Homework #7Due Tuesday, March 10th in recitation

Same instructions as usual regarding the format of your homework.

Problems 1-4. Find the general solution of the differential equation using variation of parameters. If initial conditions are given, also solve the initial value problem. The "prime" notation indicates differentiation with respect to the variable that appears on the right-hand side of the equation. (Note: most of these problems could also be solved by judicious guessing.)

- 1. $2y'' 3y' + y = (t^2 + 1)e^{2t}$
- **2**. $y'' + y = \sec t$, $-\pi/2 < t < \pi/2$
- **3**. $y'' 3y' + 2y = te^{3t} + 1$
- 4. $3y'' + 4y' + y = e^{-t} \sin t$, y(0) = 1, y'(0) = 0

Problem 5. Give short answers to the following questions.

- (a) What property must an operator L satisfy to be *linear*?
- (b) What does it mean for functions $\{y_1(x), y_2(x), \dots, y_n(x)\}$ to be *linearly dependent*?
- (c) What does it mean for functions $\{y_1(x), y_2(x), \ldots, y_n(x)\}$ to be *linearly independent*?
- (d) Why are these properties important for the solution of linear differential equations?

(e) How many linearly independent solutions does an nth order linear homogeneous equation have?

(f) Is a particular solution to a nonhomogeneous equation linearly independent from the solutions to the homogeneous solutions, or linearly dependent on them?

(g) What is Euler's formula?

(h) How would you prove Euler's formula? Don't do the proof, just describe the proof in a sentence or two.