Homework #2 **IAM 961, UNH fall 2013**
Due Monday, Oct 14th in lecture

1. Write functions for computing the QR decomposition of a matrix via

(a) classical Gram-Schmidt orthogonalization,

(b) modified Gram-Schmidt orthogonalization, and

(c) Householder triangularization.

Please do not look at the pseudo-code for the algorithms in Trefethen & Bau. Instead, start from the mathematical expressions for the algorithms and deveise the code yourself. Use whatever programming language you like. A scripted high-level language with built-in matrix functionality (e.g. Matlab, Octave, Python) will probably be easiest and most revealing. I suggest the following function names: `QRcgs, QRmgs, QRhouse`, and function signatures of the form `[Q,R] = QRcgs(A)`.

2. Write an `x = backsolve(R,b)` function for backsolving the system $Rx = b$ for upper-triangular matrices R.

3. Write a function `A = randommatrix(m, kappa)` that returns an $m \times m$ random matrix with condition number κ and exponentially graded singular values (i.e. $\sigma_1/\sigma_m = \kappa$ and $\sigma_{j+1}/\sigma_j = \text{const}$). You can use the Matlab code at the top of pg 65 in Trefethen and Bau as a starting point.

4. Solve a large number of random $Ax = b$ problems via QR decomposition using the `QRcgs` and `backsolve` sunctions from problems 1 and 2, and produce a scatter plot of the normalized solution error $\|\hat{x} - x\|/\|x\|$ versus κ. To form a random $Ax = b$ problem, construct a random A matrix with $\kappa = 10^n$ where n is a random real-valued number uniformly distributed between 0 and 16. Select a random x vector with `x = randn(m,1)`, and then set $b = Ax$. Compute the numerical solution $\hat{x}$ of $Ax = b$ via QR, and then plot $\|\hat{x} - x\|/\|x\|$ using log-log axes. Do this a for few hundred or thousand random $Ax = b$ problems and for a fairly small value of m (perhaps 10 or 20). Repeat with `QRmgs` and `QRhouse`. Turn in three scatter plots, one for each QR decomp algorithm.

5. Comment on your results. What can you explain about the scatter plots based on the algorithms and their implementation in finite-precision arithmetic? Play numerical detective by examining how accurately your QR algorithms produce QR decompositions, and relating that to the results shown in the three scatter plots.

6. Repeat problem 4 for a different value of m (perhaps $m = 100$). Does the dimensionality of the matrix (the value of m) make any difference?