Exam #1, Feb 11, 2013 Name: Solutions
Math 527, University of New Hampshire Section:

INSTRUCTIONS: PLEASE READ CAREFULLY

1. Write your name and section number above. 5 pts will deducted if either is missing or illegible.
2. Show your work and put a box or circle around your answers.
3. Always write equations.
4. Partial credit will be given only if your work is written clearly and in equations.

Problem 1. (30 pts) Find the general solution of the differential equation. Solve for $y(x)$ explicitly.

$$\frac{dy}{dx} = -x^2\, y^3$$

separable:

$$\frac{1}{y^3}\,\frac{dy}{dx} = -x^2$$

$$y^{-3}\,\frac{dy}{dx} = -x^2$$ integrating w.r.t x on both sides yields:

$$-\frac{1}{2}y^{-2} = -\frac{1}{3}x^3 + c$$ $(c_1 = -2c)$

$$y^{-2} = \frac{2}{3}x^3 + c_1$$

$$y = \pm\left(\frac{2}{3}x^3 + c_1\right)^{-1/2}$$

Problem 2. (30 pts) Find the general solution of the differential equation and then find the solution of the initial value problem. Solve for $y(t)$ explicitly.

$$\frac{dy}{dt} = te^{-t^2} - 2ty, \quad y(0) = 1$$

1st order linear

$$\frac{dy}{dt} + 2t\,y = te^{-t^2}$$

$\mu(t) = e^{\int 2t\,dt} = e^{t^2}$

$$e^{t^2}\frac{dy}{dt} + e^{t^2}(2t)y = t e^{-t^2} e^{t^2}$$

$$\frac{d}{dt}\left[e^{t^2} y\right] = t$$

integrating both sides w.r.t. t yields:

$$e^{t^2} y = \frac{1}{2}t^2 + c$$

$$y = e^{-t^2}\left(\frac{1}{2}t^2 + c\right)$$

$$1 = e^{(0)}\left(\frac{1}{2}(0) + c\right)$$

$$1 = c$$

$$y = e^{-t^2}\left(\frac{1}{2}t^2 + 1\right)$$

Problem 3. (30 pts) Find the general solution of the differential equation. An implicit solution is fine.

$$2(y^2 - e^{-x}\sin 2y)\frac{dy}{dx} = e^{-x}\cos 2y \qquad \Rightarrow 2(y^2 - e^{-x}\sin(2y))\frac{dy}{dx} - e^{-x}\cos(2y) = 0$$

Exact Equation: $M = -e^{-x}\cos(2y)$ $\qquad \frac{\partial M}{\partial y} = 2e^{-x}\cos(2y)$

$N = 2(y^2 - e^{-x}\sin(2y))$ $\qquad \frac{\partial N}{\partial x} = 2e^{-x}\cos(2y)$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, this is an exact equation, so there exists a function $f(x,y)$ s.t. $\frac{\partial f}{\partial x} = M$ and $\frac{\partial f}{\partial y} = N$.

So $f(x,y) = \int -e^{-x}\cos(2y)\,dx$

$= e^{-x}\cos(2y) + g(y).$

And $\frac{\partial f}{\partial y} = -2e^{-x}\sin(2y) + g'(y) \overset{\text{set}}{=} 2y^2 - 2e^{-x}\sin(2y)$

so $g'(y) = 2y^2$

Hence $g(y) = \frac{2}{3}y^3$

So $f(x,y) = e^{-x}\cos(2y) + \frac{2}{3}y^3$

Thus $2(y^2 - e^{-x}\sin(2y))\frac{dy}{dx} - e^{-x}\cos(2y) = 0$

$$\frac{\partial f}{\partial y}\cdot\frac{dy}{dx} + \frac{\partial f}{\partial x} = 0$$

$\frac{d}{dx}\left[f(x,y) \right] = 0$ integrating both sides w.r.t. x yields

$$e^{-x}\cos(2y) + \frac{2}{3}y^3 = c$$

Problem 4: (10 pts) Fill in this chart of substitution methods for 1st order differential equations.

type	general form	substitution	resultant ODE type
Homogeneous*	$\frac{dy}{dx} = f(y/x)$	$u = y/x$ or $y = u \cdot x$	separable
Bernoulli	$\frac{dy}{dx} + P(x)y = f(x)y^n$	$u = y^{1-n}$ $n \neq 0,1$	1st order linear
$u = Ax + By + C$	$\frac{dy}{dx} = f(Ax+By+C)$	$u = Ax+By+C$	separable