Homework #11, part 2 **Math 527, UNH fall 2011**
Due Wednesday, Nov. 23 in lecture.

Problem 5. Derive the equations $\det(\mathbf{A}-\lambda\mathbf{I}) = 0$ and $(\mathbf{A}-\lambda\mathbf{I})\mathbf{v} = 0$ from the differential equation $\mathbf{x}' = \mathbf{A}\mathbf{x}$ and the ansatz $\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$.

Problems 6,7.
(a) Express the system of differential equations as a matrix equation $\mathbf{x}' = \mathbf{A}\mathbf{x}$.
(b) Determine the linearly independent solutions $\mathbf{x}_1(t), \mathbf{x}_2(t), \ldots$ of the system by computing the eigenvalues and eigenvectors of $\mathbf{A}$.
(c) Express the general solution of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ as a linear combination of the linearly independent solutions from (b).

Problem 6.

$$\frac{dx}{dt} = x + 2y$$
$$\frac{dy}{dt} = 4x - 6y$$

Problem 7.

$$\frac{dx}{dt} = -x + y$$
$$\frac{dy}{dt} = x + 2y + z$$
$$\frac{dz}{dt} = 3y - z$$

Problem 8. Find the solution of Problem 7 with the initial conditions $x(0) = 1$, $y(0) = 0,\ z(0) = 2$, or equivalently, $(x, y, z)(0) = (1, 0, 2)$.