Math 527 Solutions of HW #3

1. $3y'' + 6y' + 2y = 0$ (10 pts)

Find the roots of the auxiliary equation $3\lambda^2 + 6\lambda + 2 = 0$

$\lambda = \frac{-6 \pm \sqrt{6^2 - 4\cdot 3\cdot 2}}{2\cdot 3} = \frac{-3 \pm \sqrt{3}}{3} = -1 \pm \frac{\sqrt{3}}{3}$

So the general solution $y = C_1 e^{(-1-\frac{\sqrt{3}}{3})x} + C_2 e^{(-1+\frac{\sqrt{3}}{3})x}$

2. $y'' + 2y' + 3y = 0$ (10 pts)

Find the roots of the auxiliary equation $\lambda^2 + 2\lambda + 3 = 0$

$\lambda = \frac{-2 \pm \sqrt{2^2 - 4\cdot 3}}{2} = -1 \pm \sqrt{2}i$

so $y = C_1 e^{(-1-\sqrt{2}i)x} + C_2 e^{(-1+\sqrt{2}i)x}$

Or it can be written as $y = e^{-x}\cdot(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x)$

3. $y'' - 6y' + 9y = 0$ (10 pts)

Find the roots of the auxiliary equation $\lambda^2 - 6y + 9 = 0$

$\lambda_{1,2} = 3$, $\therefore y = C_1 e^{3x} + C_2 x e^{3x}$

4. $4y'' - 4y' + y = 0$; $y(0) = 0$, $y'(0) = 3$ (10 pts)

Find the roots of the auxiliary equation $4\lambda^2 - 4\lambda + 1 = 0$ (+3 pts)

$\lambda_{1,2} = \frac{1}{2}$, $y = C_1 e^{\frac{1}{2}x} + C_2 x e^{\frac{1}{2}x}$ (+2 pts)

By initial conditions, $y(0) = 0 \Rightarrow 0 = C_1$ $\therefore y = C_2 x e^{\frac{1}{2}x}$ (+2 pts)

$y' = C_2 e^{\frac{1}{2}x} + C_2 x\cdot\frac{1}{2}e^{\frac{1}{2}x}$ $y'(0) = C_2 \cdot 1 = 3$ $\therefore C_2 = 3$ (+2 pts)

Then $y = 3x e^{\frac{1}{2}x}$ (+1 pts)

5. $2y'' + y' - 10y = 0$; $y(1) = 5$, $y'(1) = 2$ (10 pts)

Find the roots of the auxiliary equation $2\lambda^2 + \lambda - 10 = 0$

$\lambda = \frac{-1 \pm \sqrt{1^2 + 4\cdot 2\cdot 10}}{2\cdot 2} = \frac{-1 \pm 9}{4} = -\frac{5}{2}, 2$; or factor $(2\lambda + 5)(\lambda - 2) = 0$

$y = C_1 e^{-\frac{5}{2}x} + C_2 e^{2x}$ By I.C.s, $y(1) = C_1 e^{-\frac{5}{2}} + C_2 e^{2} = 5$ ①

$y' = -\frac{5}{2} C_1 e^{-\frac{5}{2}x} + 2C_2 e^{2x}$, $y'(1) = -\frac{5}{2} C_1 e^{-\frac{5}{2}} + 2C_2 e^{2} = 2$ ②

$2\times① - ② \Rightarrow \frac{9}{2} C_1 e^{-\frac{5}{2}} = 8 \Rightarrow C_1 = \frac{16}{9} e^{\frac{5}{2}}$

$C_2 = \frac{5 - \frac{16}{9}}{e^2} = \frac{29}{9} e^{-2}$ So $y = \frac{16}{9} e^{\frac{5}{2}(1-x)} + \frac{29}{9} e^{2(-1+x)}$