Name:

Exam #1, Math 796.05/896.05, Topics: Numerical Linear Algebra

Problem 1. The least-squares problem is to find the $x \in \mathbb{C}^n$ that minimizes $\|Ax - b\|_2$, given a matrix $A \in \mathbb{C}^{m \times n}$ and a right-hand-side vector $b \in C^m$. We have shown that, if A has full rank, the solution x can be found by solving the normal equations $A^*Ax = A^*b$, or by solving the upper-triangular system $Rx = Q^*b$, where $A = QR$ is the QR decomposition of A. Is one of these approaches better than the other? If so, why?

Problem 2. (a) What is the condition number $\kappa(A)$ of a square, nonsingular matrix A?

(b) What is the condition number $\kappa(x)$ of matrix-vector multiplication $f : x \to Ax$? Consider A to be held fixed and the condition number to vary with x.

(c) Note that $\kappa(A)$ is the least upper bound of $\kappa(x)$. For what x does $\kappa(x) = \kappa(A)$? (Hint: use the SVD.)

Problem 3. Provide an example of an algorithm $\tilde{f} : X \to Y$ with the given stability/accuracy property and explain why it has those properties. Examples from class and the text are fine, but please try to give examples that illustrate the principles (e.g. for (d), don't give $\tilde{f}(x) = 0$ as an algorithm for $f(x) = 0$).

(a) unstable

(b) stable but not backward stable

(c) backward stable but not necessarily accurate

(d) accurate

Problem 4. What is the crucial difference between the Householder tridiagonalization algorithm and the Gram-Scmidt orthogonalization algorithms for computing the QR decomposition?