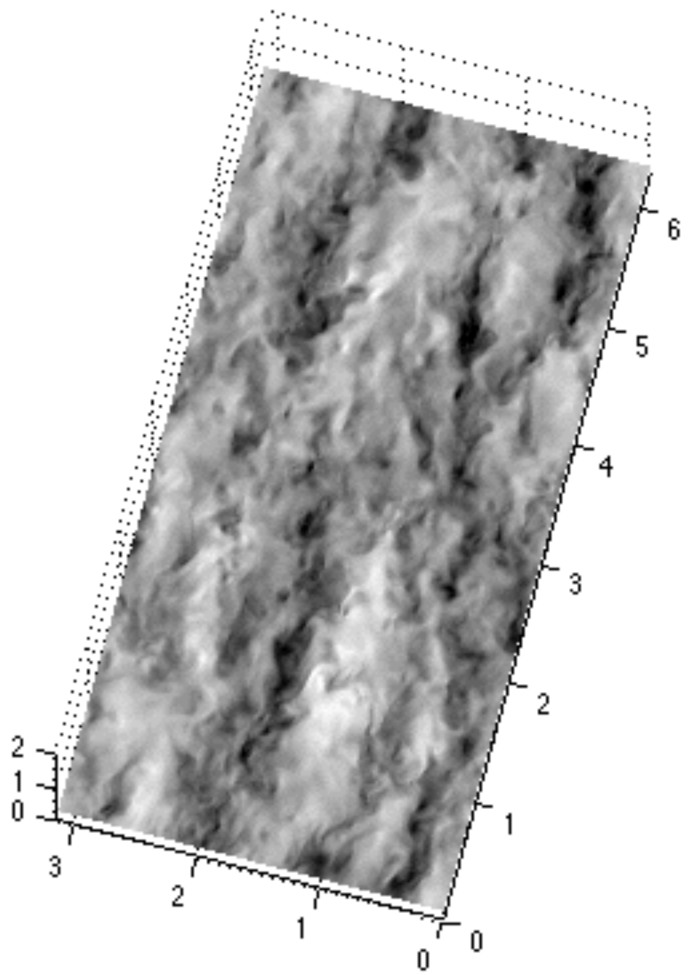

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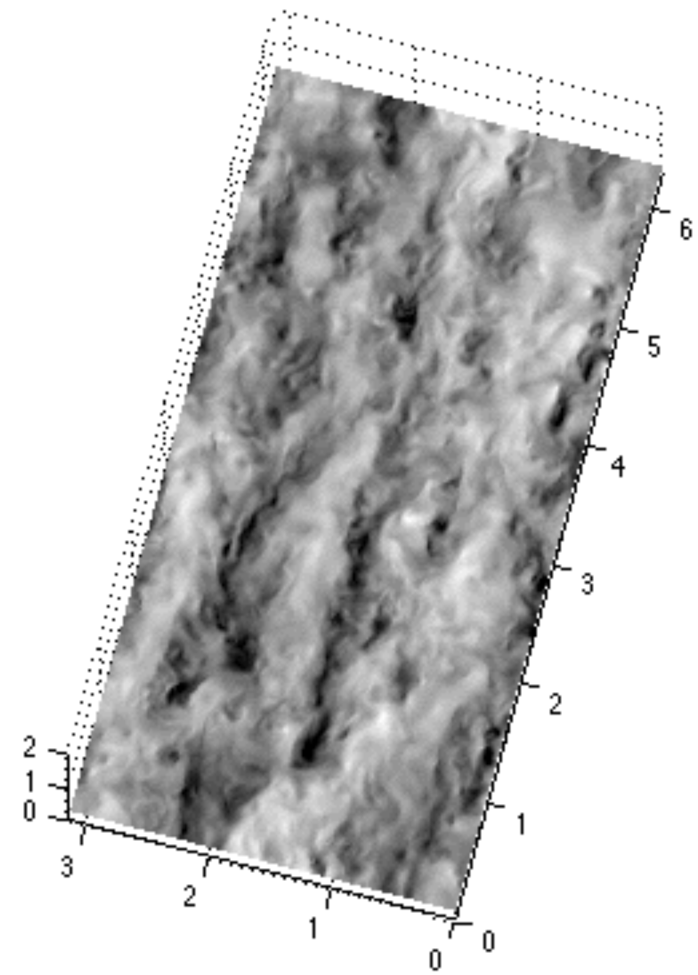

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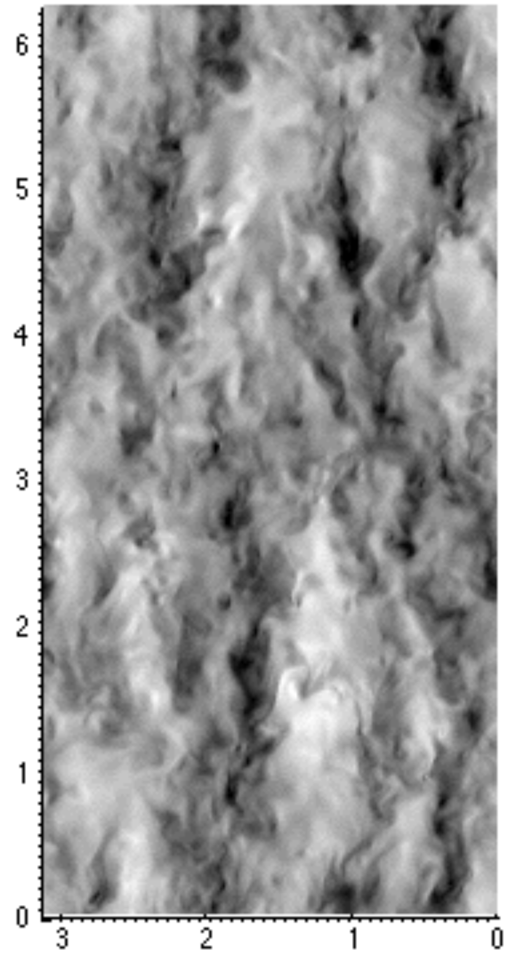

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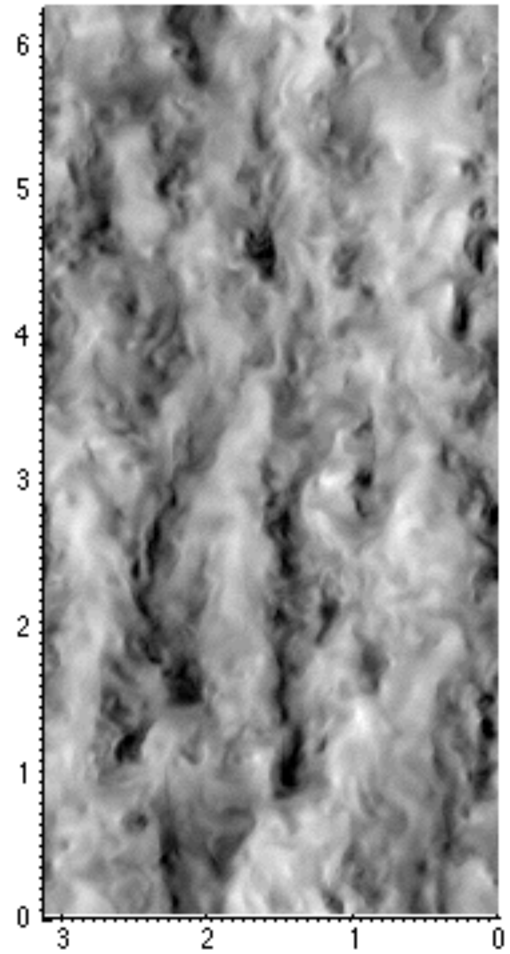

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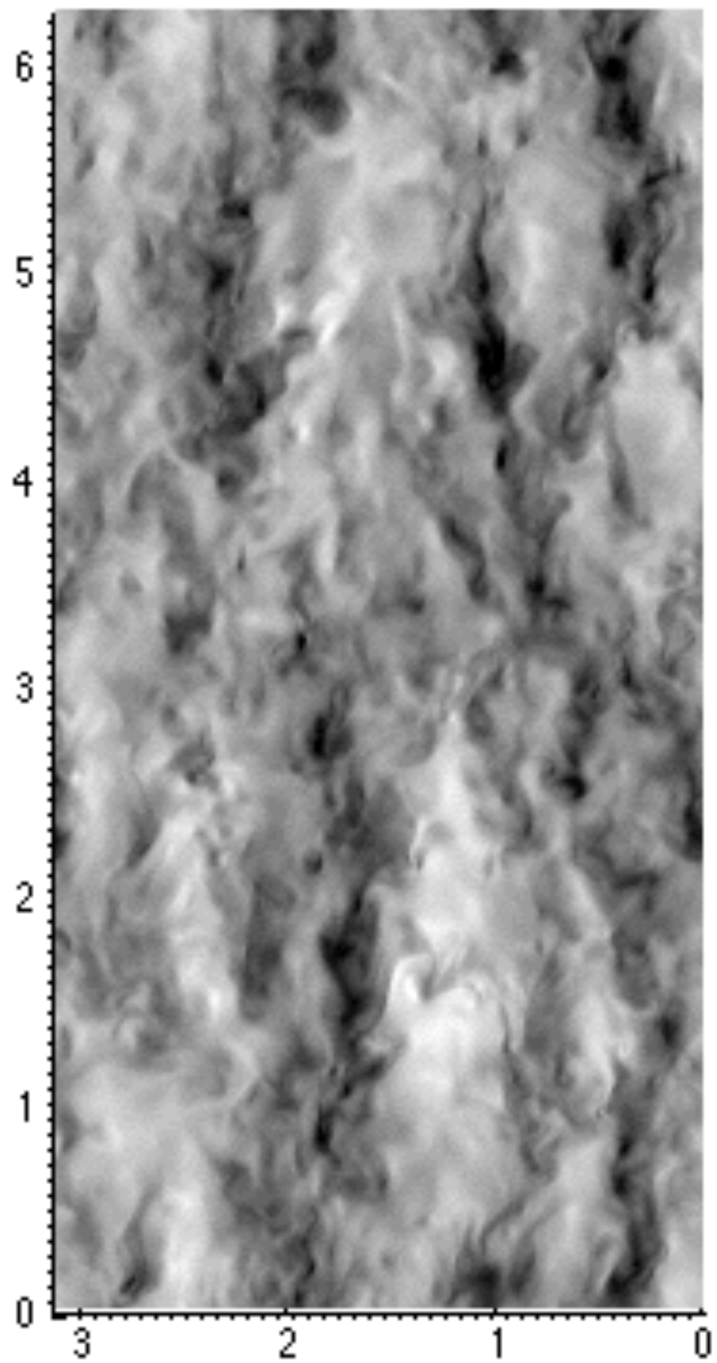

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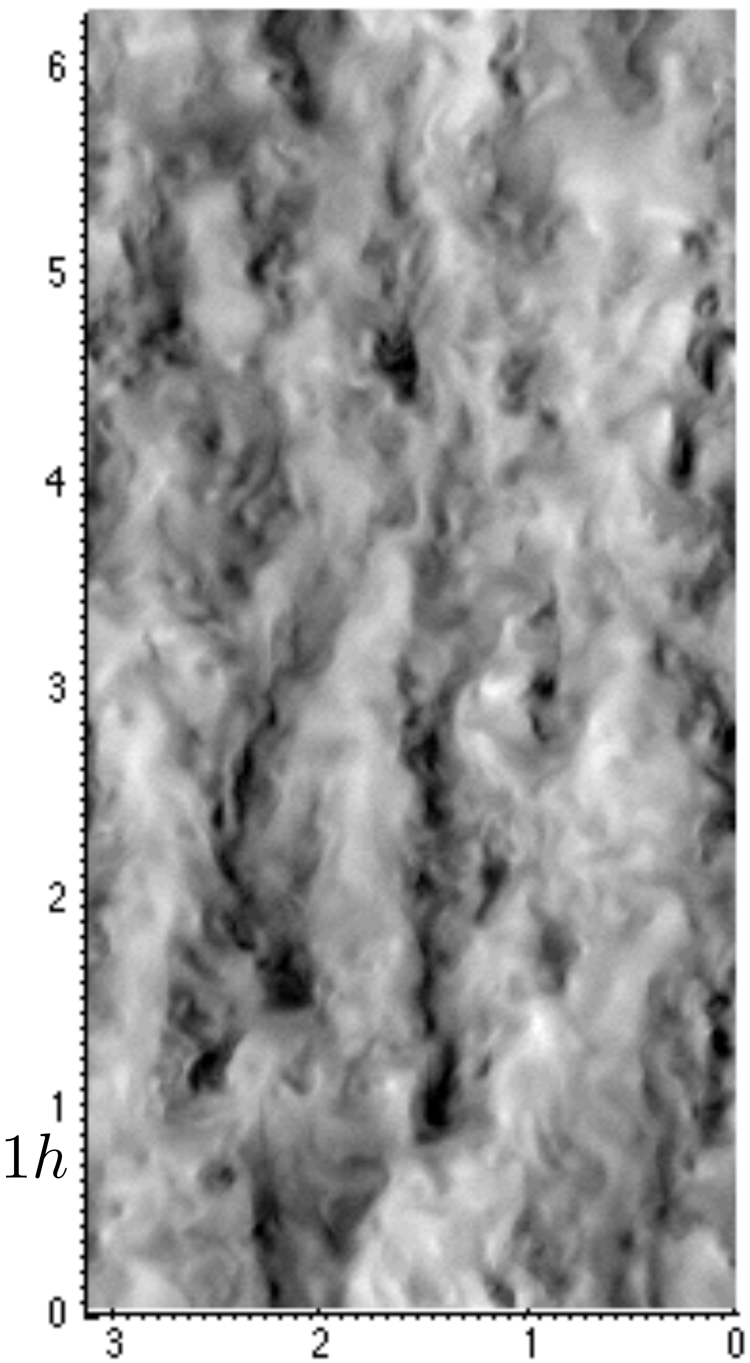
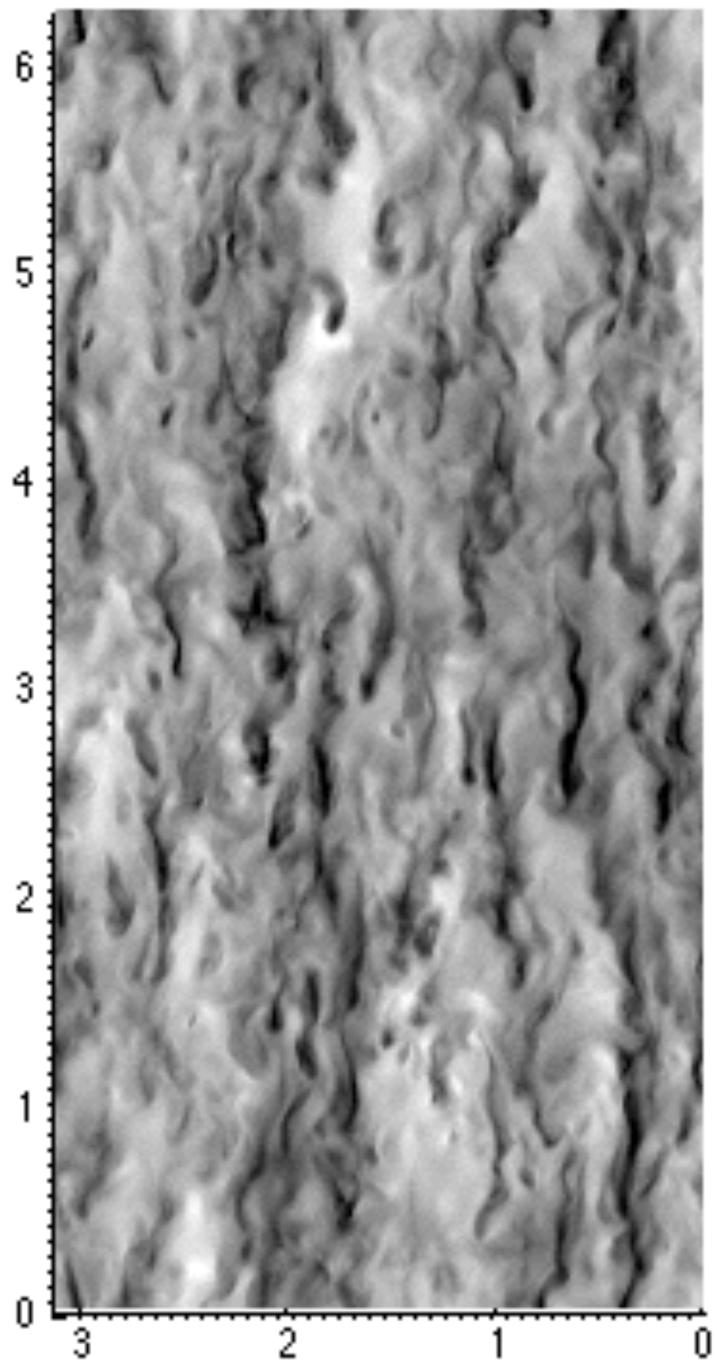


