Homework #5: Laplace transforms **Math 527, UNH spring 2014**
Due Thursday, March 27th in recitation

Problem 1. Find the Laplace transform of $F(s) = \mathscr{L}\{f(t)\}$ using algebra, linearity, trig identities, s-translation, and table-lookup.

(a) $f(t) = 4t^2 - 5\sin 3t$

(b) $f(t) = (1 + e^{5t})^2$

(c) $f(t) = t^2 e^{-3t}$

(d) $f(t) = \sin(3t + 2)$

Problem 2. Find the inverse Laplace transform $f(t) = \mathscr{L}^{-1}\{F(s)\}$ using linearity, partial fractions, complete-the-square, s-translation, and table look-up.

(a) $F(s) = \dfrac{1}{s^7}$

(b) $F(s) = \dfrac{(s+1)^2}{s^3}$

(c) $F(s) = \dfrac{s+1}{s^2+2}$

(d) $F(s) = \dfrac{10s}{s^2-16}$

(e) $F(s) = \dfrac{2s-4}{(s^2+s)(s^2+1)}$

(f) $F(s) = \dfrac{s}{s^2+2s+5}$

Problem 3. The hyperbolic functions $\sinh x$ and $\cosh x$ are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}.$$

Use these definitions to find the Laplace transforms of $\sinh kx$ and $\cosh kx$.

Problem 4. Derive the Laplace transform of t^n for positive integer n. To do this, show that $\mathscr{L}\{1\} = \frac{1}{s}$ and that $\mathscr{L}\{t^n\} = \frac{n}{s}\ \mathscr{L}\{t^{n-1}\}$. Put these together to find $\mathscr{L}\{t\}$, $\mathscr{L}\{t^2\}$, $\mathscr{L}\{t^3\}$, and then generalize to get $\mathscr{L}\{t^n\}$.

Problem 5. Solve the initial value problems using Laplace transforms

(**a**) $y' - y = 2\cos 5t, \quad y(0) = 0$

(**b**) $y'' + 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = 2$

(**c**) $y'' - 6y' + 9y = t, \quad y(0) = 0, \quad y'(0) = 1$