Practice final exam **Name:**
Math 527, University of New Hampshire **Section:**

INSTRUCTIONS: Show your work on attached sheets.

Problem 1: (15 pts) Find the general solution of the differential equation. Hint: There's is an easy way to do this problem.

$$y' - 3y = x$$

Problem 2: (15 pts) Find the general solution of the differential equation.

$$y'' - 4y = 12e^{2x}$$

Problem 3: (15 pts) Find the general solution of the differential equation using variation of parameters. Note that $\sinh x = (e^x - e^{-x})/2$.

$$y'' - y = \sinh 2x$$

Problem 4: (20 pts) Solve the initial-value problem using Laplace transforms

$$y'' + 4y = \begin{cases} 0, & 0 \le t < 2\pi \\ \sin t, & 2\pi \le t \end{cases}$$

$$y(0) = 1, \quad y'(0) = 0$$

Problem 5: (20 pts) Give both the complex-valued and real-valued form of the general solution of the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Problem 6: The eigenvalue-eigenvector method of solving the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the n-dimensional vector $\mathbf{x}$ and $n \times n$ matrix $\mathbf{A}$ requires that we solve the equations $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ and $(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = 0$.
(a) (10 pts) Derive these two equations from $\mathbf{x}' = \mathbf{A}\mathbf{x}$ and an appropriate ansatz.
(b) (5 pts) How many eigenvalue solutions λ will there be for the equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$?