$\lambda_2 = -\frac{9}{2} - i\frac{\sqrt{7}}{2}$, $(A-\lambda_2 I)\vec{v_2} = \vec{0}$

$$\begin{pmatrix} \frac{7}{2}+i\frac{\sqrt{7}}{2} & 2 \\ -7 & -\frac{7}{2}+i\frac{\sqrt{7}}{2} \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-7k_1 + (-\frac{7}{2}+i\frac{\sqrt{7}}{2})k_2 = 0$

$k_1 = (-\frac{1}{2}+i\frac{1}{2\sqrt{7}})k_2$

$\vec{v_2} = \begin{pmatrix} -\frac{1}{2}+i\frac{1}{2\sqrt{7}} \\ 1 \end{pmatrix} k_2$

8. solution. $A = \begin{pmatrix} -8 & -1 \\ 16 & 0 \end{pmatrix}$ $\det(A-\lambda I) = 0$, $\begin{vmatrix} -8-\lambda & -1 \\ 16 & -\lambda \end{vmatrix} = (8+\lambda)\lambda + 16 = 0$

$\lambda^2 + 8\lambda + 16 = 0$ $(\lambda+4)^2 = 0$ $\therefore \lambda_1 = -4, \lambda_2 = -4$ (Repeated!)

$(A-\lambda_1 I)\vec{v_1} = \vec{0} \iff \begin{pmatrix} -4 & -1 \\ 16 & 4 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow k_2 = -4k_1$, $\vec{v_1} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

pick $k_1 = 1$

Then $(A-\lambda_2 I)\vec{v_2} = \vec{v_1}$ $\begin{pmatrix} -4 & -1 \\ 16 & 4 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$ $-4k_1 - k_2 = 1$

$k_2 = -4k_1 - 1$, $\vec{v_2} = \begin{pmatrix} k_1 \\ -4k_1 - 1 \end{pmatrix}$

$\vec{v_2} = k_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ set $k_1 = 0$, $\vec{v_2} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

9. solution $A = \begin{pmatrix} -1 & 2 \\ -5 & 1 \end{pmatrix}$ $\det(A-\lambda I) = 0$, $\begin{vmatrix} -1-\lambda & 2 \\ -5 & 1-\lambda \end{vmatrix} = (\lambda+1)(\lambda-1) + 10 = \lambda^2 + 9 = 0$

$\therefore \lambda = \pm 3i$ $\lambda_1 = 3i$, $(A-\lambda_1 I)\vec{v_1} = \vec{0}$, $\begin{pmatrix} -1-3i & 2 \\ -5 & 1-3i \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $(-1-3i)k_1 + 2k_2 = 0$

$k_2 = \frac{1}{2}(1+3i)k_1$

$\vec{v_1} = \begin{pmatrix} 1 \\ \frac{1}{2}(1+3i) \end{pmatrix} k_1$, $k_1 \in \mathbb{R}$. $\lambda_2 = -3i$, $(A-\lambda_2 I)\vec{v_2} = \vec{0}$

$\begin{pmatrix} -1+3i & 2 \\ -5 & 1+3i \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $k_2 = \frac{1}{2}(1-3i)k_1$ $\therefore \vec{v_2} = \begin{pmatrix} 1 \\ \frac{1}{2}(1-3i) \end{pmatrix} k_1$.

10. solution. $A = \begin{pmatrix} 2 & -1 & 0 \\ 5 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}$, $\begin{vmatrix} 2-\lambda & -1 & 0 \\ 5 & 2-\lambda & 4 \\ 0 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^3 + 5(2-\lambda) - 4(2-\lambda)$ $\therefore \lambda_1 = 2$

$= (2-\lambda)[(\lambda-2)^2 + 1] = 0$ $\lambda_2 = 2+i, \lambda_3 = 2-i$

$(A-\lambda_1 I)\vec{v_1} = \vec{0}$ $\left(\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 5 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \Rightarrow k_2 = 0$, $5k_1 + 4k_3 = 0$ $\therefore k_3 = -\frac{5}{4}k_1$, $\therefore \vec{v_1} = \begin{pmatrix} 1 \\ 0 \\ -\frac{5}{4} \end{pmatrix} k_1$, $k_1 \in \mathbb{R}$

$(A-\lambda_2 I)\vec{v_2} = \vec{0}$, $\left(\begin{array}{ccc|c} -i & -1 & 0 & 0 \\ 5 & -i & 4 & 0 \\ 0 & 1 & -i & 0 \end{array}\right) \Rightarrow -ik_1 - k_2 = 0$, $k_1 = ik_2$, $\vec{v_2} = \begin{pmatrix} i \\ 1 \\ -i \end{pmatrix} k_2$, $k_2 \in \mathbb{R}$

$k_2 - ik_3 = 0$, $k_3 = -ik_2$

$(A-\lambda_3 I)\vec{v_3} = \vec{0}$, $\left(\begin{array}{ccc|c} i & -1 & 0 & 0 \\ 5 & i & 4 & 0 \\ 0 & 1 & i & 0 \end{array}\right)$ $ik_1 - k_2 = 0$, $k_1 = -ik_2$, $\vec{v_3} = \begin{pmatrix} -i \\ 1 \\ i \end{pmatrix} k_2$, $k_2 \in \mathbb{R}$.

$k_2 + ik_3 = 0$, $k_3 = ik_2$