Homework #2 **IAM 961, UNH fall 2011**
Due Friday, Oct 7th in lecture

1. (Trefethen exercise 4.4) Two matrices $A, B \in \mathbb{C}^{m \times m}$ are *unitarily equivalent* if $A = QBQ^*$ for some unitary matrix Q. Is it true or false that A and B are unitarily equivalent if and only if they have the same singular values?

2. (Trefethen exercise 4.5) Show that any real-valued matrix has a real-valued SVD.

3. (Trefethen Theorem 5.4) Show that the nonzero singular values of A are the square roots of the nonzero eigenvalues of A^*A or AA^*.

4. (Trefethen Theorem 5.5) Show that the singular values of a hermitian matrix are the absolute values of its eigenvalues.

5. There are two flaws in the uniqueness portion of Trefethen Theorem 4.1. What are they? Can you fix them?

6. The SVD is used in statistical data analysis to compute the *principal components* of multivariate random data. Suppose you have M observations of an N-dimensional random variable $x = (x_1, x_2, x_3, \ldots, x_N)$ gathered in an $M \times N$ matrix X, so that the element X_{ij} is the jth component of the ith observation. For example, each row of X might represent N measurements of a hospital patient's physical state (blood pressure, heart rate, etc.), and the set of such observations over M patients would result in the $M \times N$ matrix X. For simplicity's sake, assume that the mean of each column is zero.[1]

Define the *correlation* $\langle x_i x_j \rangle$ of variable x_i with x_j to be the average of the product of those variables over the M observations. That is,

$$\langle x_i x_j \rangle = 1/M \left[X^T X\right]_{ij}$$

In general, the variables of the observed data will be correlated; that is, $\langle x_i x_j \rangle$ will generally be nonzero, indicating that there is statistical dependence between x_i and x_j.

(a) Show that the right singular matrix V of the SVD $X = U\Sigma V^*$ provides a coordinate transformation $Y = XV$ that *decorrelates* the observations, that is, that the components of the transformed observations $y = x\,V$ are uncorrelated, or

$$\langle y_i y_j \rangle = \begin{cases} \langle y_i^2 \rangle & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

(b) Show how the same transformation V can be determined from the SVD of the $N \times N$ correlation matrix $C_{ij} = \langle x_i x_j \rangle$.

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[1] If it is not, you can always define X to be the difference of each observation from its mean value.