**Final exam, Mar. 16, 2013** **Name:**
**Math 527, University of New Hampshire** **Section:**

**INSTRUCTIONS: PLEASE READ CAREFULLY**

1. Write your name and section number above. Two points deducted for either if missing or illegible.
2. Always write equations.
3. Show your work and put a box or circle around your answers.
4. Final answers should be simplified as much as possible.
5. Partial credit will be given only if your work is written clearly and in equations.
6. If you have time, check your answers by differentiation and substitution.

**Problem 1:** (20 points) Find the general solution of the differential equation.

$$\frac{d^2y}{dt^2} + 9y = \sin(2t)$$

**Problem 2:** (15 points) Find the general solution of the differential equation.

$$\frac{dy}{dx} - y = \frac{11}{8}e^{-x/3}$$

**Problem 3:** (20 points) Find the solution of the initial value problem.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = \begin{cases} 1, & 0 \le t < 3 \\ 0, & 3 \le t \end{cases} \qquad y'(0) = y(0) = 0.$$

**Problem 4:** (15 points) Find the general solution of the differential equation as a power series centered about $x = 0$. Simplify your answer as much as you can.

$$(x-3)\frac{dy}{dx} + 2y = 0$$

**Problem 5:** (15 points) Find the general solution of the differential equation. Express your answer in terms of real-valued functions.

$$\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{x}$$

**Problem 6:** (10 points) Read the questions carefully and answer them. Please do not ask for any kind of help on these questions during the exam.

**(a)** For what region of $x$ will the series solution of Problem 4 converge?

**(b)** True or false: The general solution of a second-order nonlinear differential equation is an arbitrary linear combination of its two linearly independent solutions.

**(c)** Write down the general solutions of the following differential equations. You do not need to show your work.

$$\frac{dy}{dx} + ay = 0$$

$$\frac{d^2y}{dx^2} - k^2y = 0$$

$$\frac{d^2y}{dx^2} + k^2y = 0$$