Final exam, Dec. 14, 2011
Math 527, University of New Hampshire

Name: Answers
Section:

INSTRUCTIONS: PLEASE READ CAREFULLY

Write your name and section number above. 5 pts will deducted if either is missing or illegible. Write your final answers in the space provided. Show your work on attached sheets.

Problem 1: (15 points) Find the general solution of the differential equation.

$y' - 6x(y-1)^{2/3} = 0$ $\qquad y = (x^2 + c_1)^3 + 1,$ (SEPARABLE)

Problem 2: (15 points) Write down the general solution of each equation. For (b) and (c), assume $k > 0$. It is not necessary to show your work.

(a) $y' + ky = 0$ $\qquad y = c_1 e^{-kt}.$

(b) $y'' + ky = 0$ $\qquad y = c_1 \cos(\sqrt{k}\, t) + c_2 \sin(\sqrt{k}\, t).$

(c) $y'' - ky = 0$ $\qquad y = c_1 e^{\sqrt{k} t} + c_2 e^{-\sqrt{k}\, t}.$

Problem 3: (20 points) Find the solution of the initial value problem.

$y'' + 4y = \sin 3x, \quad y(0) = y'(0) = 0$ $\qquad y = \frac{3}{10} \sin(2x) - \frac{1}{5} \sin(3x).$

(note: can use var. of param., judicious guessing or Laplace transforms).

Problem 4: (20 points) Find the solution of the initial value problem.

$y'' + 2y' + 5y = \delta(t-3), \quad y(0) = 1,\ y'(0) = 0$

(Laplace)

$y(t) = \frac{1}{2} \mathcal{U}(t-3) e^{-(t-3)} \sin(2(t-3)) + e^{-t}\cos(2t) + \frac{1}{2} e^{-t} \sin(2t).$

Problem 5: (15 points) Find the general solution of the differential equation as a power series centered about $x = 0$. The first three terms of each linearly independent solution are enough.

$y'' - (x+1)y' - y = 0$

$$y = c_0\left[1 + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + \dots\right] + c_1\left[x + \tfrac{1}{2}x^2 + \tfrac{1}{2}x^3 + \dots\right].$$

Problem 6: (15 points) Find the general solution of the differential equation. Express your answer in terms of real-valued functions.

$\mathbf{x}' = \begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x}$

$$\vec{x} = c_1 e^{4t}\left[\begin{pmatrix} 1 \\ -1 \end{pmatrix}\cos(t) - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\sin(t)\right] + c_2 e^{4t}\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\cos(t) + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\sin(t)\right].$$