$$z = 123\nu/U_\tau = 0.21h$$

Contours: 3 std. dev. from mean

Streamwise velocity visualisations

No slip



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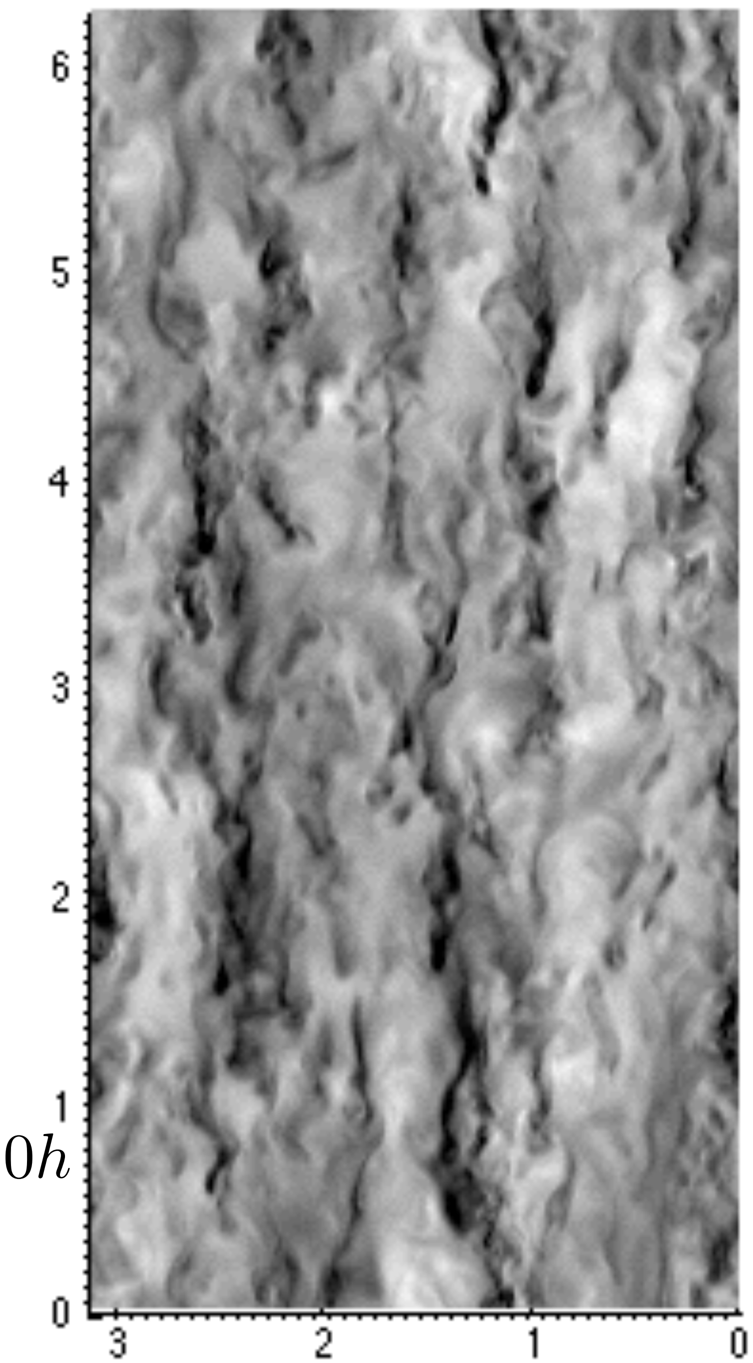
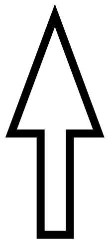


