More on Why linearity is important to the solutions of linear ODEs, based on your lecture notes.

Linearity is important for the solution of ODEs because it allows us to solve for the particular solution to nonhomogeneous problems by breaking up the right-hand side of the differential equation into its linearly independent components and solving for the individual particular solutions. The entire particular solution is then formed from adding the individual particular solutions.

$L(y_{p1} + y_{p2} + y_{p3}) = Ly_{p1} + Ly_{p2} + Ly_{p3}$