Homework #5 **Math 527, UNH spring 2015**
Due Tuesday, February 24th in recitation

Same instructions as usual regarding writing your name, section number, etc.

Problems 1-6. Find the general solution. If initial conditions are given, also solve the initial value problem. The "prime" notation indicates differentiation: $y' = dy/dt$, etc.

1. $y'' - 3y' + y = 0$

2. $2y'' + 3y' + 4y = 0$

3. $4y'' - 12y' + 9y = 0$

4. $9y'' + 6y' + y = 0; \quad y(0) = 1,\ y'(0) = 0$

5. $5y'' + 5y' - y = 0; \quad y(0) = 0,\ y'(0) = 1$

6. $y'' + 2y' + 5y = 0; \quad y(0) = 0,\ y'(0) = 2$

Problem 7.
(a) Show that $y_1(t) = e^{i\omega t}$ and $y_2(t) = e^{-i\omega t}$ are linearly independent complex-valued solutions of the ODE $y'' + \omega^2 y = 0$.

(b) Let

$$\hat{y}_1(t) = a_1\, y_1(t) + a_2\, y_2(t)$$
$$\hat{y}_2(t) = b_1\, y_1(t) + b_2\, y_2(t)$$

Find complex-valued constants a_1, a_2, b_1, b_2 such that $\hat{y}_1(t) = \cos \omega t$ and $\hat{y}_2(t) = \sin \omega t$.

(c) Show that $\hat{y}_1(t) = \cos \omega t$ and $\hat{y}_2(t) = \sin \omega t$ are also linearly independent solutions of the ODE.

(d) Express the general solution of the ODE in terms of the real-valued solutions $\hat{y}_1(t) = \cos \omega t$ and $\hat{y}_2(t) = \sin \omega t$.

Problem 8. Use Euler's formula $e^{ix} = \cos x + i \sin x$ to show that $(\cos x + i \sin x)^n = \cos nx + i \sin nx$, and then use this result to obtain the double-angle formulae $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$.

Problem 9. Find the general solution to the following ODE, using the ansatz $y(t) = e^{\lambda t}$ and reduction of order.

$$t\frac{d^2y}{dt^2} - (1 + 3t)\frac{dy}{dt} + 3y = 0$$

Problem 10. Find two linearly independent solutions of

$$t^2\frac{d^2y}{dt^2} + 5t\frac{dy}{dt} - 5y = 0$$

using the ansatz $y(t) = t^{\lambda}$.