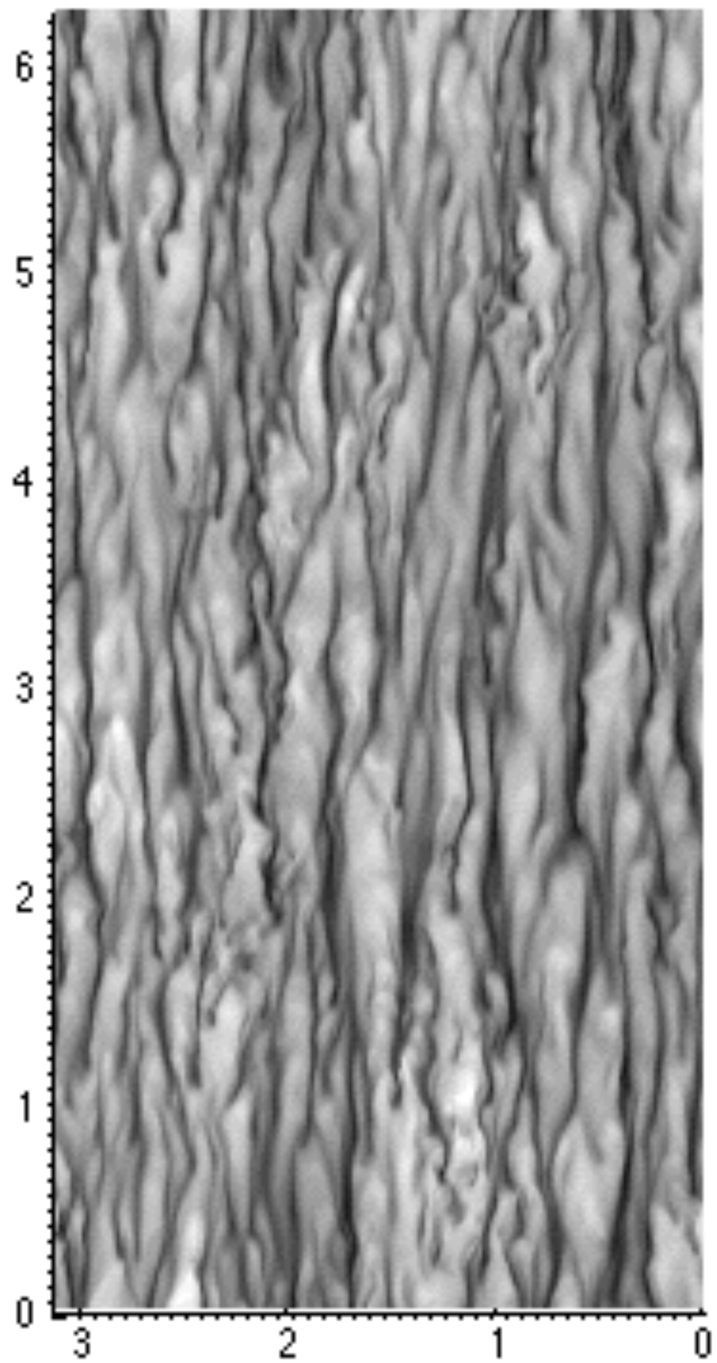


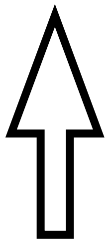
$$z = 58\nu/U_\tau = 0.10h$$

Contours: 3 std. dev. from mean

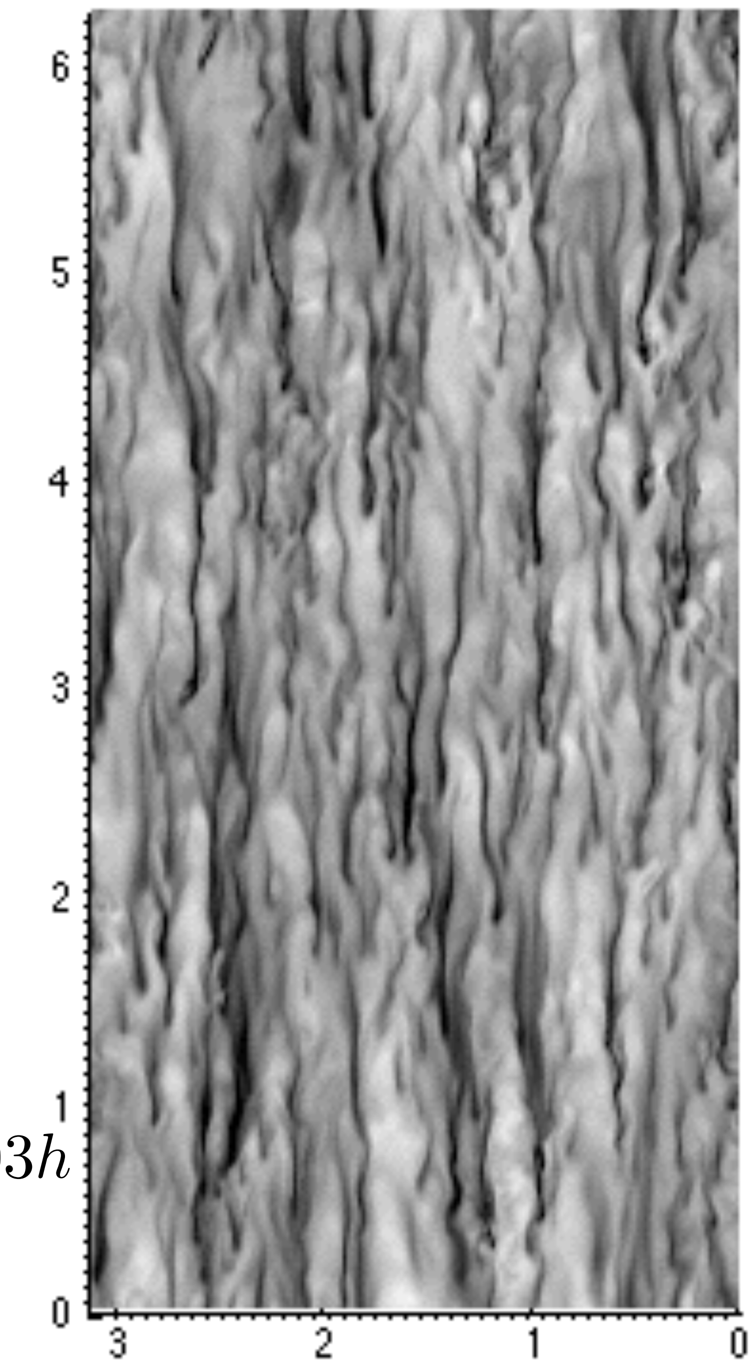
Streamwise velocity visualisations

No slip





Shear stress

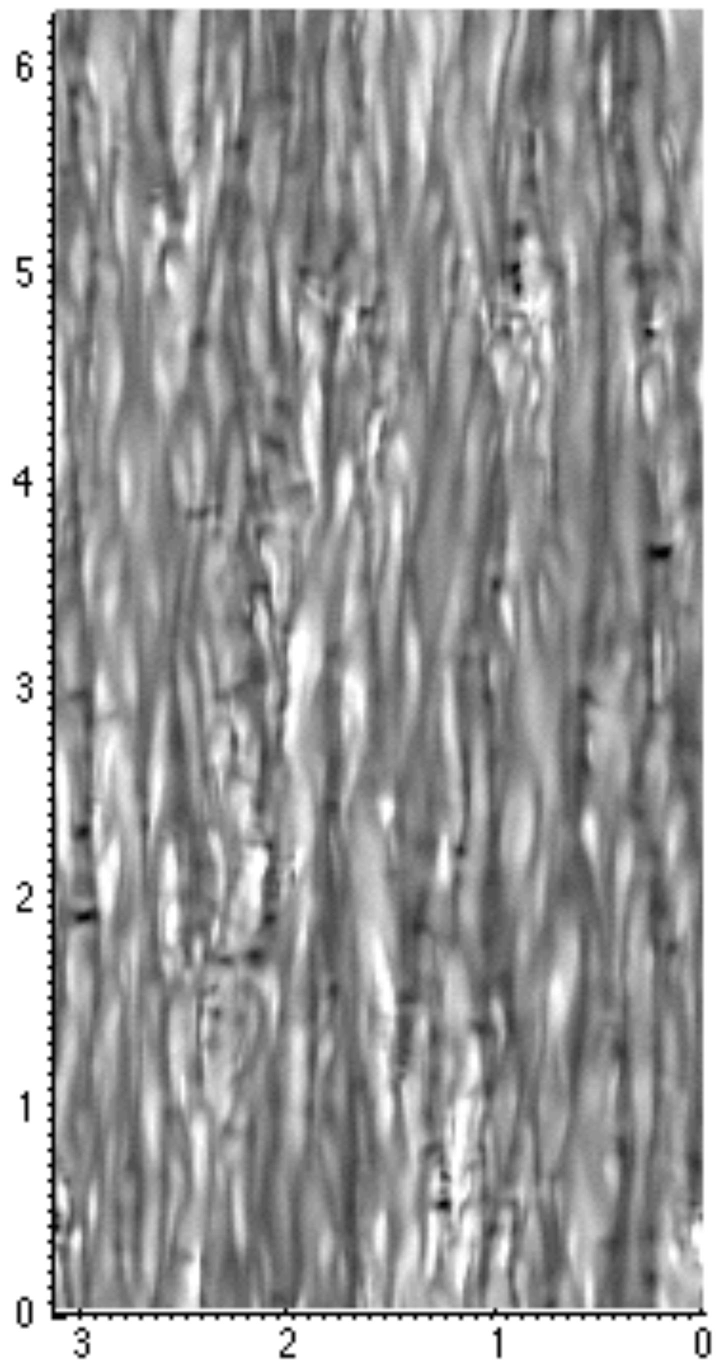


$z = 15\nu/U_\tau = 0.03h$

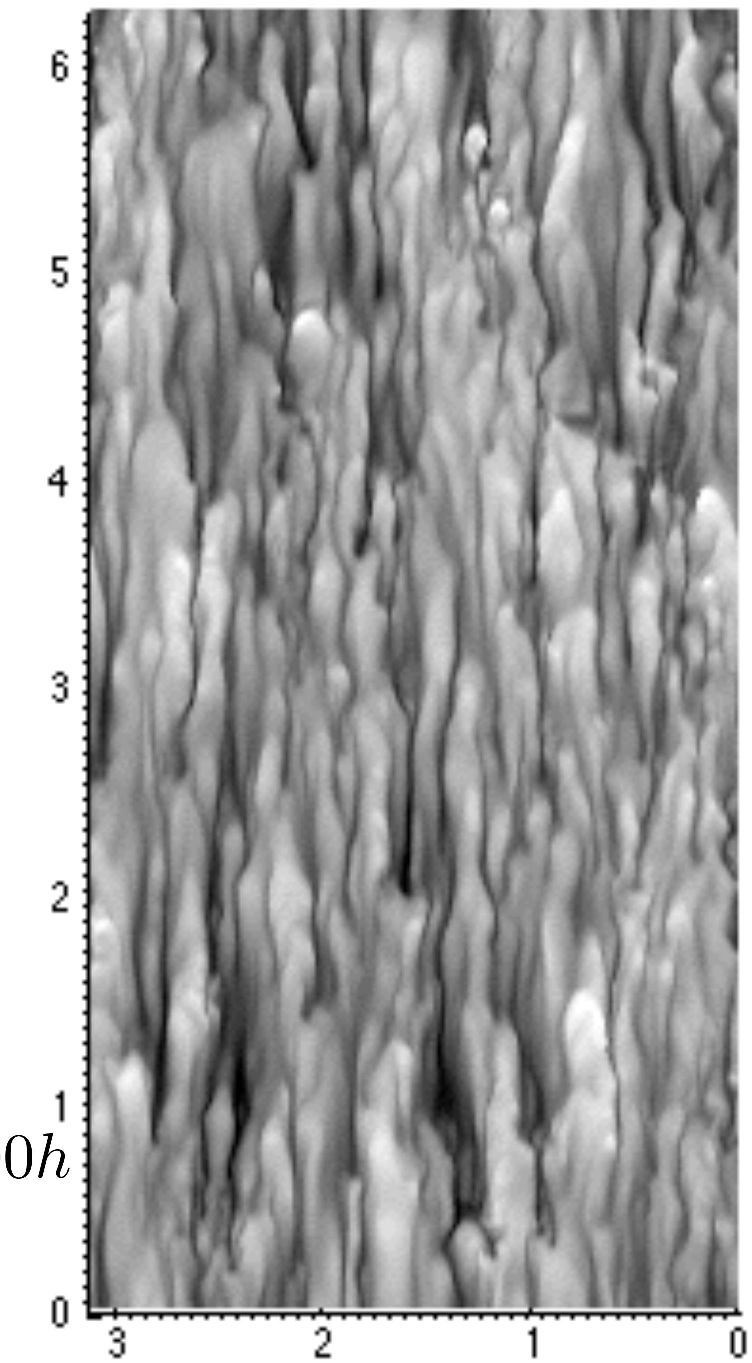
Contours: 3 std. dev. from mean

Streamwise velocity visualisations

No slip



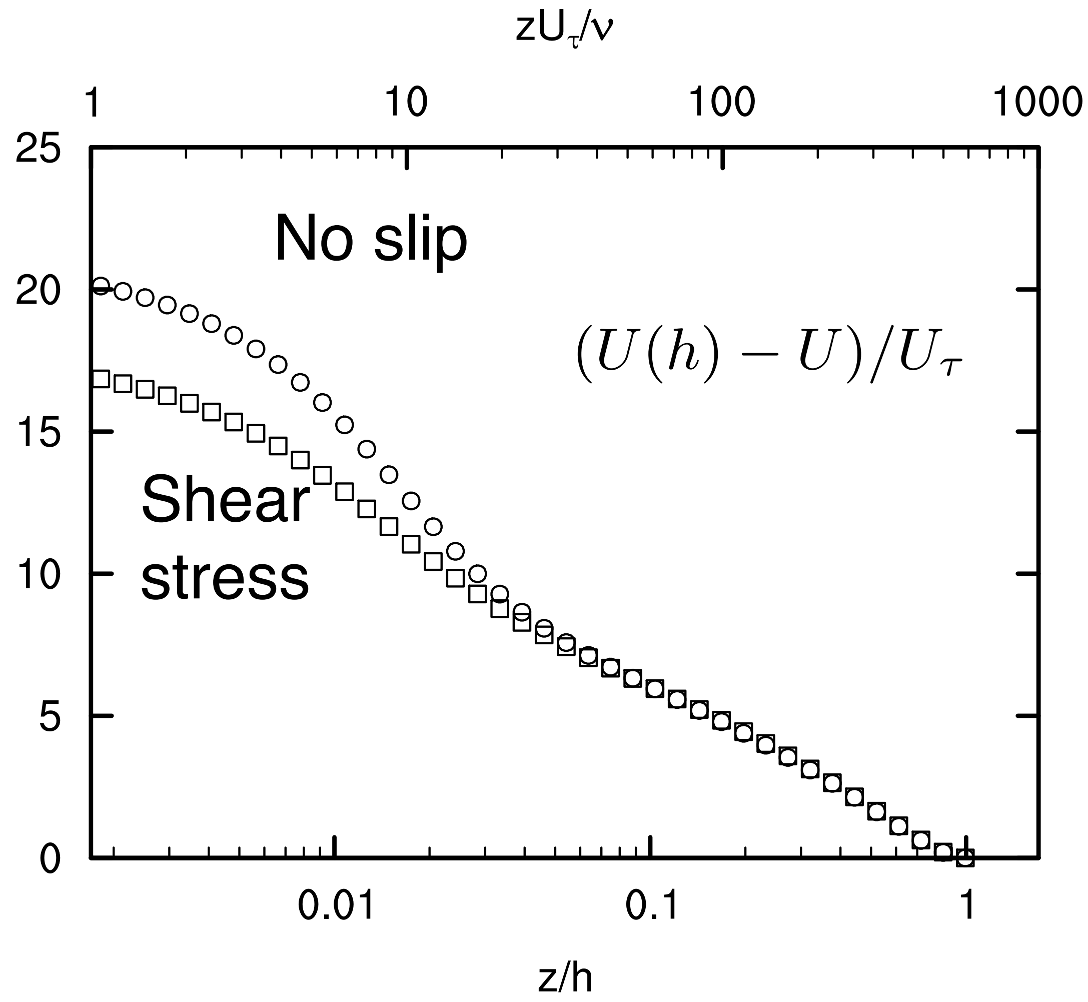
Shear stress



$z = 1\nu/U_\tau = 0.00h$

Contours: 3 std. dev. from mean

Mean velocity



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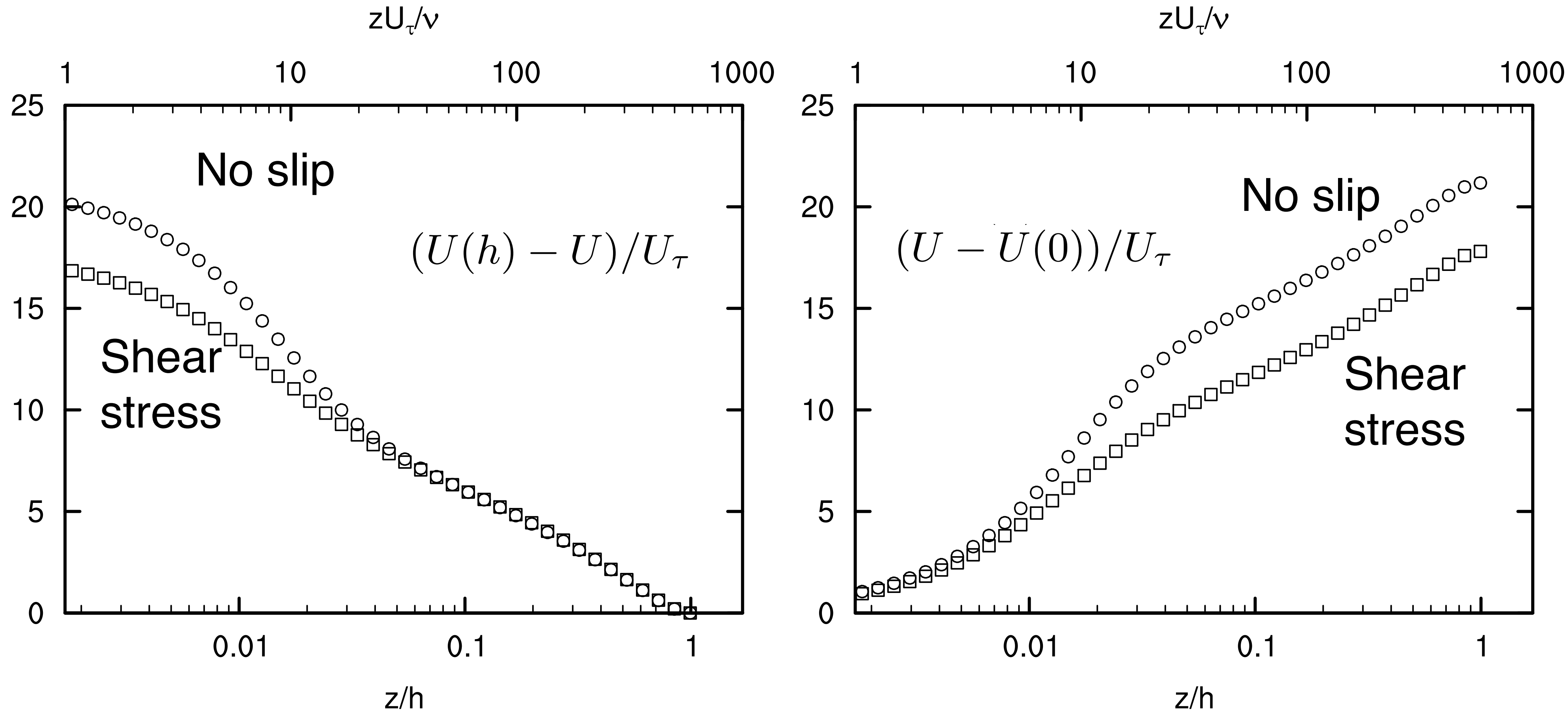

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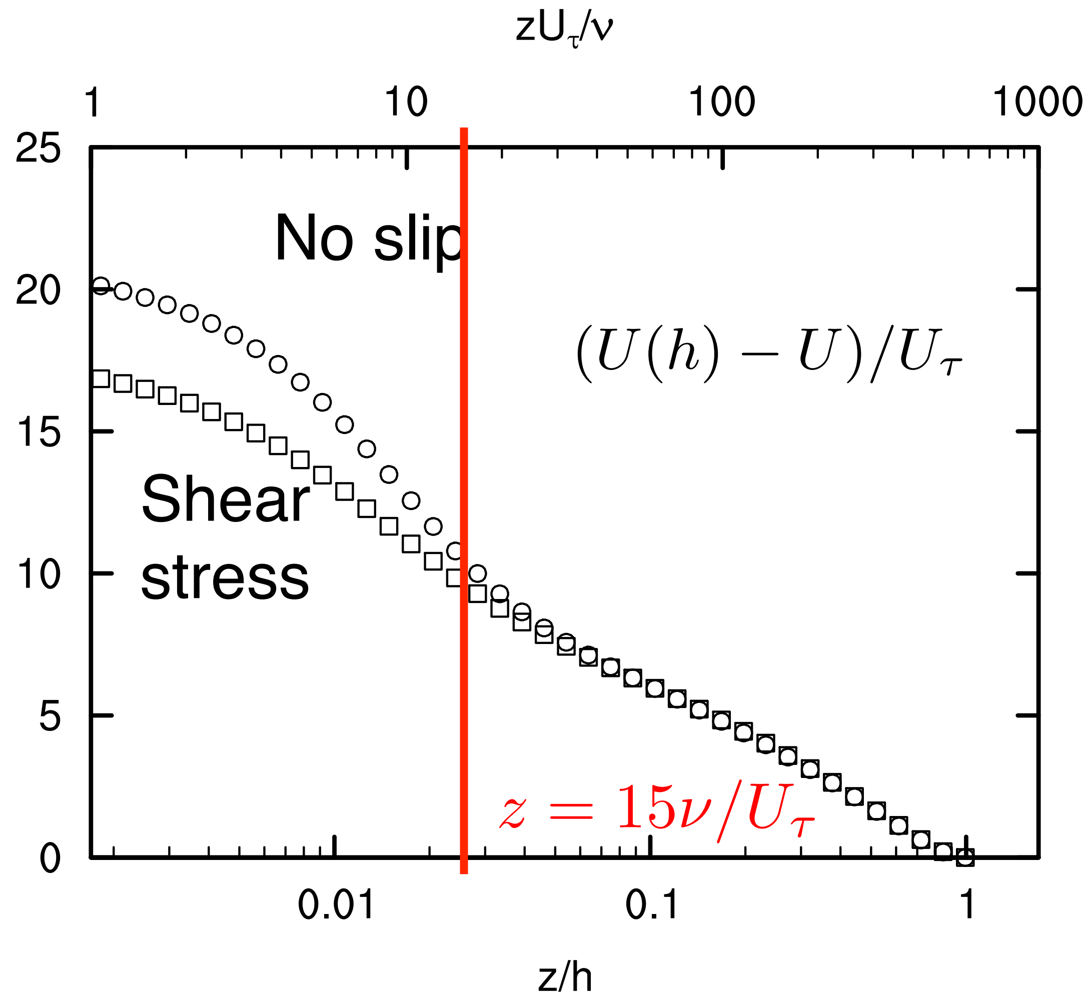

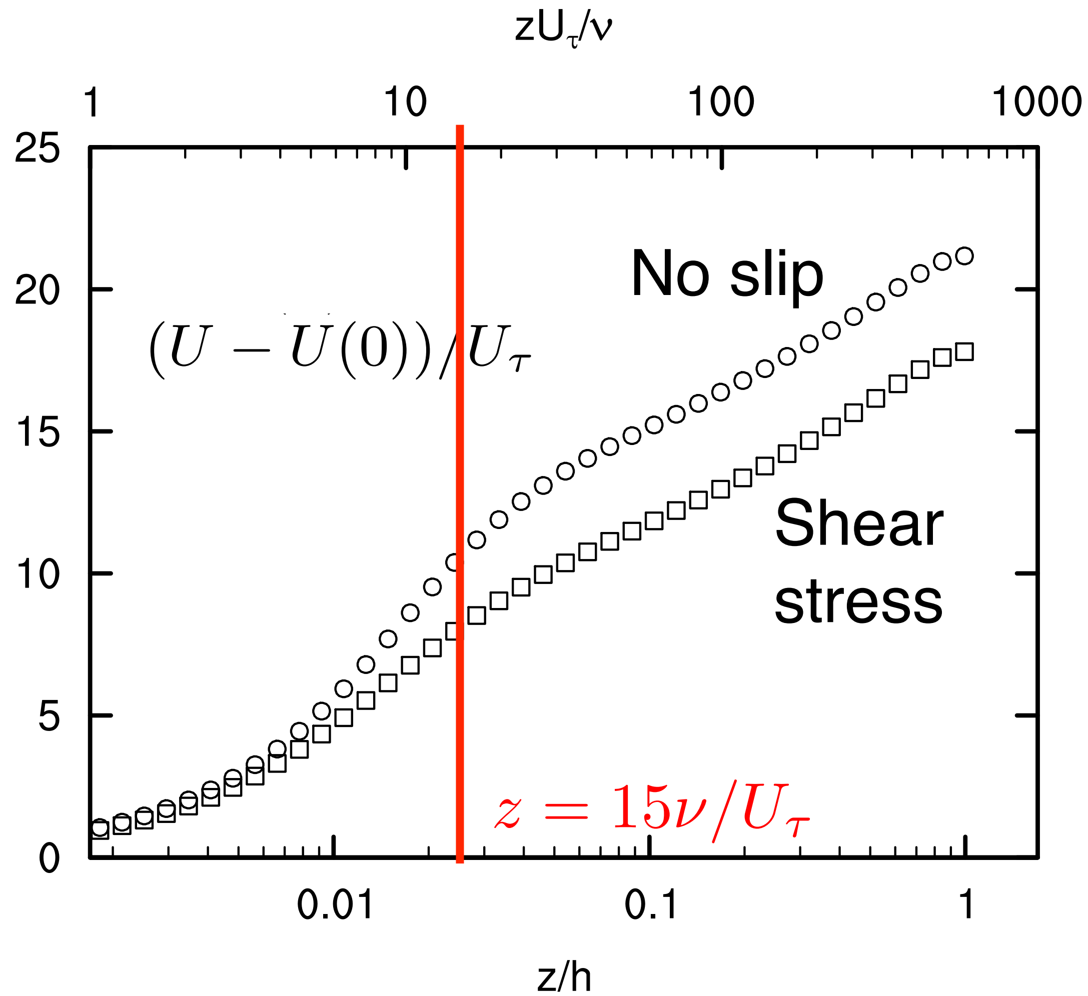

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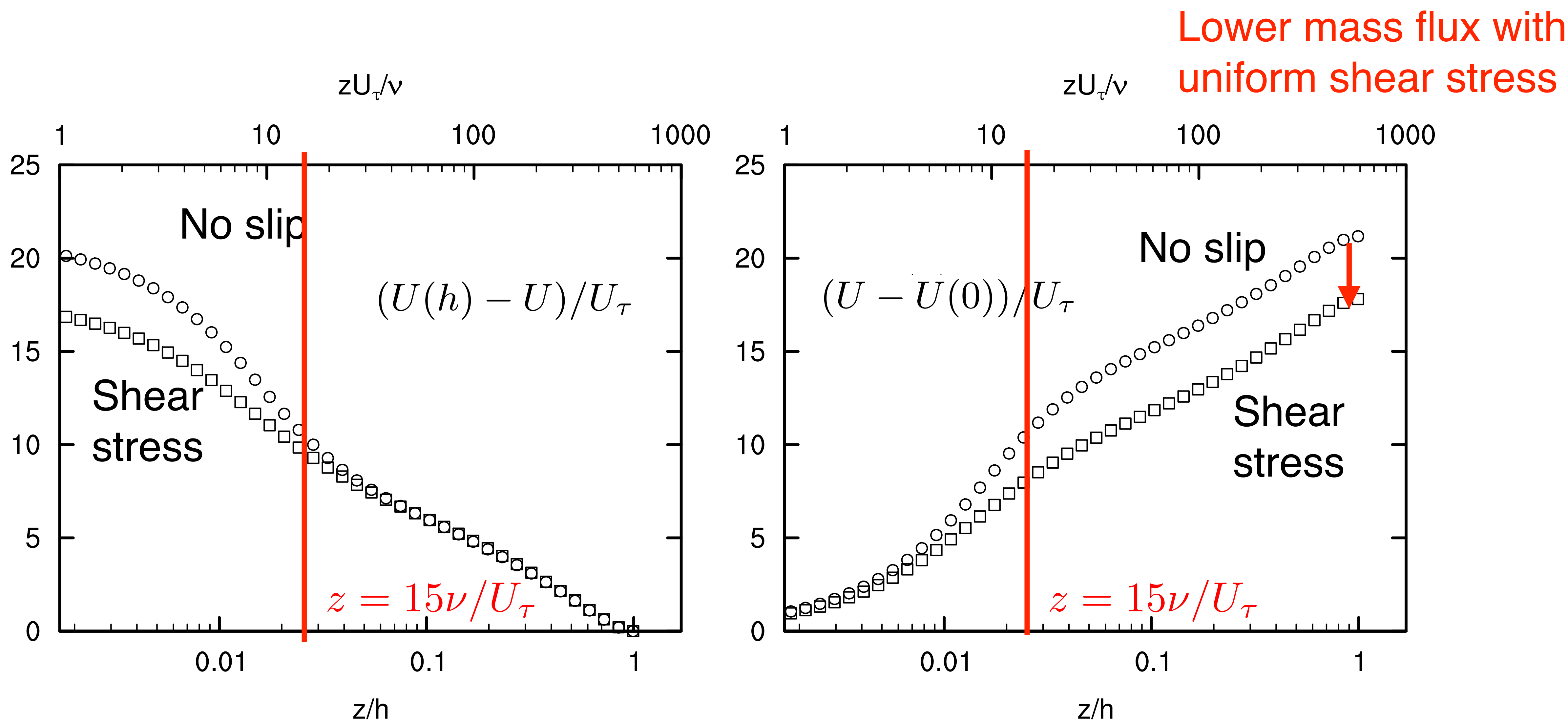
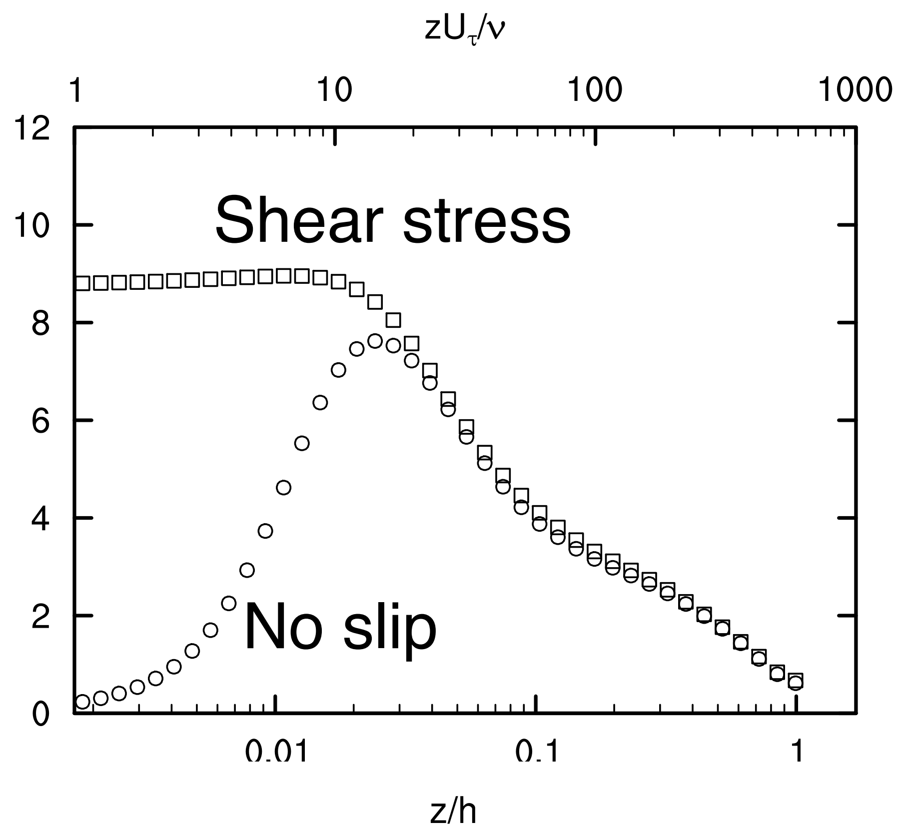


