Tim A. Fill
Section # X
Homework # 0
MATH 527

1.) $\frac{d}{dx}(6x^3) = 18x^2$ (power rule)

2.) $\frac{d}{dx}(2x^{-1}) = -2x^{-2}$ (power rule)

3.) $\frac{d}{dx}(ax^n) = an\,x^{n-1}$ (power rule)

4.) $\frac{d}{dx}\left(\sum_{n=0}^{N} a_n x^n\right) = \frac{d}{dx}\left(a_0 + a_1 x + a_2 x^2 + \dots + a_N x^N\right)$

$= a_1 + 2a_2 x + \dots + N a_N x^{N-1}$

$= \sum_{n=1}^{N} n a_n x^{n-1}$ (note: subscript changes!)

5.) $\frac{d}{dt}\left(a\cos(\omega t) + b\sin(\omega t)\right) = -a\omega\sin(\omega t) + b\omega\cos(\omega t)$ (chain rule)

6.) $\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$ (chain rule)

7.) $\frac{d}{dx}\left(\ln(\mu x)\right) = \frac{1}{\mu x}\cdot\mu = \frac{1}{x}$ (chain rule)

8.) $\frac{d}{dx}\left(\sin(\alpha x^2)\right) = 2\alpha x\cos(\alpha x^2)$ (chain rule)

9.) $\frac{d}{dx}\left(x^2\sin(\alpha x)\right) = 2x\sin(\alpha x) + \alpha x^2\cos(\alpha x)$ (product rule)

10.) $\frac{d}{dx}\left(\frac{x^2}{\sin(\alpha x)}\right) = \frac{2x\sin(\alpha x) - \alpha x^2\cos(\alpha x)}{(\sin(\alpha x))^2}$ (quotient rule)

OR

$\frac{d}{dx}\left(\frac{x^2}{\sin(\alpha x)}\right) = \frac{d}{dx}\left(x^2(\sin(\alpha x))^{-1}\right)$

$= 2x(\sin(\alpha x))^{-1} + x^2\left(-1(\sin(\alpha x))^{-2}(\alpha\cos(\alpha x))\right)$ (product + chain)

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11.) $\frac{d}{dx}\left(\sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n x^n\right) = \sum_{n=1}^{\infty} n\left(\frac{1}{n!}\right)\lambda^n x^{n-1}$

$= \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \lambda^n x^{n-1}$

12.) $\frac{d}{dx}\left(\int f(x)\,dx\right) = \frac{d}{dx}\left(F(x)+c\right) = f(x)$ where $\frac{d}{dx}(F(x)) = f(x)$.

13.) $\frac{d}{dx}\left(\int_0^x f(s)\,ds\right) = f(x)$ (Fund. Thm of Calc.)

14.) $\int 8x^3\,dx = 2x^4 + c$

15.) $\int_0^1 8x^3\,dx = 2x^4 \Big|_0^1 = 2(1)^4 - 2(0)^4 = 2$

16.) $\int_0^y 8x^3\,dx = 2x^4 \Big|_0^y = 2y^4 - 2(0)^4 = 2y^4$

17.) $\int \sum_{n=0}^{N} a_n x^n\,dx = \sum_{n=0}^{N} \frac{a_n}{n+1} x^{n+1} + c$

18.) $\int \frac{1}{x}\,dx = \ln|x| + c$

19.) $\int \frac{d}{dx}(f(x))\,dx = \int \frac{df}{dx}\,dx = f(x) + c$

20.) $\int \frac{dy}{dx}\,dx = y(x) + c$

21.) $\int \frac{d^n y}{dx^n}\,dx = \frac{d^{(n-1)}y}{dx^{(n-1)}} + c$

22.) $\int y\,dx = \int y\,dx$ (not enough information).

23.) $\int \ln(x)\,dx$

Integration by parts: let $u = \ln(x)$ $v = x$

$du = \frac{1}{x}\,dx$ $dv = dx$

$\int \ln(x)\,dx = \ln(x)(x) - \int x\left(\frac{1}{x}\right)dx$

$= x\ln(x) - \int dx = x\ln(x) - x + c$

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24.) $\int \tan^{-1}(x)\,dx$

Integration by parts: let $u = \tan^{-1}(x)$ $\quad v = x$

$du = \frac{1}{1+x^2}\,dx \quad dv = dx$

$$\int \tan^{-1}(x)\,dx = \tan^{-1}(x)(x) - \int x\left(\frac{1}{1+x^2}\right)dx$$

$$= x\tan^{-1}(x) - \int \frac{x\,dx}{1+x^2}$$ let $w = 1+x^2$, $dw = 2x\,dx$

$$= x\tan^{-1}(x) - \frac{1}{2}\int \frac{dw}{w}$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln|w| + c = x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + c$$

25.) $\int \sum_{n=0}^{\infty} \frac{1}{n!}\lambda^n x^n \, dx = \left[\sum_{n=0}^{\infty} \frac{1}{n!}\lambda^n \left(\frac{x^{n+1}}{n+1}\right)\right] + c$

$$= c + \sum_{n=0}^{\infty} \frac{1}{(n+1)!}\lambda^n x^{n+1}$$

26.) Solve the system $3x^2 - 2y = 0$, $4x + y = 1$ for x and y.

By eq'n 2, $y = 1 - 4x$.

Substitution into eq'n 1 yields: $3x^2 - 2(1-4x) = 0$

$3x^2 + 8x - 2 = 0$

Applying the quadratic equation gives us: $x = \dfrac{-8 \pm \sqrt{64 - 4(3)(-2)}}{6}$

$$x = \frac{-8 \pm \sqrt{88}}{6} = \frac{1}{3}(-4 \pm \sqrt{11})$$

If $x = \frac{1}{3}(-4 + \sqrt{11})$, then $y = 1 - 4\left(\frac{1}{3}(-4+\sqrt{11})\right)$

$= 1 - \frac{4}{3}(-4 + \sqrt{11}) = 1 + \frac{16}{3} - \frac{4}{3}\sqrt{11} = \frac{19}{3} - \frac{4}{3}\sqrt{11}$

If $x = \frac{1}{3}(-4 - \sqrt{11})$, then $y = 1 - 4\left(\frac{1}{3}(-4-\sqrt{11})\right) = \frac{19}{3} + \frac{4}{3}\sqrt{11}$.