Exam #2, Oct 7, 2011
Math 527, University of New Hampshire

Name: Solutions
Section:

INSTRUCTIONS: PLEASE READ CAREFULLY
Write your name and section number above. 5 pts will deducted if either is missing or illegible.
Write your final answers in the space provided. Show your work on attached sheets. Staple together in the upper-left corner.

Problem 1 (20 pts): **DO NOT SOLVE THE DIFFERENTIAL EQUATION.**
Just give an appropriate guess for the particular solution of the nonhomogeneous equation.

(a) $y'' - 4y' + 4y = \cos 2x$ $\quad y_p = A\sin(2x) + B\cos(2x)$

(b) $y'' - 4y' + 4y = e^{2x}$ $\quad y_p = Ax^2e^{2x}$

(c) $y'' + 4y = \cos 2x$ $\quad y_p = x(A\sin(2x) + B\cos(2x))$

(d) $y'' + 4y = x^2 + e^x \cos 2x$ $\quad y_p = (Ax^2 + Bx + C) + e^x(D\sin(2x) + E\cos(2x))$

Problem 2 (30 pts): Find the general solution of the ODE

$y'' + 2y' + 4y = 3\cos x$

$y(x) = e^{-x}[c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x)] + \frac{6}{13}\sin(x) + \frac{9}{13}\cos(x)$

Problem 3 (30 pts): Find the general solution of the ODE

$y'' + 4y' + 4y = x^{-2}e^{-2x}$

$y(x) = c_1e^{-2x} + c_2xe^{-2x} - \ln|x|e^{-2x} - e^{-2x}$

Problem 4 (20 pts): Consider the forced mass-spring-dashpot ODE with $m > 0$, $k > 0$, and $\beta \geq 0$:

$my'' + \beta y' + ky = f(t)$

(a) If $\beta = 0$ and $f(t) = 0$, what is the frequency of oscillation ω? $\quad \omega = \sqrt{k/m}$

(b) If $\beta = 0$, give a simple bounded function $f(t)$ that will cause unbounded growth in $y(t)$ as $t \to \infty$.

$f(t) = \alpha\sin(\sqrt{k/m}\,t) + \beta\cos(\sqrt{k/m}\,t)$ for $\alpha, \beta \in \mathbb{R}$

(c) Will the same $f(t)$ cause unbounded growth if β is increased slightly from zero? Why or why not?

No; if $\beta \neq 0$, then $f(t)$ will no longer be part of the homogeneous solution, so the guess will no longer need to be multiplied by t, and hence no longer exhibit unbounded growth.