Reynolds stresses

Streamwise

$\overline{u^2}/U_\tau^2$



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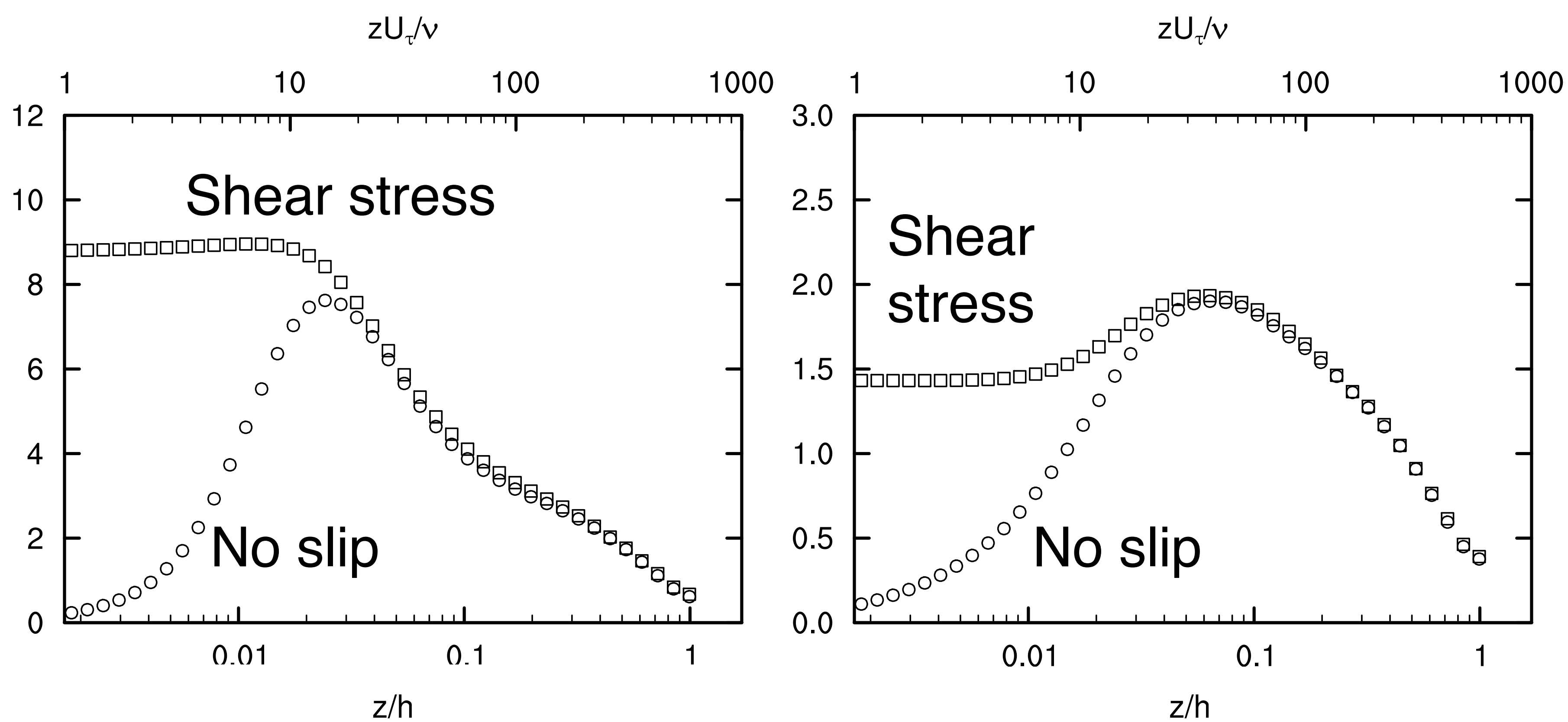


Spanwise

$\overline{v^2}/U_\tau^2$

Reynolds stresses

Streamwise

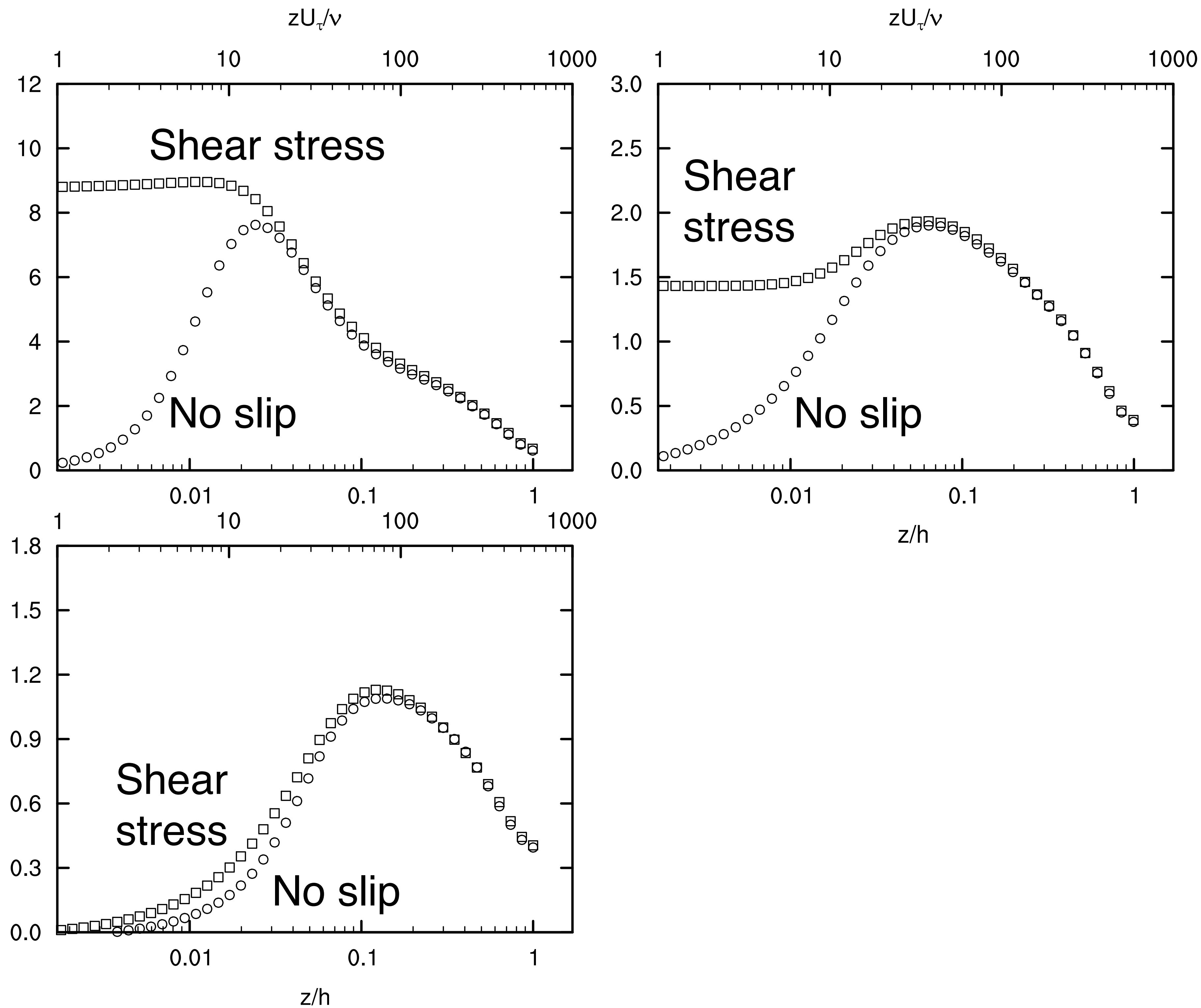
$\overline{u^2}/U_\tau^2$

Spanwise

$\overline{v^2}/U_\tau^2$

Wall-normal

$\overline{w^2}/U_\tau^2$



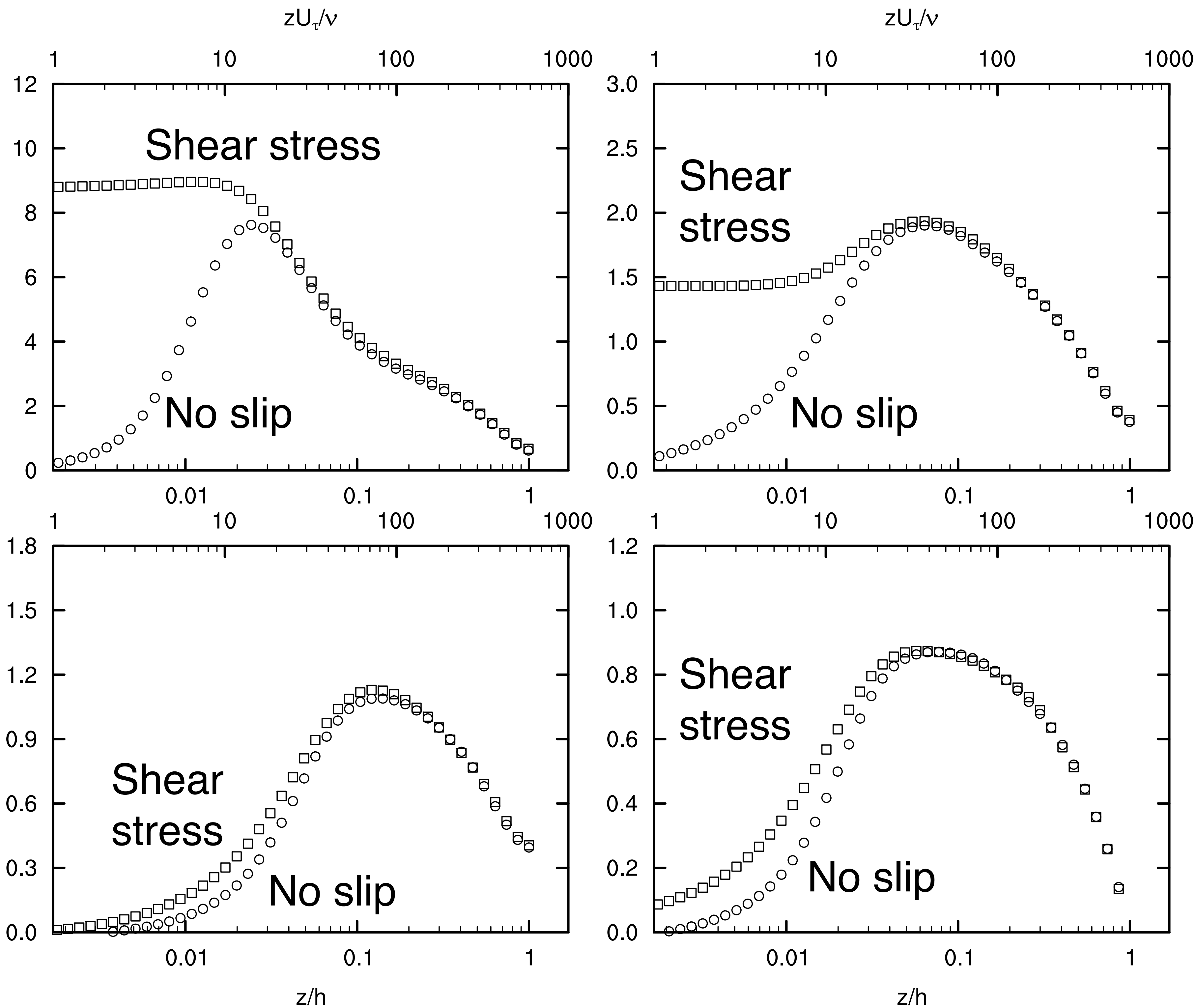
Reynolds stresses

Streamwise

$\overline{u^2}/U_\tau^2$

Spanwise

$\overline{v^2}/U_\tau^2$



Wall-normal

$\overline{w^2}/U_\tau^2$

Streamwise–
wall-normal

$-\overline{wu}/U_\tau^2$

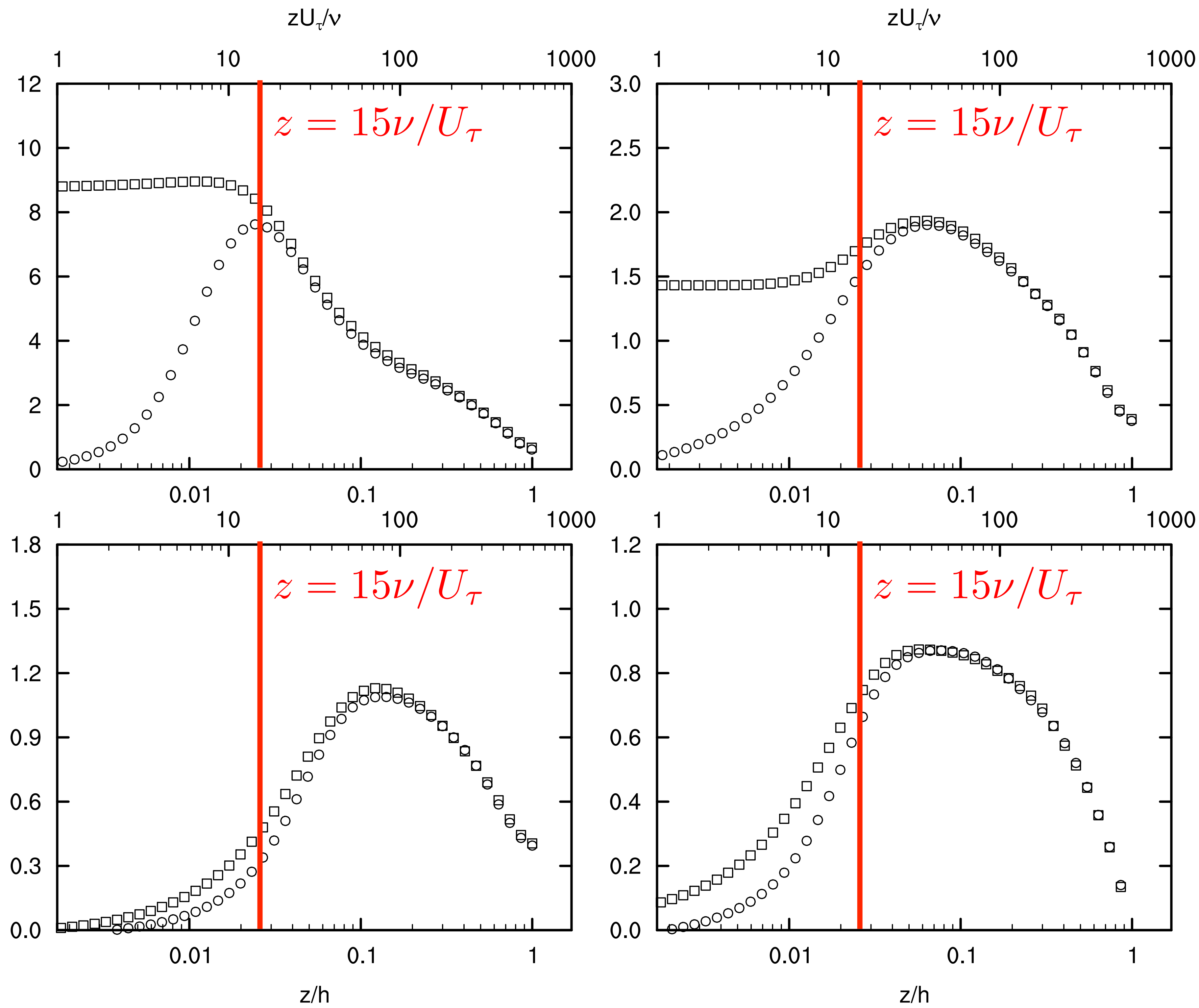
Reynolds stresses

Streamwise

$\overline{u^2}/U_\tau^2$

Spanwise

$\overline{v^2}/U_\tau^2$



Wall-normal

$\overline{w^2}/U_\tau^2$

Streamwise–wall-normal

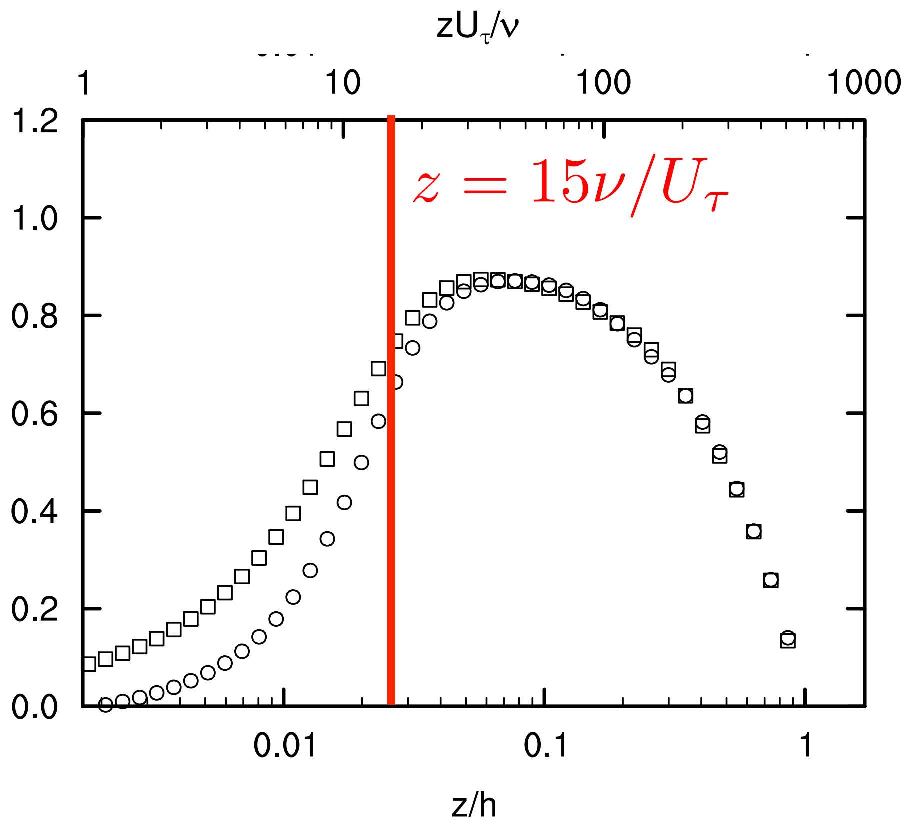
$-\overline{wu}/U_\tau^2$

Reynolds stresses

Outer-layer (where Reynolds stresses dominate)

