Homework #3: Applications of 1st-order equations Math 527, UNH spring 2013 Due Friday, February 15th in lecture

Problem 1. Newton's empirical law of heating/cooling is

$$\frac{dT}{dt} = k(T - T_m)$$

where $T(t)$ is the temperature of an object as a function of time, k is a constant of proportionality, and T_m is the ambient temperature. Suppose that you make a cup of coffee at 200 F at 8:00am in a room with ambient temperature $T_m = 70$ F, and that the coffee cools to 190 F at 8:02am. (a) What is the value of the constant k? (b) At what time will the coffee be 175 F?

Problem 2. A body falling through air and subject to gravity obeys the equation

$$m\frac{dv}{dt} = mg - kv$$

if we assume that the force of air resistance is linear in the velocity (an appropriate assumption if the flow around the body remains laminar). In this equation, v is the velocity of the body (positive is downwards), m is the mass of the body, g is the acceleration due to gravity, and k is a positive constant of proportionality. (a) What is the *terminal velocity* of the body, that is, the limit of $v(t)$ as t goes to infinity? (b) What is v as a function of time t, assuming the initial velocity is $v(0) = 0$? (c) Given that the the velocity $v(t)$ and position $s(t)$ are related by $v = ds/dt$, derive an explicit expression for $s(t)$ assuming $s(0) = 0$,

Problem 3. A body falling through air obeys the equation

$$m\frac{dv}{dt} = mg - \alpha v^2$$

with some constant $\alpha > 0$ if we assume that the force of air resistance is proportional to the square velocity and in the opposite direction (an appropriate assumption if the flow around the body becomes turbulent). (a) What is the terminal velocity of the body in this case? (b) What is v as a function of time t, assuming the initial velocity is $v(0) = 0$? (Hint: the equation is separable, but the resulting integral is trickier than most we've seen in this course. Use partial fractions.) (c) Derive an explicit expression for $s(t)$ assuming $s(0) = 0$.