IAM 961 HW3: QR decomp

Due Monday Nov 2, 2015.

Problem 1.

Write the following Julia functions for computing the QR decomposition of a matrix

- `qrcgs(A)` via Classical Gram-Schmidt orthogonalization,
- `qrmgs(A)` via Modified Gram-Schmidt orthogonalization, and
- `qrhouse(A)` via Householder triangularization.

In [110]:
```
function qrcgs(A)
    # fill in
end

function qrmgs(A)
    # fill in
end

function qrhouse(A)
    # fill in
end
```

Out[110]:
```
qrhouse (generic function with 1 method)
```

Problem 2.

Test that your QR algorithms work correctly on a fairly small and well-conditioned matrix (e.g. a 5 x 5 matrix with normally distributed elements, `A = randn(5,5)`). You should test that Q is unitary and that $QR \approx A$. Verify to your own satisfaction that R is upper-triangular. Make these tests as comapct and readable as you can!

In [ ]:
```

```

Problem 3.

Write a `backsub(R,b)` function that computes a solution of $Rx = b$ by backsubstitution. You can assume that R is square and nonsingular. Test your backsubstitution function by solving an $Ax = b$ problem with your 5 x 5 A matrix, one of your QR algorithms, and a known solution x.

In [111]:
```
function backsub(R, b)
    # fill in
end
```

Out[111]: `backsub (generic function with 1 method)`

Problem 4.

Write a function `A = randommatrix(m,n kappa)` function that returns an m x n random matrix with condition number kappa and exponentially graded singular values (i.e. $\sigma_1/\sigma_m = \kappa$ and $\sigma_{j+1}/\sigma_j = \text{const}$). You can use the Matlab code at the top of pg 65 in Trefethen and Bau as a starting point. Test that it works by constructing a 4 x 4 matrix with kappa=10^8 and then computing its condition number.

In [45]:
```
function randommatrix(m,n, kappa)
    # fill in
end
```

Out[45]: `randommatrix (generic function with 1 method)`

Problem 5.

Solve a large number of random $Ax = b$ problems using your QR decompositions and `randommatrix` and `backsubstitution` functions, and produce a scatter plot of the normalized solution error $\|\hat{x} - x\|/\|x\|$ versus κ. Plot data points from CGS in blue, MGS in red, and Householder in green.

Specifically: Construct a random A matrix with $\kappa = 10^n$ where n is a random real-valued number uniformly distributed between 0 and 16. Select a random x vector with `x = randn(m,1)`, and then set $b = Ax$. For each of the CGS, MGS, and Householder QR algorithms, compute the numerical solution $\hat{x}$ of $Ax = b$ via QR and then plot $\|\hat{x} - x\|/\|x\|$ versus κ using log-log axes and the color scheme specified above. Do this for one hundred random $Ax = b$ problems and for a fairly small value of m (perhaps 10 or 20).

In [ ]:
```

```

Problem 6

Comment on your results. What can you explain about the scatter plots based on the algorithms and their implementation in finite-precision arithmetic? Or, contrariwise, what can you say about the algorithms based on the scatter plots?

Problem 7

If you are curious, repeat problem 5 for a different value of m (perhaps $m = 100$). Does the dimensionality of the matrix (the value of m) make any difference?

In [ ]: