Homework #5 **Math 527, UNH spring 2013**
Due March 1 in lecture

Problems 1-6: Find the general solution of the linear nonhomogeneous ODE using judicious guessing or variation of parameters. (Differentiation on the left-hand side is with respect to the same variable as the function on the right-hand side.)

1. $y'' + 4y = \sin x$

2. $y'' + 4y = \sin 2x$

3. $y'' - 4y' + 10y = e^{-t} \sin t$

4. $y''' - 2y'' - 4y' + 8y = 6xe^{2x}$

Hints: (a) The polynomial equation for λ is easily factorable; try some obvious guesses. (b) Calculation of the derivatives is much easier if you simply polynomials as much as possible each time you differentiate. (c) It is a lot easier to keep track of all the terms in the equations if your work is neatly organized.

5. $y'' + 2y' + y = e^{-t} \ln t$

6. $y'' - 2y' + 2y = e^{x} \sec x$

Problem 7:
Find the solution of the forced mass-spring system $my'' + ky = F \sin(\sqrt{k/m}\, t)$ with initial conditions $y(0) = y'(0) = 0$ and sketch the solution $y(t)$. Note that the solution grows without bound as $t \to \infty$. This is called *resonant forcing*: since the forcing frequency is the same as the natural frequency of oscillation, the pushing is always in synch with the motion, and the oscillations always grow in time.

Hint: Divide the equation by m and make the substitutions $\omega = \sqrt{k/m}$ and $F' = F/m$ at the outset. That'll save you from writing $\sqrt{k/m}$ over and over.