Exam #1, Feb 11, 2015 **Name:** J. Gibson
Math 527, University of New Hampshire **Section:** all

INSTRUCTIONS: PLEASE READ CAREFULLY

1. Write your name and section number above. 1 pt each will deducted if either is missing or illegible.
2. Show your work and put a box or circle around your answers.
3. Always write equations.
4. Partial credit will be given only if your work is written clearly and in equations.
5. Solve for $y(x)$ or $y(t)$ explicitly if possible.

Problem 1. (40 pts) Identify the type of equation, find the general solution, and solve the initial value problem.

$$\frac{dy}{dt} - te^y = 0, \quad y(0) = -3$$

$$e^{-y}\frac{dy}{dt} = t \qquad \boxed{\text{separable}}$$

$$\int e^{-y}\frac{dy}{dt}\,dt = \int t\,dt$$

$$\int \frac{d}{dt}\left(-e^{-y}\right)dt = \frac{1}{2}t^2 + c$$

$$-e^{-y} = \frac{1}{2}t^2 + c$$

$$e^{-y} = c - \frac{1}{2}t^2$$

$$-y = \ln(c - t^2/2)$$

valid for $\frac{t^2}{2} < c$ or $-\sqrt{2c} < t < \sqrt{2c}$

$$\boxed{y(t) = -\ln(c - t^2/2)}$$

$$y(0) = -3 = -\ln c$$

$$3 = \ln c$$

$$e^3 = c$$

$$\boxed{y(t) = -\ln(e^3 - t^2/2)}$$

Problem 2. (30 pts) Identify the type of the equation and find the general solution. An implicit solution is fine.

$$\frac{dy}{dx} = -\frac{2x^3 + y}{x + y}$$

$$\underbrace{y + 2x^3}_{M} + \underbrace{(x+y)}_{N}\frac{dy}{dx} = 0 \qquad \frac{\partial M}{\partial y} = 1 \;,\; \frac{\partial M}{\partial x} = 1$$

$$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial x}$$ so this is an $\boxed{\text{exact eqn}}$

So $\exists$ $f(x,y)$ s.t $M = \frac{\partial f}{\partial x}$ and $N = \frac{\partial f}{\partial y}$

$$M = \frac{\partial f}{\partial x} \Rightarrow \frac{\partial f}{\partial x} = y + 2x^3$$

integrating w.r.t x ... $f(x,y) = xy + \frac{1}{4}2x^4 + g(y)$

$$= xy + \frac{1}{2}x^4 + g(y)$$

$$N = \frac{\partial f}{\partial y} \Rightarrow \cancel{x} + y = \cancel{x} + g'(y)$$

$$g'(y) = y$$

integrating w.r.t y ... $g(y) = \frac{1}{2}y^2$

so $f(x,y) = xy + \frac{1}{2}x^4 + \frac{1}{2}y^2$

and solns to ODE are level curves of $f(x,y)$, ie solns of

$$\boxed{xy + \frac{1}{2}x^4 + \frac{1}{2}y^2 = c}$$

Problem 3. (30 pts) Identify the type of the equation and find the general solution.

$$3\frac{dy}{dx} + 6y - 2x = 0$$

$$\frac{dy}{dx} + \underset{\substack{\uparrow \\ p(x) = 2}}{2y} = \frac{2}{3}x \qquad \boxed{\text{1st order linear}}$$

$$\mu(x) = e^{\int p(x)\,dx} = e^{\int 2\,dx} = e^{2x}$$

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = \frac{2}{3}xe^{2x}$$

$$\frac{d}{dx}\left[e^{2x}y\right] = \frac{2}{3}xe^{2x}$$

$$e^{2x}y = \frac{1}{3}\int \underset{u}{x}\, \underbrace{2e^{2x}dx}_{dv} \qquad v = e^{2x} \quad \frac{dv}{dx} = 2e^{2x}$$

$$= \frac{1}{3}\left[\underset{u}{x}\,\underset{v}{e^{2x}} - \int \underset{v}{e^{2x}}\,\underset{du}{dx}\right]$$

$$e^{2x}y = \frac{1}{3}\left[xe^{2x} - \frac{1}{2}e^{2x}\right] + C$$

$$\boxed{y(x) = \frac{1}{3}x - \frac{1}{6} + Ce^{-2x}}$$