Homework #1 **IAM 961, UNH fall 2011**
Due Friday, Sept 16th in lecture

1. Prove that any linear map $\mathcal{L} : \mathbb{C}^n \to \mathbb{C}^m$ can written as an $m \times n$ matrix.

2. Prove that $A \in \mathbb{C}^{m\times n}$ with $m \geq n$ has full rank iff $Ax \neq Ay \; \forall x, y \in \mathbb{C}^n$, using just the basic definition of rank.

3. Prove that $\|AB\|_p \leq \|A\|_p \|B\|_p$.

4. Prove that $\|A\|_1 = \max_{1\leq j\leq n} \|a_j\|_1$ where $\{a_j\}$ are the columns of the $A \in \mathbb{C}^{m\times n}$.

5. If u and v are m-vectors the matrix $A = I + uv^*$ is known as a *rank-one perturbation of the indentity.* Show that if A is nonsingular, then its inverse has the form $A^{-1} = I + \alpha uv^*$ for some scalar α, and give an expression for α. For what u and v is A singular? If it is singular, what is null(A)? (Trefethen exercise 2.6).

Note that a couple of these are proofs are outlined verbally in the text. You can look at these proofs and follow the general strategy. But see if you can improve on the presentation.