Homework #10 **Math 527, UNH spring 2013**

Problem 1: Write the system of equations in matrix-vector form. Find the solution using augmented matrix notation and Gaussian elimination. Verify that your solution satisfies the original equations.

$$\begin{aligned} x_2 - 3x_3 &= -5 \\ 2x_1 + 3x_2 - x_3 &= 7 \\ 4x_1 + 5x_2 - 2x_3 &= 10. \end{aligned}$$

Problem 2: Calculate $\det(A)$. Based on the value of $\det(A)$, state whether or not the equation $A\mathbf{x} = \mathbf{0}$ has a nonzero solution for $\mathbf{x}$.

(a)

$$A = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} 1 & 3 & 4 \\ -2 & -5 & -3 \\ 1 & 4 & 9 \end{pmatrix}$$

(d)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{pmatrix}$$

Problems 3-6: Find the general solution of these systems of equations. If the system has complex eigenvalues, express the solution in terms of complex exponentials and also in terms of sines and cosines. Boldface indicates a vector of appropriate dimension, e.g.

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Problem 3: Write the system in the form $\mathbf{x}' = A\mathbf{x}$ and then find the general solution.

$$\begin{aligned} x_1' &= 4x_1 - 3x_2 \\ x_2' &= 2x_1 - 3x_2 \end{aligned}$$

Problem 4: Find the general solution of the differential equation.

$$\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \mathbf{x}$$

Problem 5: Find the general solution of the differential equation.

$$\mathbf{x}' = \begin{pmatrix} 1 & -3 \\ 3 & 7 \end{pmatrix} \mathbf{x}$$

Problem 6: Find the general solution of the differential equation.

$$\mathbf{x}' = \begin{pmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{pmatrix} \mathbf{x}$$