Homework #5 **Math 527, UNH fall 2015**
Due Thursday, October 8th in recitation

Same instructions as usual regarding writing your name, section number, etc.

Problems 1-6. Find the general solution. If initial conditions are given, also solve the initial value problem. The "prime" notation indicates differentiation: $y' = dy/dt$, etc.

1. $6y'' - 7y' + y = 0$

2. $y'' + 2y' + 3y = 0$

3. $y'' - 6y' + 9y = 0$

4. $2y'' + y' - 10y = 0; \quad y(1) = 5,\ y'(1) = 2$

5. $4y'' - 4y' + y = 0; \quad y(0) = 0,\ y'(0) = 3$

6. $y'' + y' + 2y = 0; \quad y(0) = 1,\ y'(0) = -2$

Problem 7. Consider the the initial value problem

$$y'' + 9y = 0; \quad y(0) = 2,\ y'(0) = -3/2.$$

(a) Express the general solution of $y'' + 9y = 0$ in terms of sines and cosines.

(b) Find the constants in (a) that satisfy $y(0) = 2,\ y'(0) = -3/2$, and then use those constants to find the solution of the initial value problem.

(c) Express the general solution of $y'' + 9y = 0$ in terms of complex exponentials.

(d) Find the constants in (c) that satisfy $y(0) = 2,\ y'(0) = -3/2$, and then use those constants to find the solution of the initial value problem.

Your answers for (b) and (d) should be the same. This demonstrates (for a particular case) that the real-valued and complex-valued forms of the general solution are just two different expressions for the same thing.

Problem 8. Use Euler's formula $e^{ix} = \cos x + i \sin x$ to show that $(\cos x + i \sin x)^n = \cos nx + i \sin nx$, and then use this result to obtain the double-angle formulae $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$.

Problem 9. Find two linearly independent solutions of

$$t^2 \frac{d^2 y}{dt^2} + 5t \frac{dy}{dt} - 5y = 0$$

using the ansatz $y(t) = t^\lambda$.