Exam #3, April 15, 2015 **Name:** John Gibson
Math 527, University of New Hampshire **Section:**

READ ALL INSTRUCTIONS CAREFULLY

1. Write your name on each page where indicated and your section number above.
2. Show your work and put a box or circle around your answers.
3. If you need more space continue on the back side of the paper.
4. Always write equations.
5. Partial credit will be given only if your work is written clearly and in equations.

Problem 1. (30 pts) Find the inverse Laplace transform.

(a) $\mathscr{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$ +5 complete the square

$= \frac{1}{2} e^{-t} \mathscr{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$ +5 invert s-shift

$= \frac{1}{2} e^{-t} \sin 2t$ +5 invert $\frac{k}{s^2+k^2}$

< 1 min

(b) $\mathscr{L}^{-1}\left\{e^{-4s}\frac{1}{s^2+s-6}\right\} = \mathcal{U}(t-4)\left[\mathscr{L}^{-1}\left\{\frac{1}{(s+3)(s-2)}\right\}\right]_{t\to t-4}$ +3 factor

+3 invert e^{-4s}

$\frac{1}{(s+3)(s-2)} = \frac{A}{s+3} + \frac{B}{s-2}$ $A = -1/5$ $B = 1/5$ by cover-up +3 partial frac

$= \frac{1}{5}\mathcal{U}(t-4)\left[\mathscr{L}^{-1}\left\{\frac{1}{s-2}\right\} - \mathscr{L}^{-1}\left\{\frac{1}{s+3}\right\}\right]_{t\to t-4}$

$= \frac{1}{5}\mathcal{U}(t-4)\left[e^{2t} - e^{-3t}\right]_{t\to t-4}$ +3 invert $\frac{1}{s-a}$

$= \frac{1}{5}\mathcal{U}(t-4)\left[e^{2(t-4)} - e^{-3(t-4)}\right]$ +3 t shift

3 min

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Problem 2. (30 pts) Solve the initial value problem.

$$y'' + 16y = 2 + \delta(t-3), \quad y(0) = y'(0) = 0$$

$$(s^2+16)Y(s) = \frac{2}{s} + e^{-3s}$$

$$Y(s) = \frac{2}{s(s^2+16)} + e^{-3s}\frac{1}{s^2+16}$$

$$\frac{2}{s(s^2+16)} = \frac{A}{s} + \frac{Bs+C}{s^2+16} \qquad \text{cover-up} \Rightarrow A = 1/8$$

$$2 = \frac{1}{8}(s^2+16) + Bs^2 + Cs \qquad \Rightarrow C=0, \ B = -1/8$$

$$Y(s) = \frac{1/8}{s} - \frac{1/8\, s}{s^2+16} + e^{-3s}\frac{1}{s^2+16}$$

$$y(t) = \frac{1}{8}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{8}\mathcal{L}^{-1}\left\{\frac{s}{s^2+16}\right\} + \mathcal{L}^{-1}\left\{e^{-3s}\frac{1}{s^2+16}\right\}$$

$$= \frac{1}{8} - \frac{1}{8}\cos 4t + \mathcal{U}(t-3)\left[\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{4}{s^2+16}\right\}\right]_{t \to t-3}$$

$$\boxed{y(t) = \frac{1}{8} - \frac{1}{8}\cos 4t + \frac{1}{4}\mathcal{U}(t-3)\sin 4(t-3)}$$

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Problem 3. (40 pts) Find two linearly independent solutions of the differential equation. One can be determined exactly. For the other, provide the first three nonzero terms. Specify the region of convergence for each.

$$(2+x^2)y'' - xy' - 3y = 0$$

$$(2+x^2)\sum_{n=0}^{\infty} n(n-1)c_n x^{n-2} - x\sum_{n=0}^{\infty} n c_n x^{n-1} - 3\sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} 2n(n-1)c_n x^{n-2} + \sum_{n=0}^{\infty} n(n-1)c_n x^n - \sum_{n=0}^{\infty} n c_n x^n - \sum_{n=0}^{\infty} 3c_n x^n = 0$$

$m = n-2, \quad n = m+2$

$$\sum_{m=0}^{\infty} 2(m+2)(m+1)c_{m+2} x^m + \sum_{n=0}^{\infty} \left[n(n-1) - n - 3\right] c_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[2(n+2)(n+1)c_{n+2} + (n^2 - 2n - 3)c_n\right] x^n = 0$$

each coeff of x^n must be zero for $n = 0, 1, \dots$

$$2(n+2)\cancel{(n+1)}c_{n+2} + (n-3)\cancel{(n+1)}c_n = 0$$

$$\boxed{c_{n+2} = -\frac{n-3}{2(n+2)} c_n}$$

$y_0(x)$: $c_0 = 1, \quad c_1 = 0$

$n=0 \quad c_2 = -\frac{-3}{2^2}c_0 \qquad c_2 = \frac{3}{2^2}$

$n=2 \quad c_4 = -\frac{(-1)}{2\cdot 4}\frac{3}{2^2} = \frac{3}{2!\,2^4}$

$n=4 \quad c_6 = \frac{1}{2\cdot 6}\frac{3}{2^5} = \frac{3}{3!\,2^6}$

$y_1(x)$: $c_0 = 0, \quad c_1 = 1$

$n=1 \quad c_3 = -\frac{-2}{2\cdot 3} = \frac{1}{3}$

$n=3 \quad c_5 = 0$

$n=5 \quad c_7 = 0$

$$\boxed{y_0(x) = 1 + \frac{3}{2^2}x^2 + \frac{3}{2!\,2^4}x^4 + \frac{3}{3!\,2^6}x^6 + \dots}$$

converges at least for

$-\sqrt{2} < x < \sqrt{2}$

$$\boxed{y_1(x) = x + \frac{1}{3}x^3}$$

a polynomial, converges $\forall x$