Homework #7 **Math 527, UNH spring 2014**
Due Tuesday, Apr. 22 in recitation.

Note: Problems 1 and 2 are warm-up/review problems for power series.

Problem 1: Find the power series expansions of $\sin x$ and $\cos x$ about $x = 0$ by using the Taylor expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \left. \frac{d^n f}{dx^n} \right|_{x=0} x^n$$

That is, plug $f(x) = \sin x$ into the above equation and evaluate the derivatives to derive a power series expansion of $\sin x$. Then do the same for $\cos x$.

Problem 2: Use the power series expansions of $\sin x$ and $\cos x$ to show that

$$\frac{d}{dx} \sin x = \cos x$$

That is, differentiate the power series of $\sin x$ and show it equals the power series of $\cos x$.

Problem 3: Find the general solution of the ODE using the ansatz $y = e^{\lambda x}$, and then find it again using the power series method.

$$y'' + k^2 y = 0$$

Problem 4: Find two linearly independent power-series solutions of the ODE, centered about $x = 0$. If the power series does not simplify to a known function or have a simple expression for the coefficients, provide the first four terms of each solution.

$$y'' + x^2 y' + xy = 0$$

Problem 5: Use the power series method to solve the initial value problem and specify the solution's interval of convergence (Zill 6.1 problem 29).

$$(x-1)y'' - xy' + y = 0, \quad y(0) = -2, \quad y'(0) = 6$$