Practice problems for exam #3 **Math 527, UNH spring 2015**

Problem 1: Find the Laplace transform or inverse Laplace transform as indicated.

(**a**) $\mathscr{L}\{(3t+1)\mathscr{U}(t-1)\}$

(**b**) $\mathscr{L}\{e^{2t}(t-1)^2\}$

(**c**) $\mathscr{L}^{-1}\left\{\dfrac{2s+5}{s^2+6s+34}\right\}$

(**d**) $\mathscr{L}^{-1}\left\{\dfrac{se^{-\pi s/2}}{s^2+4}\right\}$

Problem 2: Express the function $f(t)$ in terms of the Heaviside function $\mathscr{U}(t-a)$ and then find the Laplace transform $\mathscr{L}\{f(t)\}$.

(**a**) $f(t) = \begin{cases} \sin t & 0 \le t < 2\pi \\ 0 & 2\pi \le t \end{cases}$

(**b**) $f(t) = \begin{cases} 0 & 0 \le t < 1 \\ t^2 & 1 \le t \end{cases}$

Problems 5-7: Use Laplace transforms to solve the initial-value problems.

5. $y' + 2y = f(t), \quad y(0) = 0, \quad \text{where } f(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t \end{cases}$

6. $y'' + 2y + y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \quad \text{where } f(t) = \begin{cases} 0 & 0 \le t < 3 \\ 2 & 3 \le t \end{cases}$

7. $y'' + 4y' + 5y = \delta(t - 2\pi), \quad y(0) = y'(0) = 0$

Problem 8: Find two linearly independent power-series solutions of the ODE, centered about $x = 0$. If the power series does not simplify to a known function or have a simple expression for the coefficients, provide the first four terms of each solution.

$$y'' + x^2 y' + xy = 0$$

Problem 9: Use the power series method to solve the initial value problem and specify the solution's interval of convergence (Zill 6.1 problem 29).

$$(x-1)y'' - xy' + y = 0, \quad y(0) = -2, \quad y'(0) = 6$$