statistics of turbulent relative motion

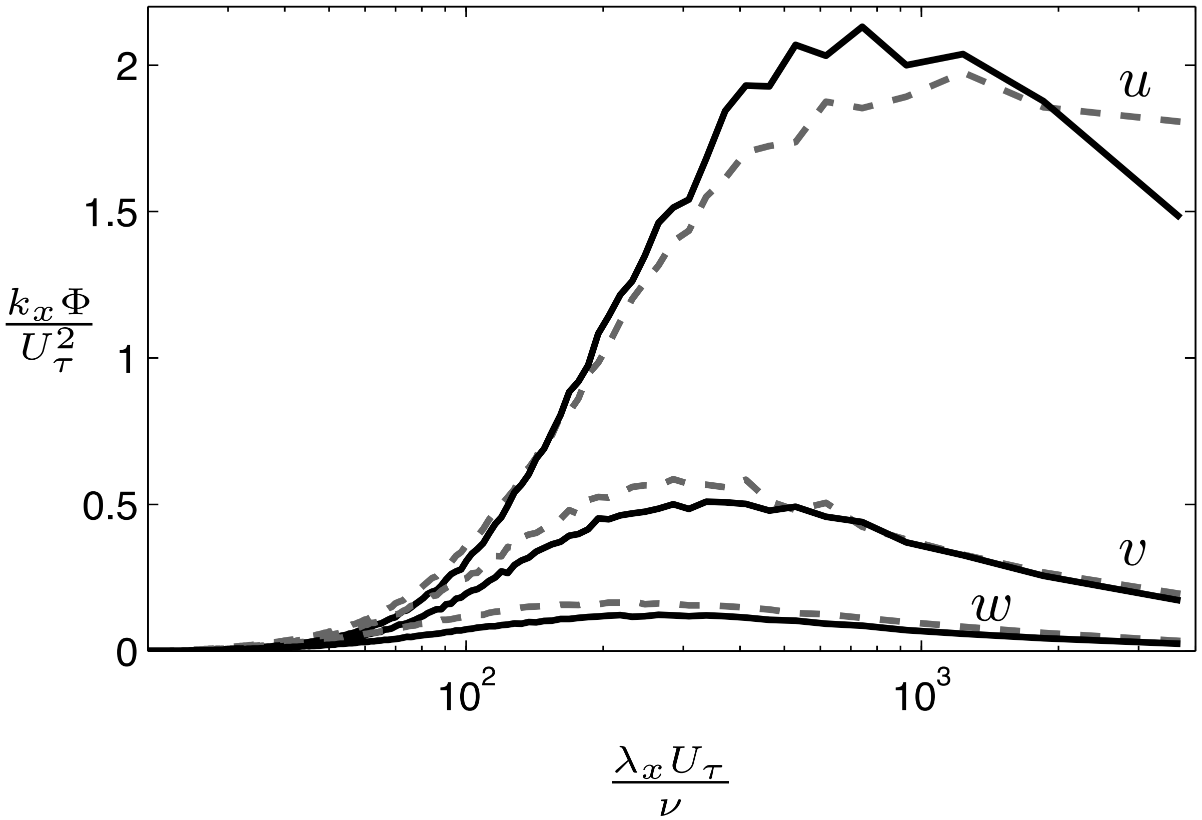
depend only on D and τ_0.



Streamwise–wall-normal $-\overline{wu}/U_\tau^2$

Spectra of streamwise velocity fluctuations

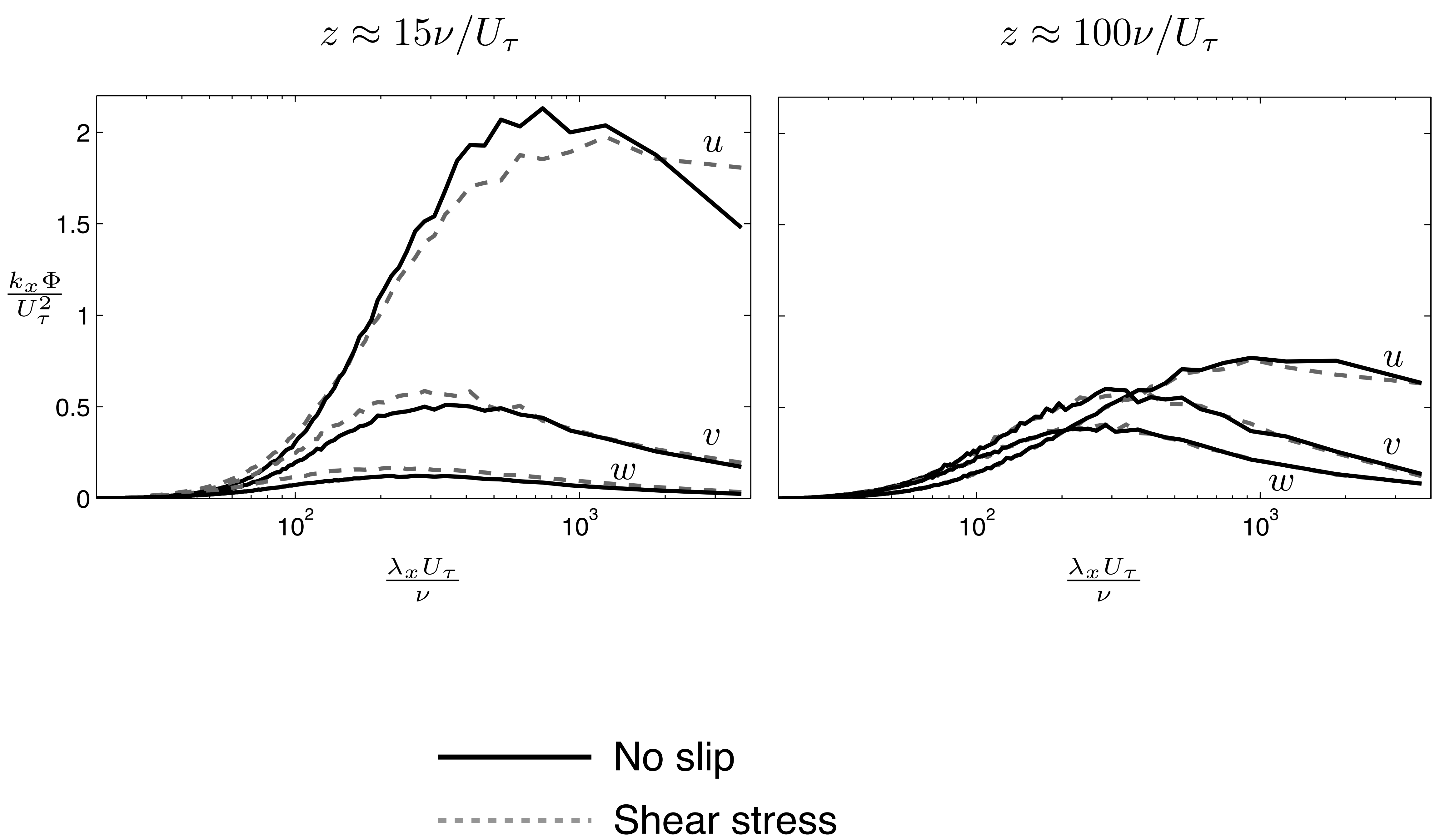
$z \approx 15\nu/U_\tau$



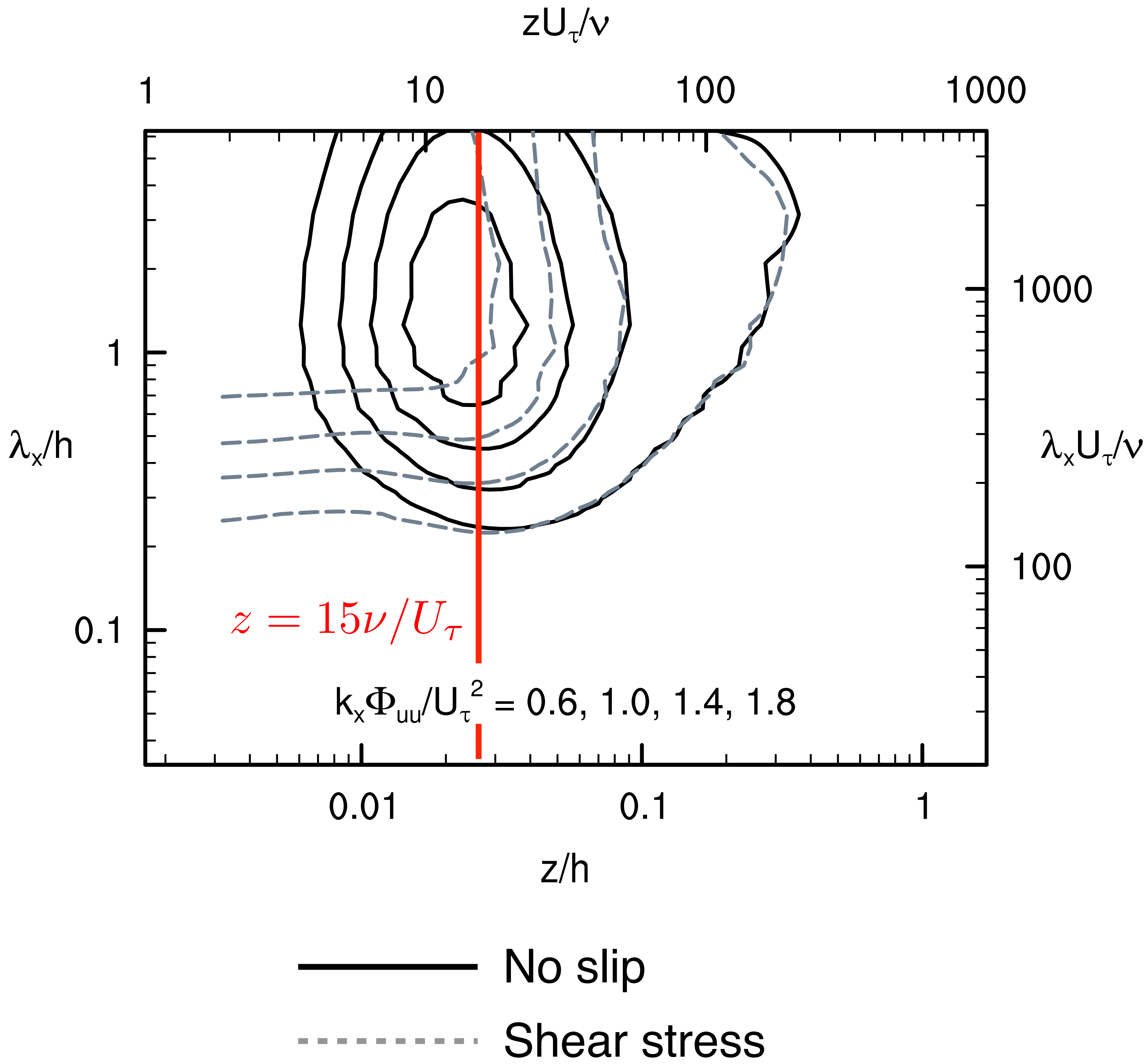
No slip

Shear stress

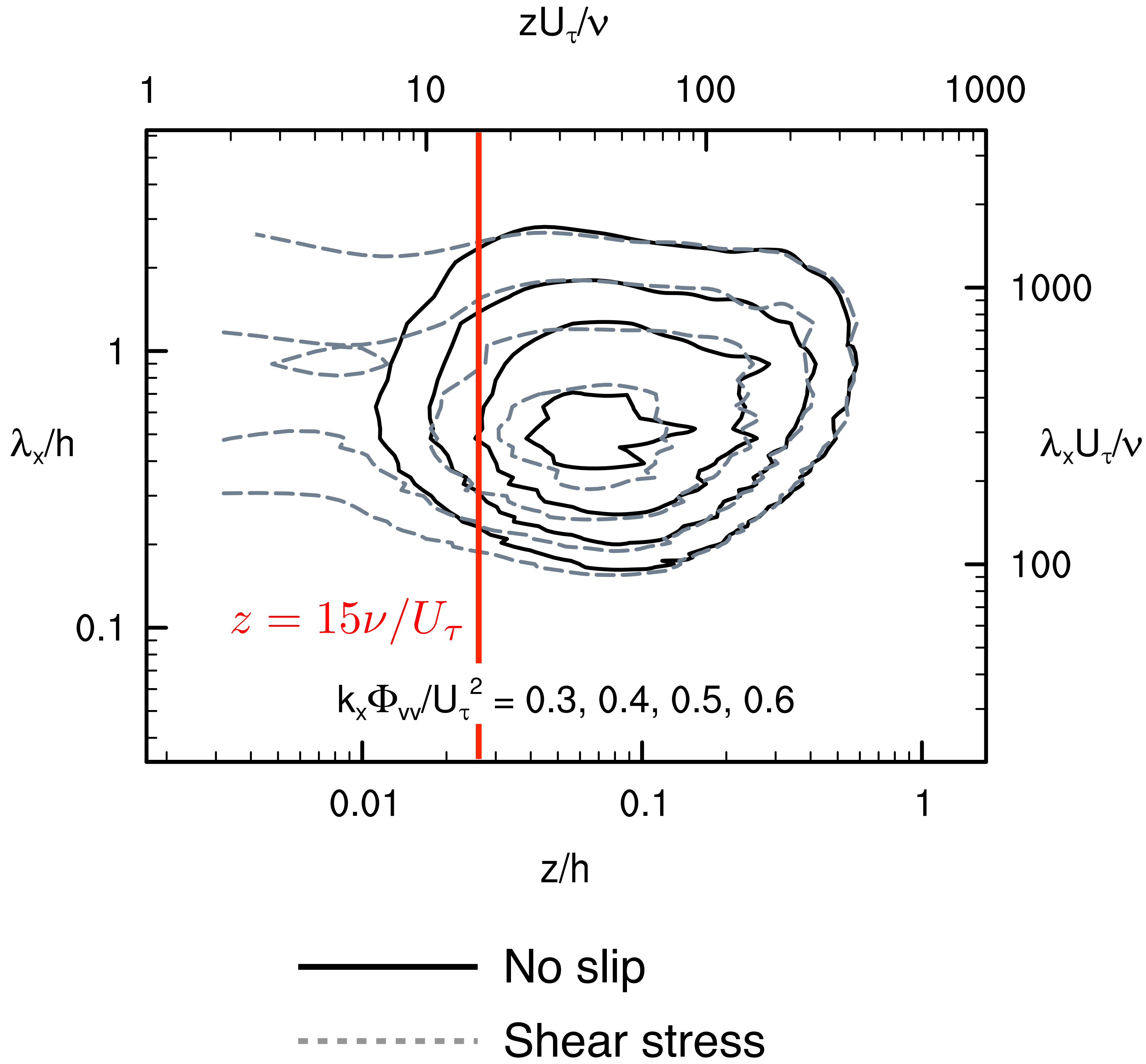
Spectra of streamwise velocity fluctuations



