

Homework #9

Math 527, UNH spring 2013

Due Monday, Apr. 22 in lecture.

Note: Problems 1 and 2 are warm-up/review problems for power series.

Problem 1: Find the power series expansions of $\sin x$ and $\cos x$ about $x = 0$ by using the Taylor expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \left. \frac{d^n f}{dx^n} \right|_{x=0} x^n$$

That is, plug $f(x) = \sin x$ into the above equation and evaluate the derivatives to derive a power series expansion of $\sin x$. Then do the same for $\cos x$.

Problem 2: Use the power series expansions of $\sin x$ and $\cos x$ to show that

$$\frac{d}{dx} \sin x = \cos x$$

That is, differentiate the power series of $\sin x$ and show it equals the power series of $\cos x$.

Problems 3–5: Find two linearly independent power-series solutions of the ODE, centered about $x = 0$, and give a lower bound on the radius of convergence of each solution. If the power series does not simplify to a known function or have simple expressions for the coefficients, provide the first four terms of each solution. These problems are from the Zill textbook, exercises 6.1 problems 17, 21, 23. You can check your answers in the back of the book.

3. $y'' - xy = 0$

4. $y'' + x^2 y' + xy = 0$

5. $(x - 1)y'' + y' = 0$

Problem 6: Use the power series method to solve the initial value problem (Zill 6.1 problem 29).

6. $(x - 1)y'' - xy' + y = 0, \quad y(0) = -2, \quad y'(0) = 6$