Homework #6: Laplace transforms **Math 527, UNH spring 2013**
Due Friday, March 22nd in lecture

Problem 1. Derive the Laplace transform of e^{at} using the definition of the Laplace transform.

Problem 2. Derive the Laplace transform of $\sin kt$ and $\cos kt$, starting from the definition of the Laplace transform. We covered most of this in lecture, getting $\mathscr{L}\{\sin kt\} = k/(s^2+k^2)$ but not the corresponding formula for $\mathscr{L}\{\cos kt\}$. I want you to repeat the work we did in lecture and then finish off by finding the formula for $\mathscr{L}\{\cos kt\}$.

Problem 3. Derive the Laplace transform of t^n for positive integer n. To do this, show that $\mathscr{L}\{1\} = \frac{1}{s}$ and that $\mathscr{L}\{t^n\} = \frac{n}{s}\ \mathscr{L}\{t^{n-1}\}$. Put these together to find $\mathscr{L}\{t\}$, $\mathscr{L}\{t^2\}$, $\mathscr{L}\{t^3\}$, and then generalize to get $\mathscr{L}\{t^n\}$.

Problem 4. Find the inverse Laplace transform

$$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\} =$$

Problem 5. Find the inverse Laplace transform

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2+s-20}\right\} =$$

Problem 6. Find the inverse Laplace transform

$$\mathscr{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\} =$$

Problem 7. Solve the initial value problem using Laplace transforms

$$\frac{dy}{dt} - y = 1, \quad y(0) = 0$$