Lab 4 **Math 445, UNH fall 2014**
Due Tuesday, Oct 27th

Problem 1: Write a function `x = newtonsearch(f, xguess)` that finds the solution x of the equation $f(x) = 0$ for an input function f and an initial guess $xguess$ using the Newton search algorithm.

- Use a `for` loop to perform the Newton-search iteration $x_{n+1} = x_n + \Delta x$. Take up to ten Newton steps.
- Use a `if` statement inside the `for` loop to test if either $|f(x)| < \epsilon$ or $|\Delta x| < \epsilon$ for some specified tolerance ϵ. If so, use a `break` statement to terminate the iteration and return from the function. For our purposes $\epsilon = 10^{-7}$ is a decent choice.

Problem 2: Test your Newton-search algorithm by solving the following problems. Check your answers by plugging the answer x back into f and verifying that $f(x)$ is approximately zero.

(a) Find an x for which $x^3 - 7x - 13 = 0$.

(b) Find the cube root of 54. Hint: devise an equation of theform $f(x) = 0$ whose solution is $x = \sqrt[3]{72}$.

(c) Find an x for which $\sqrt{4 - x^2} = x \tan x$.

Hint: find good initial guesses for the Newton search by plotting each function and roughly estimating an x position at which $f(x)$ is zero.

Problem 3: Use your Newton-search algorithm to solve the following problem. Utility companies must avoid freezing water mains in cold weather. If we assume uniform soil conditions, the temperature $T(x, t)$ at distance x below the surface and time t after the beginning of a cold spell is given approximately by

$$\frac{T(x,t) - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{\sqrt{2\alpha t}}\right)$$

where

- T_s is the constant surface temperature during the cold spell,
- T_i is the initial soil temperature before the cold spell started,
- α is the thermal conductivity of the soil, and
- erf is the *error function*, computed by the built-in Matlab function `erf`.

If x is in meters and t is in seconds, the $\alpha = 0.138 \times 10^{-6}\ m^2/s$. Let $T_i = 20\,C$ and $T_S = -15\,C$, and recall that water freezes at $T = 0\,C$. Use your Newton-search algorithm to determine how deep a water main must be buried so that it will not freeze until at least 60 days' exposure to these conditions.