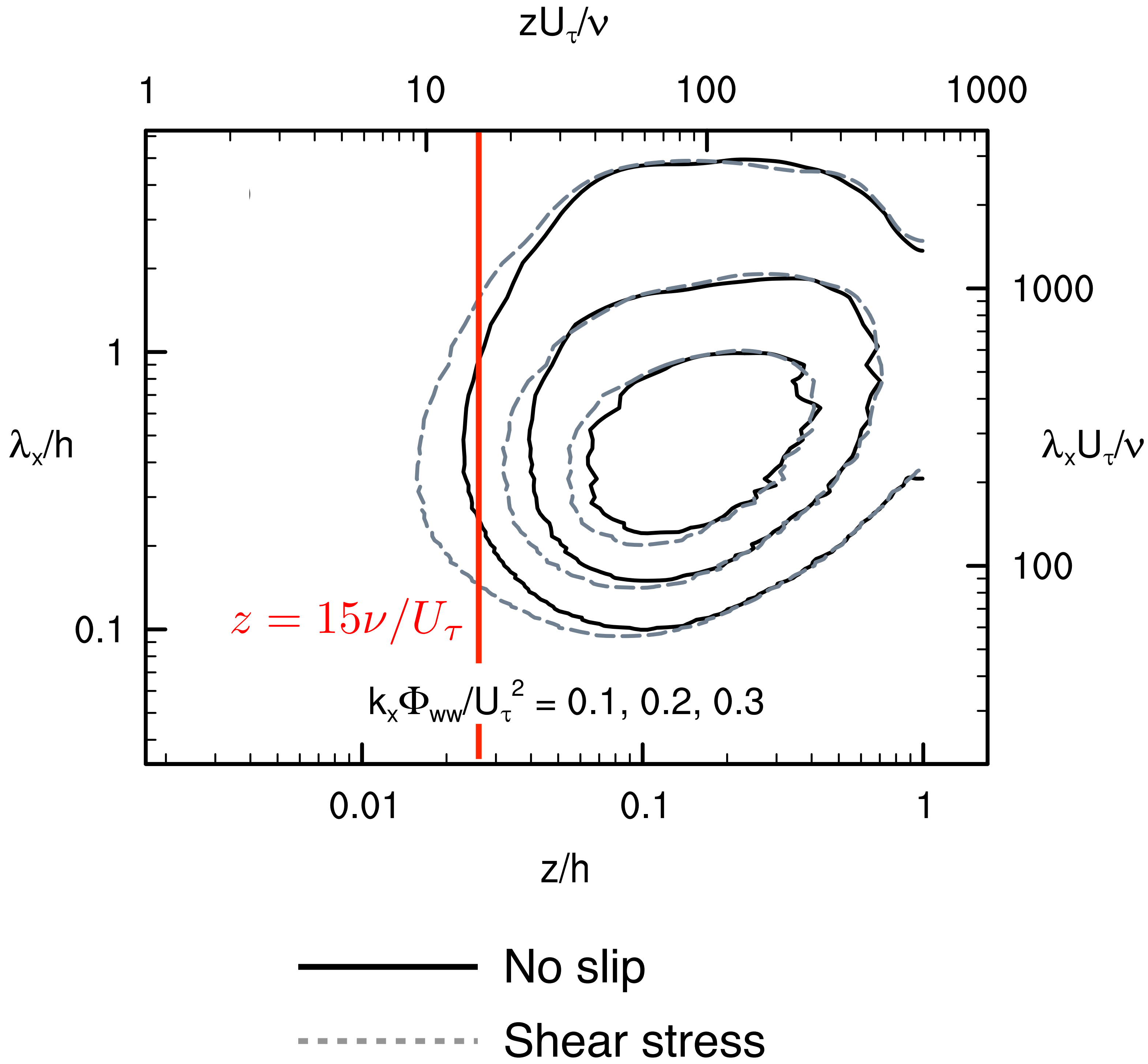
Spectra of streamwise velocity fluctuations



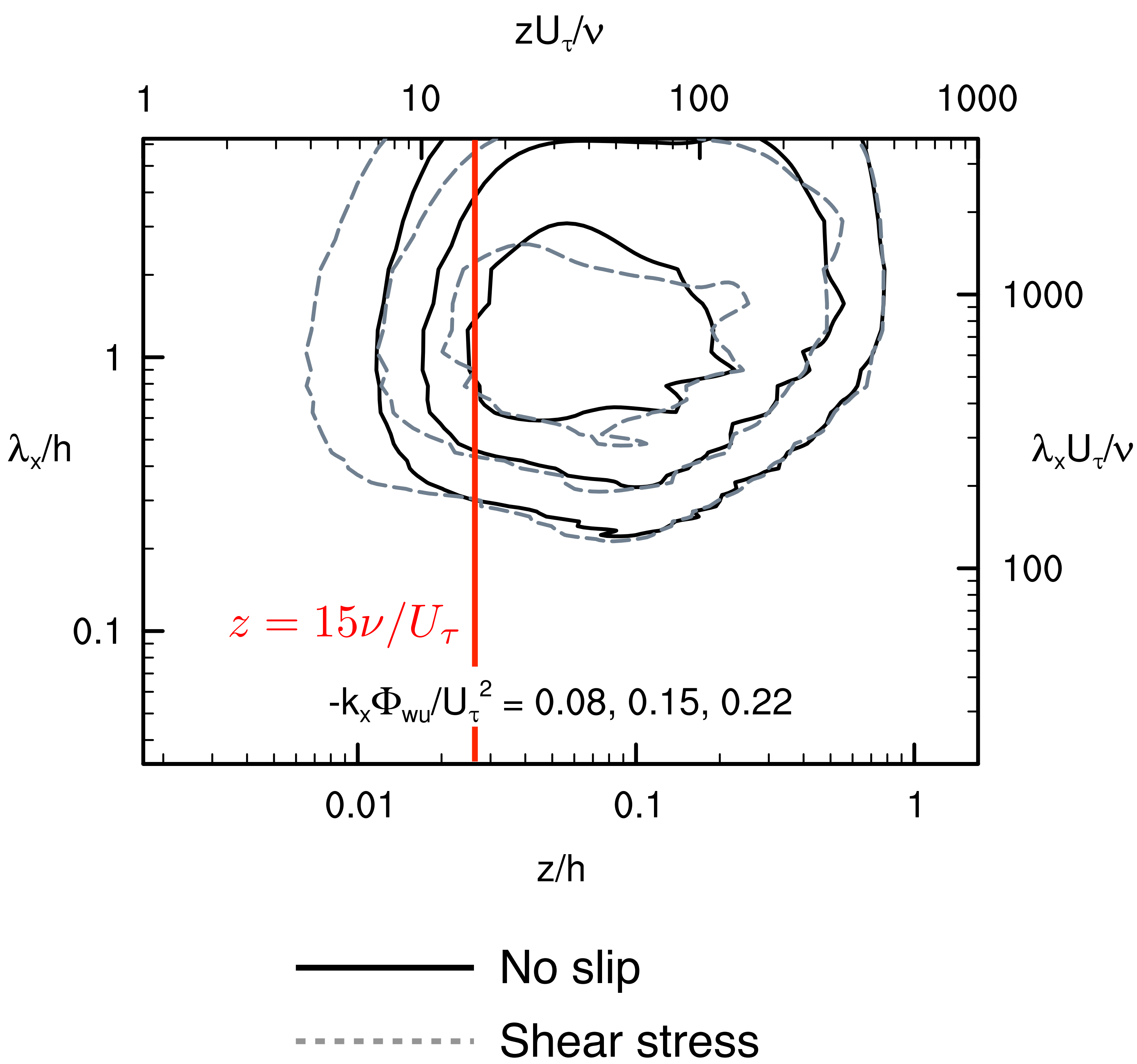
Spectra of spanwise velocity fluctuations



Spectra of wall-normal velocity fluctuations



Cospectra of streamwise–wallnormal velocity fluctuations



Townsend's hypothesis

Outer-layer (where Reynolds stresses dominate)

statistics of turbulent relative motion bounded by rigid walls

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Townsend's hypothesis

Idealised assessment shows that

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Higher-order statistics

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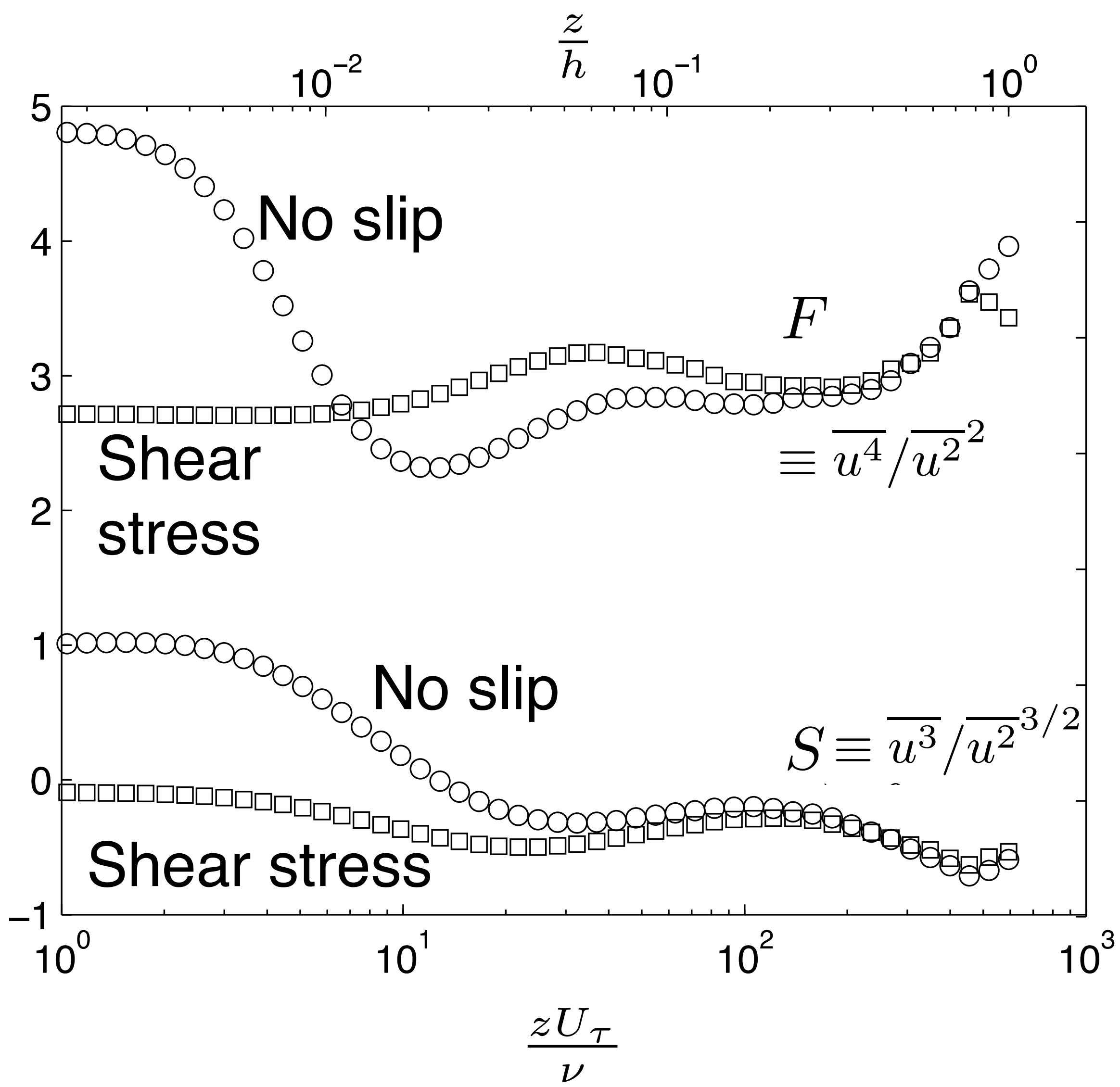
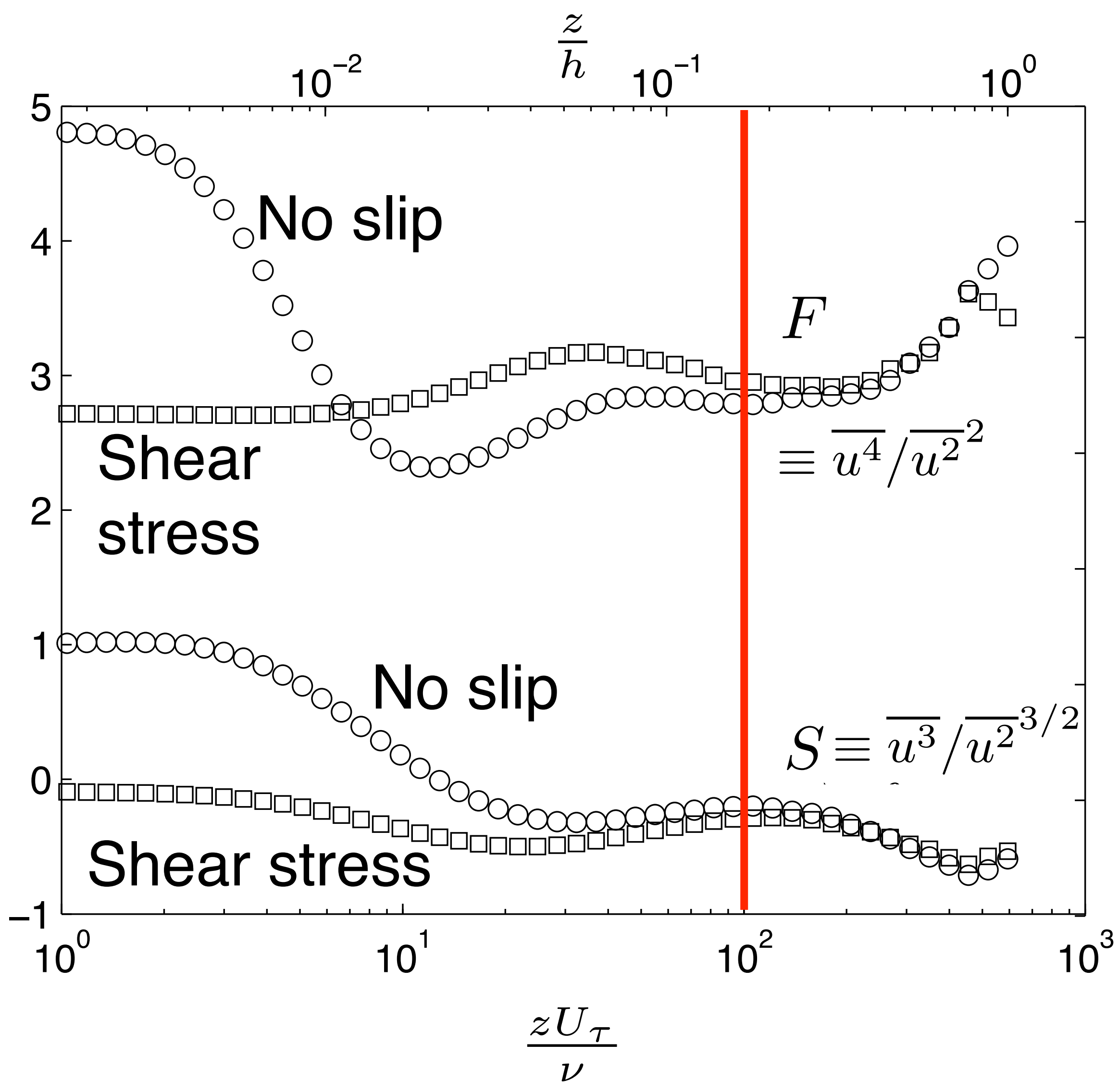
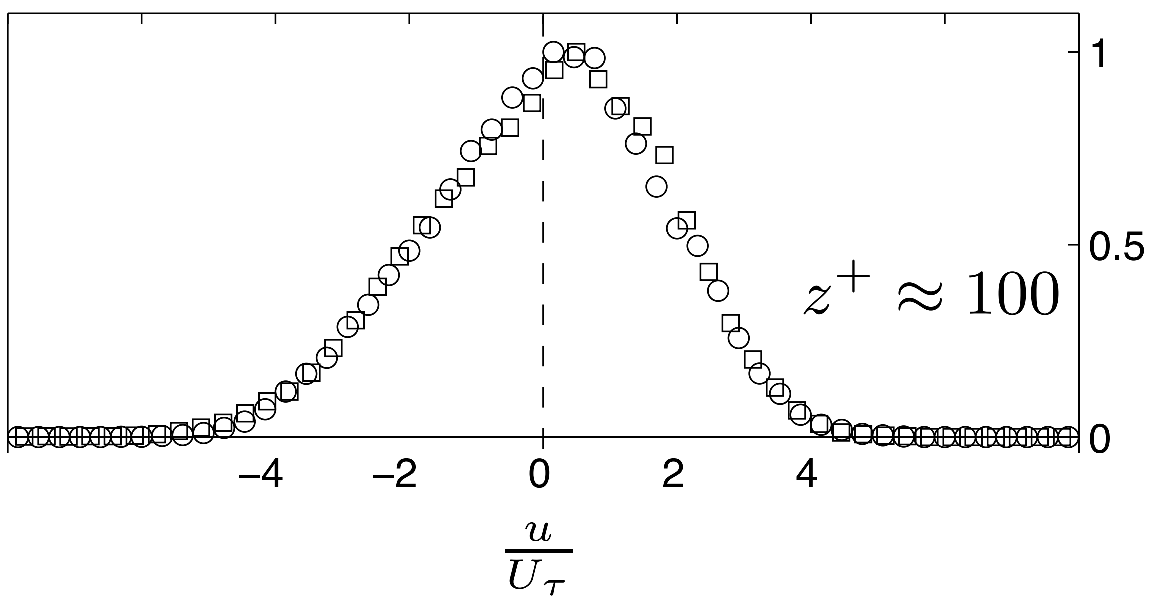


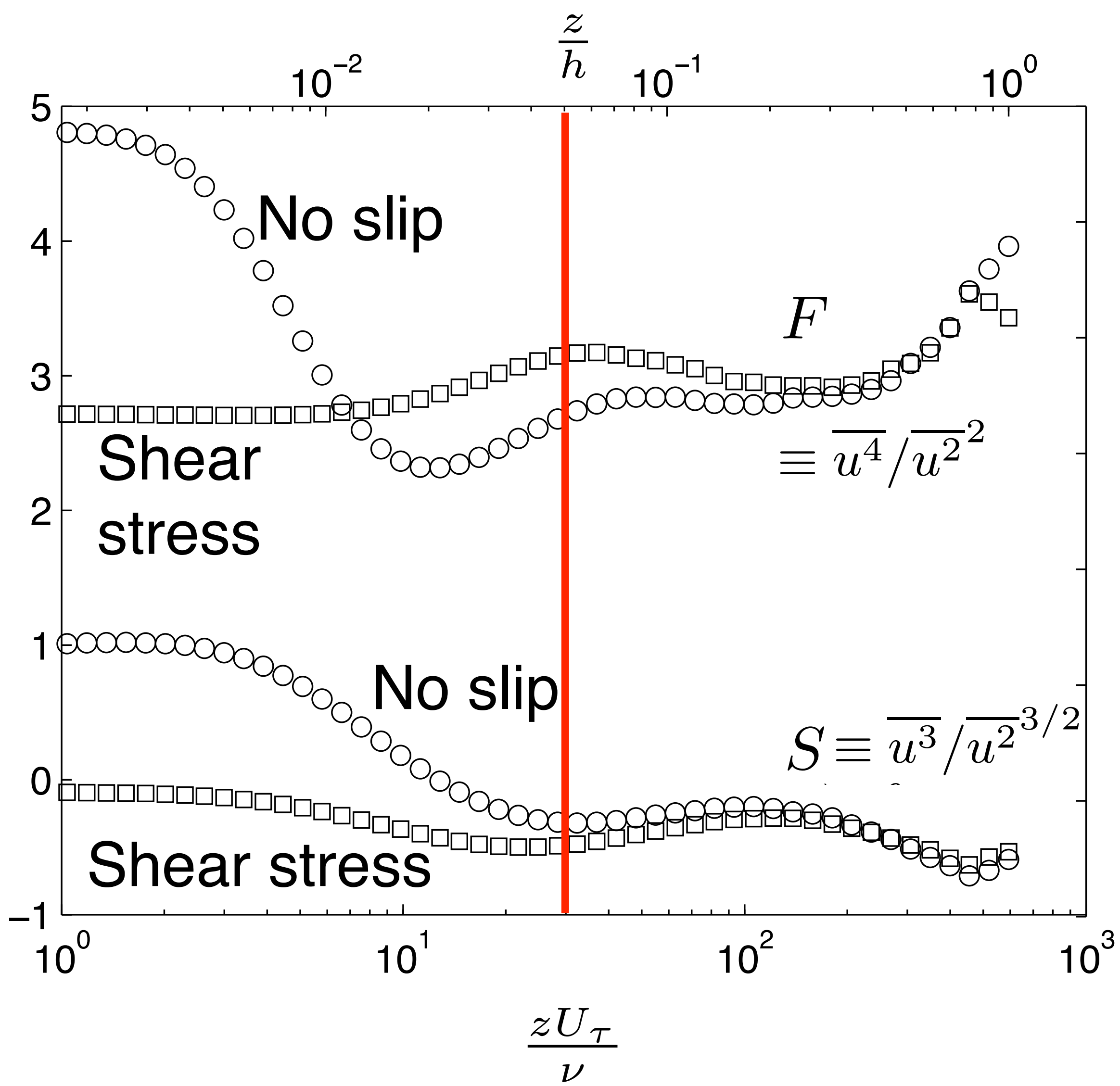
Higher-order statistics



Normalisation: max(PDF) = 1



Higher-order statistics



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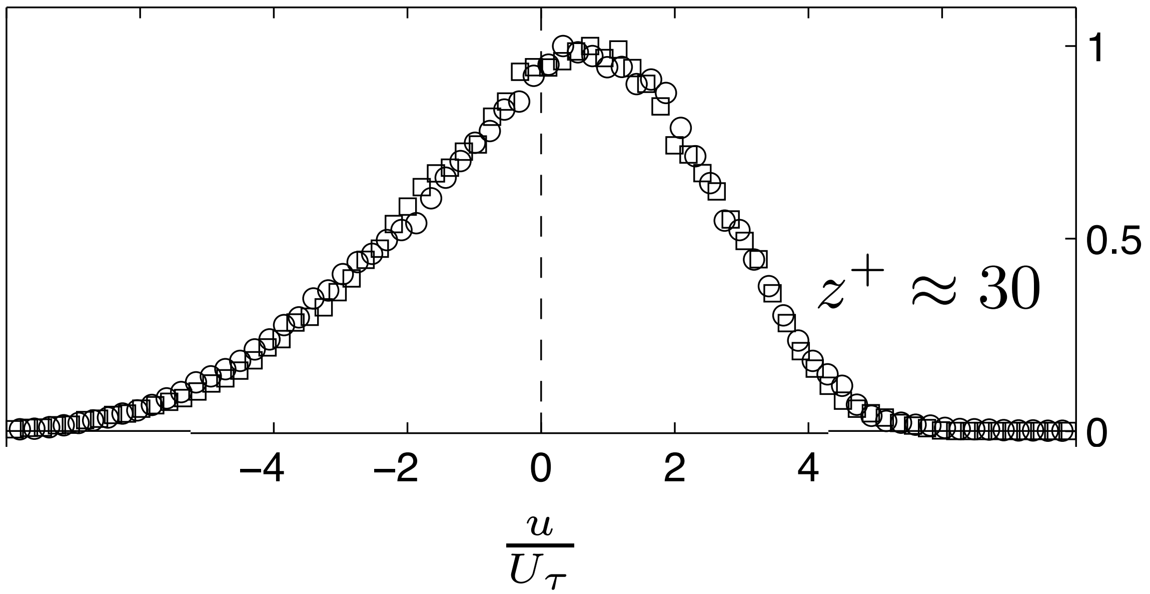

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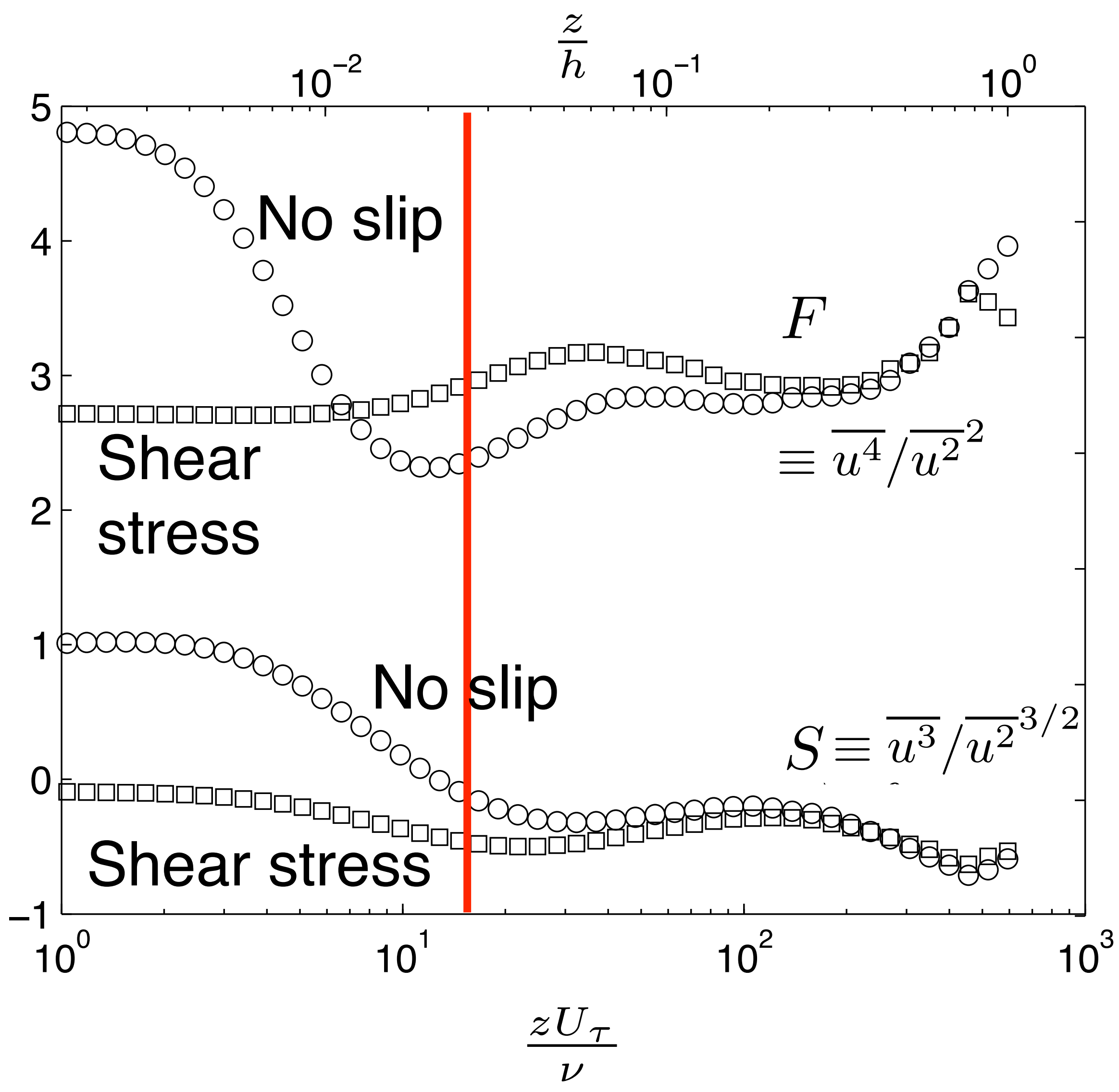

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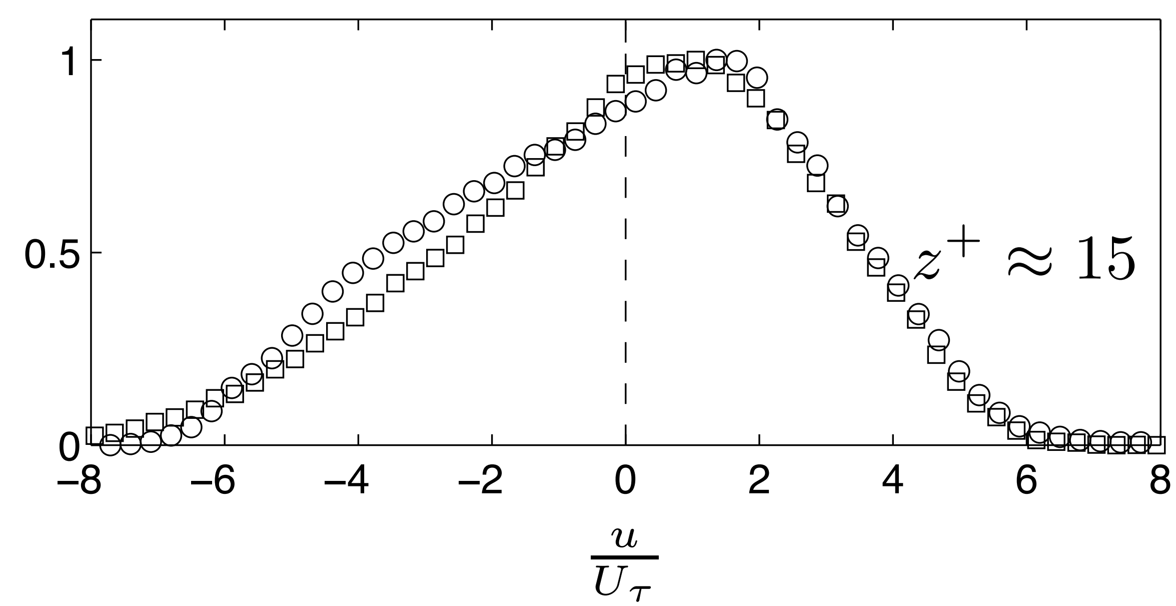

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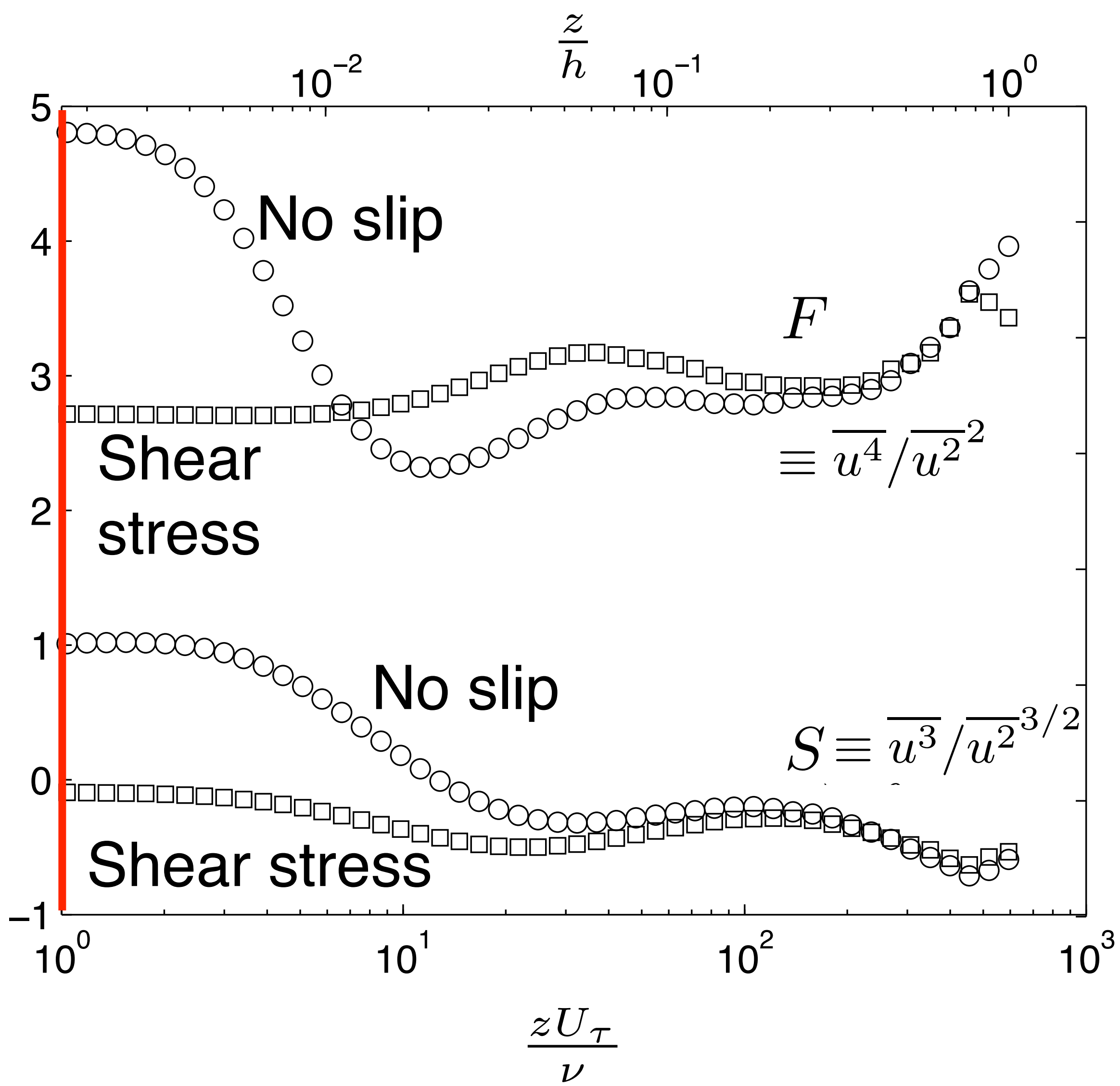

