Sample exam #1, Mar. 01, 2018
Math 445, University of New Hampshire

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1. Write one line of Matlab code that returns the even numbers from 48 to 764, inclusive.

```
48:2:764
```

2. Given a matrix A, write one line of Matlab code that returns a matrix consisting of the 3rd, 11th, and 1st columns of A, in that order.

```
A(:, [3 11 1])
```

3. Write a few lines of Matlab code that would plot $y = x^{-2} \sin 4x - 1$ versus x for $1 \leq x \leq 2\pi$ as a magenta dashed line with a superimposed grid. Label your axes.

```
x = linspace(1, 2*pi, 100);
plot(x, (x.^-2) .* sin(4*x-1), 'm--')
xlabel('x')
ylabel('y')
grid on
```

some of these parantheses are not necessary but they don't hurt

4. Calculate the product of the matrix and vector.

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+3+0 \\ -2+9+4 \\ 10+0+6 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ 16 \end{bmatrix}$$

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5. Write a few lines of Matlab code that could calculate the product of the matrix and vector.

$$y = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

```
A = [2 1 0; -1 3 4; 5 0 6];
x = [2; 3; 1];
y = A*x
```

6. Write two lines of Matlab code that would evaluate this sum.

$$\sum_{n=0}^{10} \frac{n}{(n+1)^2}$$

```
n = 0:10
sum(n./(n+1).^2)
```

7. Write Matlab code that would solve the system of equations.

$$\begin{aligned} 3y - x + 4z - 6 &= 0 \\ 5z + 2x - 7 &= 0 \\ y - 2x + 1 &= 0 \end{aligned}$$

$$\begin{aligned} -x + 3y + 4z &= 6 \\ 2x + 5z &= 7 \\ -2x + y &= -1 \end{aligned}$$

or $\begin{bmatrix} -1 & 3 & 4 \\ 2 & 0 & 5 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ -1 \end{bmatrix}$

Matlab code

```
A = [-1 3 4; 2 0 5; -2 1 0];
b = [6; 7; -1];
x = A\b
```

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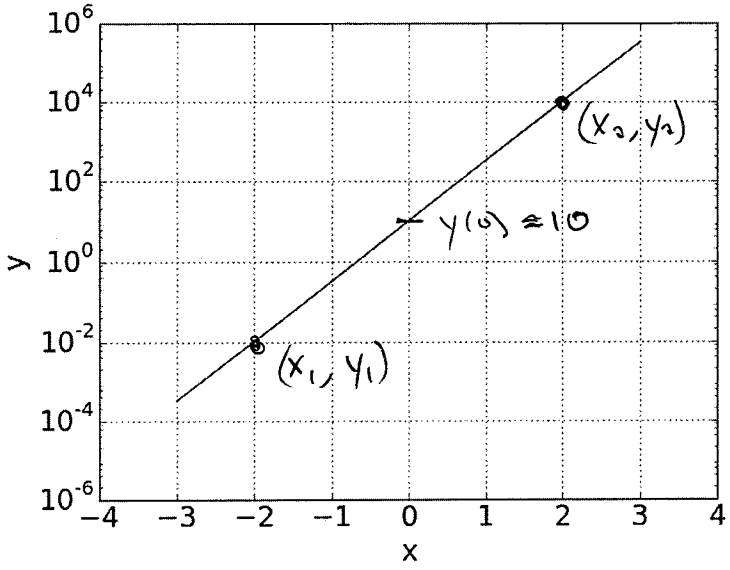
8. What is y as a function of x? Give an explicit formula for $y(x)$ with specific numerical constants.



log y is linear in x

therefore

$\log y = mx + b$

$y = 10^{mx+b}$

$y = c\,10^{mx}$

m is the slope of log y vs x

$$m = \frac{\log y_2 - \log y_1}{x_2 - x_1} = \frac{4 - (-2)}{2 - (-2)} = \frac{6}{4} = \frac{3}{2}$$

$y(0) = c\,10^{m \cdot 0} = c$ so $c = 10$, judging from graph

$\Rightarrow$ $\boxed{y(x) = 10 \cdot 10^{3x/2}}$ or $10^{\frac{3}{2}x+1}$

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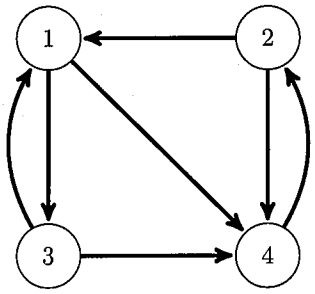
9. Write Matlab code that defines a function named `myfactorial` that uses a **for** loop to compute $n!$ (the factorial of n) according to the formula

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 = \prod_{k=1}^{n} k$$

```
function p = myfactorial(n)
    p=1;
    for k=1:n
        p = p*k;
    end
end
```

10 (a) Write down the matrix A for hamster dynamics $x^{n+1} = A\,x^n$, where $x = (x_1, x_2, x_3, x_4)$, assuming hamsters choose and run through a random tunnel from their current house at each time step.



$$A = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 1/2 & 0 \end{bmatrix}$$

(A_{ij} represents fraction of hamsters going from house j to house i)

(b) Write a few lines of Matlab code that would estimate the steady-state distribution of hamsters $x = \lim_{n\to\infty} x^n$ from twenty iterations of $x^{n+1} = A\,x^n$.

```
A = [0 1/2 1/2 0; 0 0 0 1; 1/2 0 0 0; 1/2 1/2 1/2 1/2];
x = [100; 0; 0; 0]   % or any other initial distrib. you like
for n=1:20
    x = A*x;
end
```