1.) $y'' + 2y' + 10y = 2\delta(t-2), \quad y(0) = y'(0) = 0$

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 10\mathcal{L}\{y\} = 2\mathcal{L}\{\delta(t-2)\}$$

$$s^2Y(s) - sy(0) - y'(0) + 2(Y(s) - y(0)) + 10(Y(s)) = 2e^{-2s}$$

$$(s^2+2s+10)\,Y(s) = 2e^{-2s}$$

$$Y(s) = 2e^{-2s}\left(\frac{1}{s^2+2s+10}\right)$$

$$Y(s) = 2e^{-2s}\left(\frac{1}{(s+1)^2+9}\right)$$

$$\mathcal{L}^{-1}\{Y(s)\} = 2\mathcal{L}^{-1}\left\{e^{-2s}\left(\frac{1}{(s+1)^2+9}\right)\right\}$$

$$y(t) = 2u(t-2)\,\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+9}\right\}\Bigg|_{t\to t-2}$$

$$y(t) = 2u(t-2)\left[\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\Bigg|_{s\to s+1}\right\}\right]\Bigg|_{t\to t-2}$$

$$y(t) = \frac{2}{3}u(t-2)\left[e^{-t}\,\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}\right]\Bigg|_{t\to t-2}$$

$$y(t) = \frac{2}{3}u(t-2)\left[e^{-t}\sin(3t)\right]\Big|_{t\to t-2}$$

$$y(t) = \frac{2}{3}u(t-2)\left[e^{-(t-2)}\sin(3(t-2))\right].$$

2.) (a.) $\mathcal{L}^{-1}\{e^{-cs}\}$

$$\mathcal{L}^{-1}\{e^{-cs}\} = \delta(t-c)$$

(b.) $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s+3)}\right\}$

Note: $\frac{1}{(s-2)(s+3)} = \frac{A}{(s-2)} + \frac{B}{(s+3)}$

$1 = (s+3)A + (s-2)B$

@ $s=2$, $1 = 5A$, $A = 1/5$

@ $s=-3$, $1 = -5B$, $B = -1/5$

SO

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{1/5}{s-2}\right\} - \mathcal{L}^{-1}\left\{\frac{1/5}{s+3}\right\}$$

$$= 1/5\,\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 1/5\,\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= 1/5\, e^{2t} - 1/5\, e^{-3t}$$

(c.) $\mathcal{L}^{-1}\left\{e^{-bs}\frac{1}{(s-c)^m}\right\}$ for positive integer m

$$\mathcal{L}^{-1}\left\{e^{-bs}\frac{1}{(s-c)^m}\right\} = \mathcal{U}(t-b)\,\mathcal{L}^{-1}\left\{\frac{1}{(s-c)^m}\right\}\Bigg|_{t\to t-b}$$

$$= \mathcal{U}(t-b)\,\mathcal{L}^{-1}\left\{\frac{1}{s^m}\Bigg|_{s\to s-c}\right\}\Bigg|_{t\to t-b}$$

$$= \mathcal{U}(t-b)\left[e^{ct}\,\mathcal{L}^{-1}\left\{\frac{1}{s^m}\right\}\right]\Bigg|_{t\to t-b}$$

$$= \frac{1}{(m-1)!}\,\mathcal{U}(t-b)\,e^{c(t-b)}(t-b)^{m-1}$$

(d.) $\mathcal{L}\{\mathcal{U}(t-\pi/2)\sin(2t)\}$

$$\mathcal{L}\{\mathcal{U}(t-\pi/2)\sin(2t)\} = e^{-\pi/2\, s}\,\mathcal{L}\{\sin(2(t+\pi/2))\}$$

$$= e^{-\pi/2\, s}\,\mathcal{L}\{\sin(2t+\pi)\}$$

$$= e^{-\pi/2\, s}\,\mathcal{L}\{-\sin(2t)\}$$

$$= -e^{-\pi/2\, s}\left(\frac{2}{s^2+4}\right)$$

3.) (a.) $f(t) = \begin{cases} 0 & 0 \le t < \pi \\ \sin(3t) & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$

$f(t) = \mathcal{U}(t-\pi)\sin(3t) - \mathcal{U}(t-2\pi)\sin(3t)$

(b.) $f(t) = \begin{cases} t^2 - 2 & 0 \le t < 5 \\ 0 & t \ge 5 \end{cases}$

$f(t) = t^2 - 2 + \mathcal{U}(t-5)(2-t^2)$