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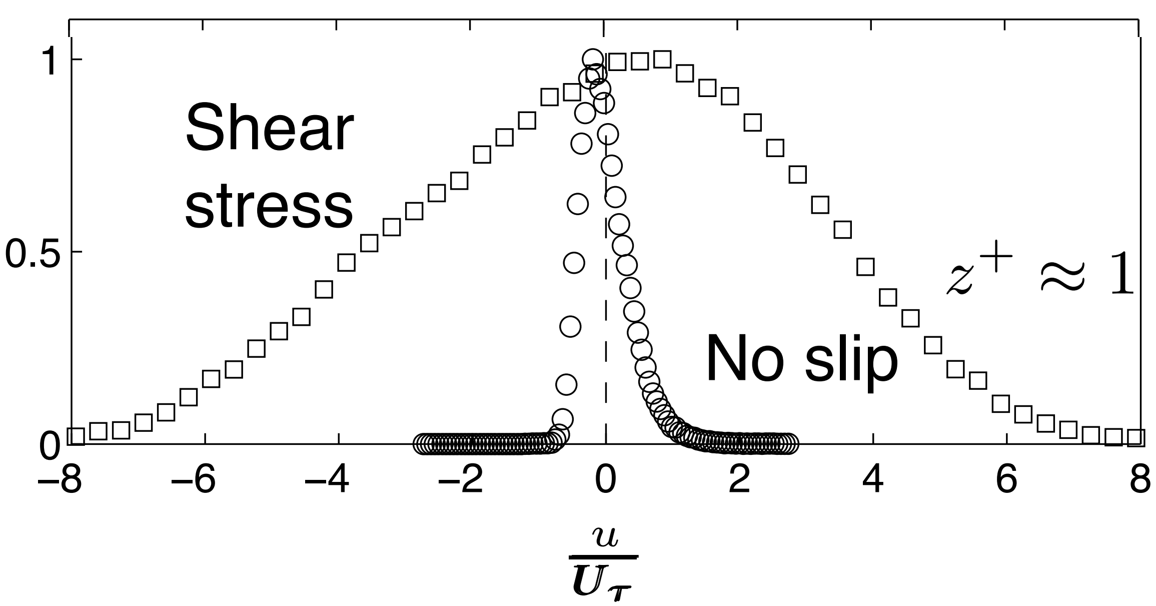

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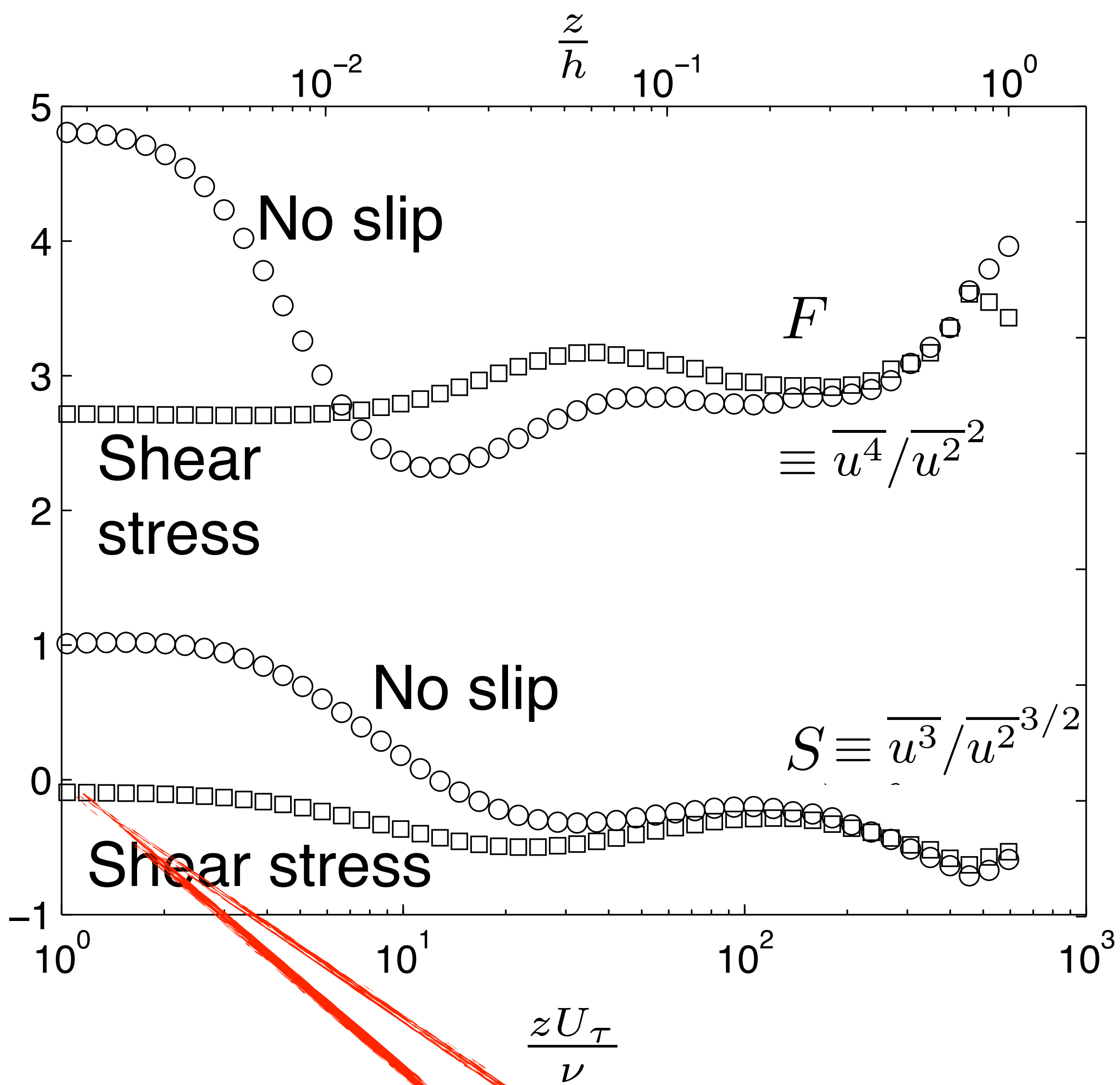

Viscous-sublayer motion not modulated by large scales.

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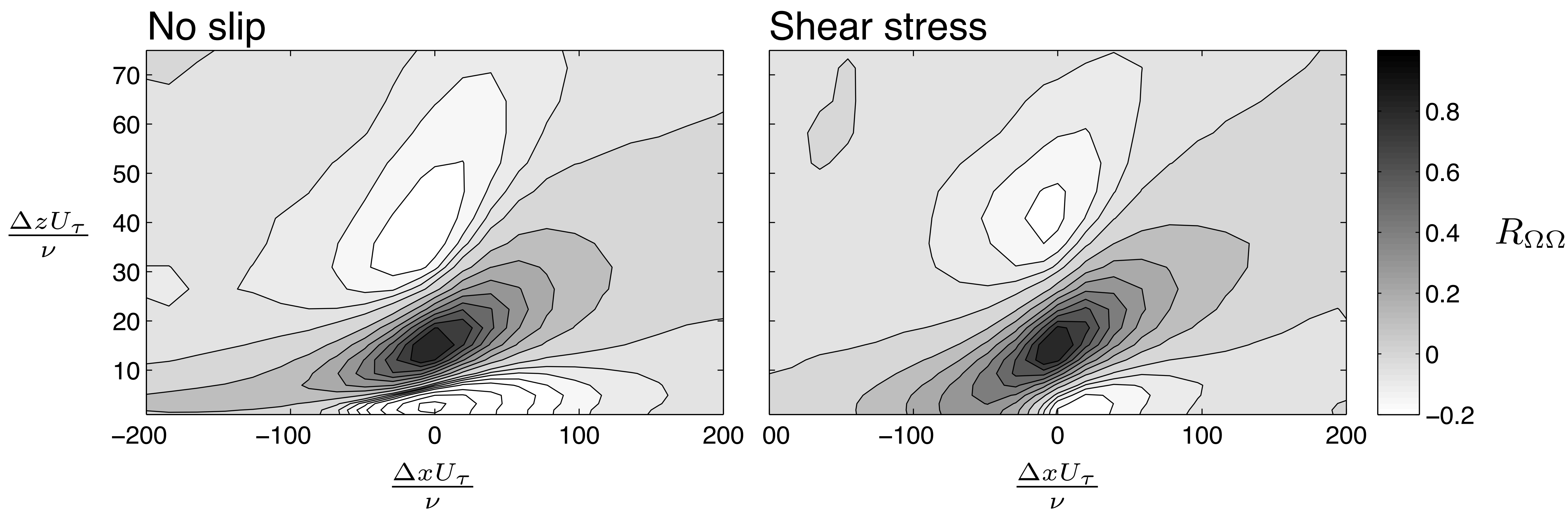
The viscous sublayer

Autocorrelation of streamwise vorticity

$$R_{\Omega\Omega}(\Delta x, \Delta z) \equiv \overline{\Omega_x(x, 15\nu/U_\tau)\Omega_x(x+\Delta x, 15\nu/U_\tau + \Delta z)}$$



Autocorrelation of streamwise vorticity

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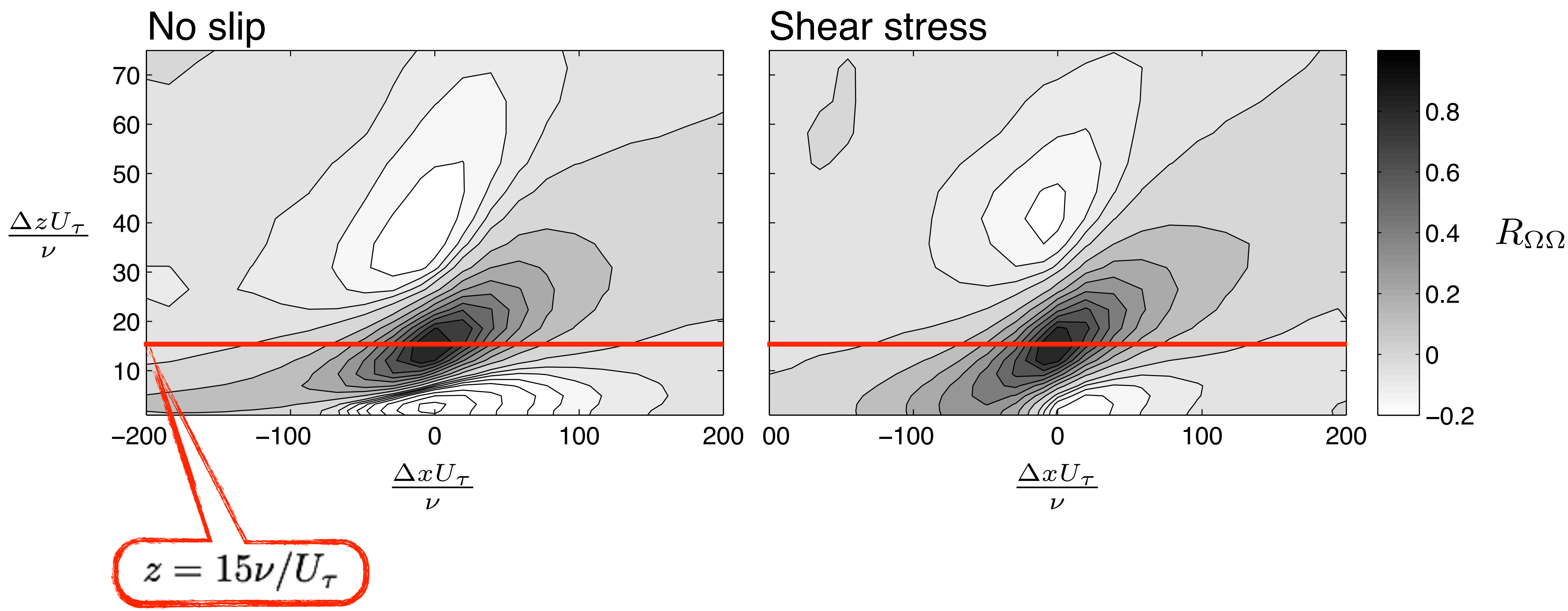

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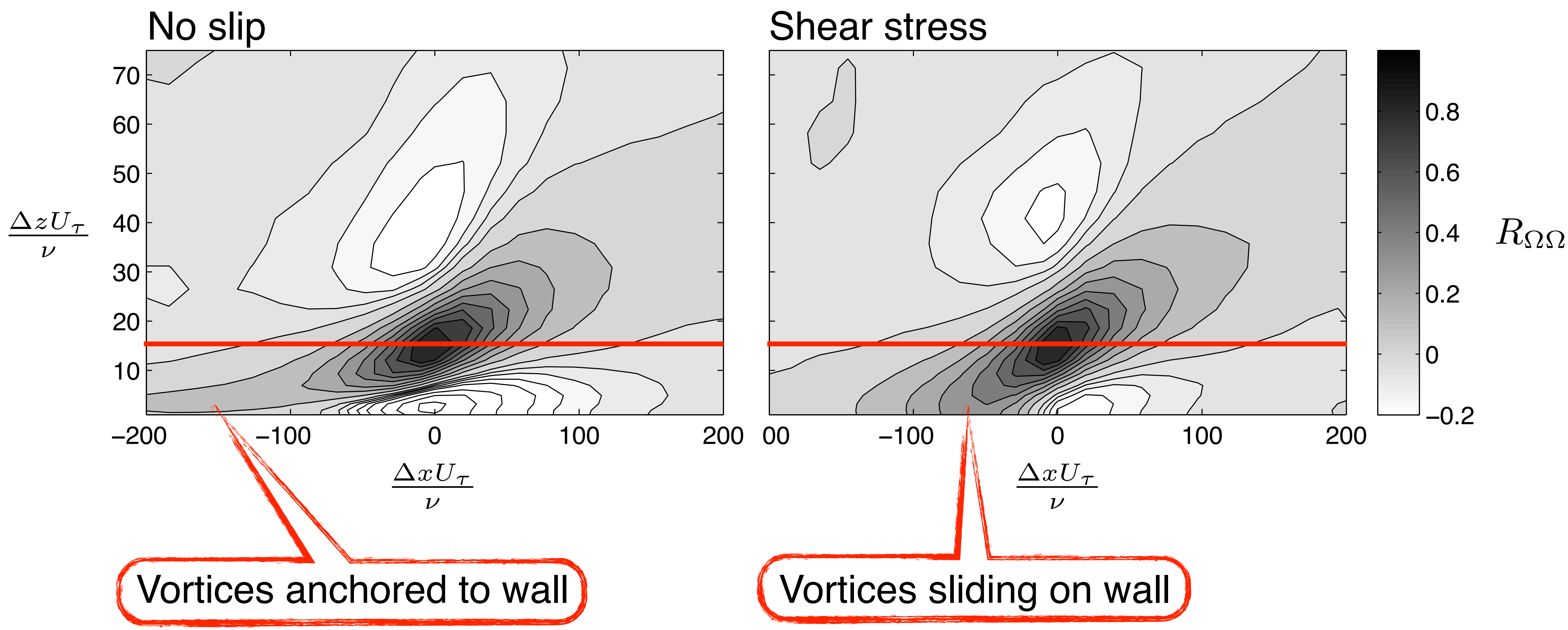
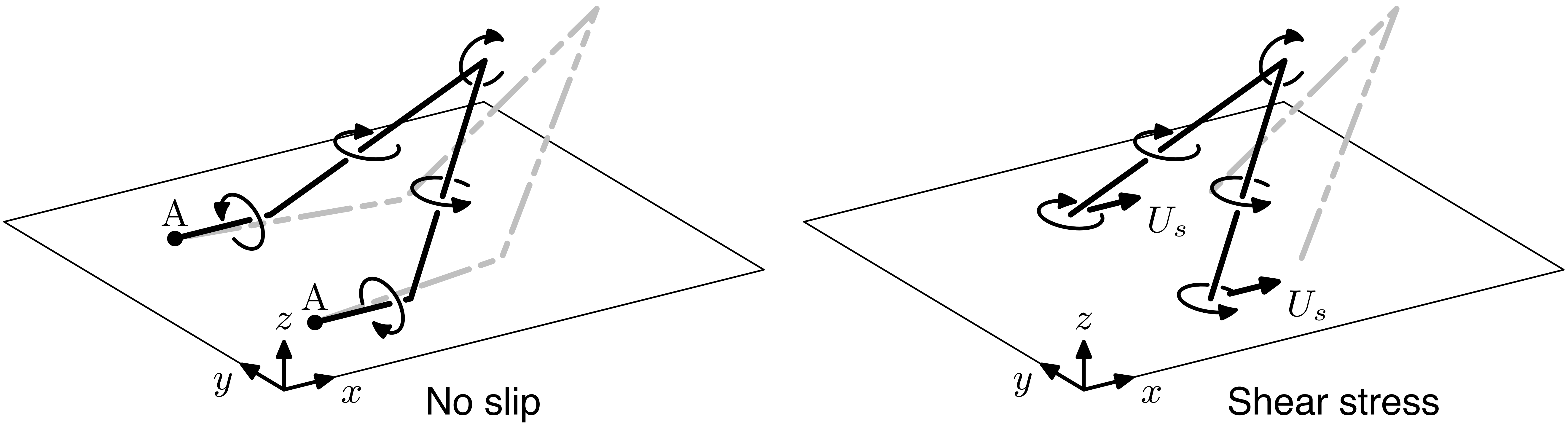


Illustration of differences in viscous sublayer



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The viscous sublayer exhibits structural differences that are nevertheless confined to the sublayer, suggesting that the no-slip boundary condition is not critical.

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An idealised assessment of Townsend's outer-layer similarity hypothesis for wall turbulence

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Direct numerical simulations of turbulent channel flow at the matched friction Reynolds number of 590, comparing the effect of no-slip versus shear-stress boundary conditions, reveal that the outer flow of wall turbulence, in accord with Townsend's outer-layer similarity hypothesis, remains largely independent of the viscous sublayer. First- and second-order statistics, including spectra, agree closely from the buffer region out to the centre of the channel. Higher-order statistics also appear to obey the hypothesised similarity, although the influence of boundary conditions is more pronounced than in the lower-order statistics. The statistical agreement in the outer layer, in spite of the structural differences in the viscous sublayer, support Townsend's idea that the primary effect of the wall is not the no-slip condition, but the impermeability condition imposed by a solid wall.

Key words: turbulent boundary layers, boundary layer structure, turbulence simulation