Homework #8 **Math 527, UNH spring 2013**

Problem 1: Determine the Laplace transform or inverse Laplace transform by combining rules from the table of Laplace transform.

$$(a) \quad \mathscr{L}^{-1}\left\{\frac{1}{(s-a)^n}\right\} = \qquad \text{(for positive integer } n\text{)}$$

$$(b) \quad \mathscr{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} =$$

$$(c) \quad \mathscr{L}^{-1}\left\{e^{-as}\frac{1}{(s-b)^2+k^2}\right\} =$$

$$(d) \quad \mathscr{L}\{t^2 \sin kt\} =$$

Problem 2: Find the inverse Laplace transform using convolution

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\} =$$

Optional: show that you get the same answer if you use partial fractions.

Problem 3: Find the Laplace transform of $\cosh kt$ by rewriting cosh as a sum of exponentials, and then using $\mathscr{L}\{e^{at}\} = 1/(s-a)$

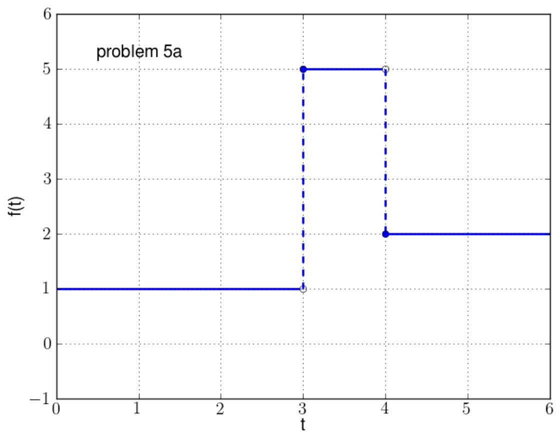
$$\mathscr{L}\{\cosh kt\} =$$

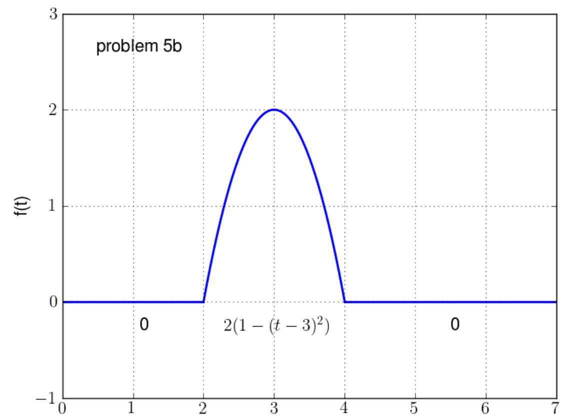
Problem 4: Starting from

$$\mathscr{L}\{U(t-a)f(t-a)\} = e^{-as}\mathscr{L}\{f(t)\}$$

show that

$$\mathscr{L}^{-1}\{e^{-as}F(s)\} = U(t-a)\left[\mathscr{L}^{-1}\{F(s)\}\right]_{t\to t-a}$$





where $F(s) = \mathscr{L}\{f(t)\}$.

Problem 5: For each of the figures above, re-express $f(t)$ in terms of Heaviside functions and then determine the Laplace transform $\mathscr{L}\{f(t)\}$.

Problems 6-8. Use Laplace transforms to solve these initial value problems.

Problem 6:

$$y'' + y' + y = 1 + e^{-t}, \quad y(0) = 3, \quad y'(0) = -5$$

Problem 7:

$$y'' + 2y' + y = 3\,\delta(t-1), \quad y(0) = y'(0) = 0$$

Problem 8:

$$y'' + 4y = f(t) = \begin{cases} \cos t & \text{for } 0 \le t < \pi/2 \\ 0 & \text{for } \pi/2 \le t \end{cases} \qquad y'(0) = y(0) = 0$$