30 pts for effort/completeness
Remaining points for correctness with question one detail graded.

Math 527
Homework 10 Due 11/24/15 Pt 1

"Correctness" Points per question: #1: 20 pts, #2–#6: 10 pts

Total 70pts earned through "correctness"

Name
Course
Section Number
Professor
Date
Assignment
↑ Some Extra But Solid Layout

Problems 1-3: Write the system of equations as an $A\mathbf{x}=b$ problem, and then find the solution $\mathbf{x}$ by Gaussian Elimination.

20pts 1.) $x+y-2z=14$
$2x-y+z=0$
$6x+3y+4z=1$

$$\overset{A}{\begin{bmatrix}1&1&-2\\2&-1&1\\6&3&4\end{bmatrix}}\overset{X}{\begin{bmatrix}x\\y\\z\end{bmatrix}}=\overset{b}{\begin{bmatrix}14\\0\\1\end{bmatrix}}$$ +5 for $Ax=b$

$$\left[\begin{array}{ccc|c}1&1&-2&14\\2&-1&1&0\\6&3&4&1\end{array}\right]\xrightarrow[-6R_1+R_3\to R_3]{-2R_1+R_2\to R_2}\left[\begin{array}{ccc|c}1&1&-2&14\\0&-3&5&-28\\0&-3&16&-83\end{array}\right]\xrightarrow{-R_2+R_3\to R_3}\left[\begin{array}{ccc|c}1&1&-2&14\\0&-3&5&-28\\0&0&11&-55\end{array}\right]\xrightarrow{\frac{1}{11}R_3\to R_3}$$

+10 for proper Gaussian Elimination, partial credit for "mostly" correct with slight algebra errors.

$$\left[\begin{array}{ccc|c}1&1&-2&14\\0&-3&5&-28\\0&0&1&-5\end{array}\right]\Rightarrow$$

$x+y-2z=14$
$-3y+5z=-28$
$\boxed{z=-5}$

$-3y+5(-5)=-28$
$-3y-25=-28$
$-3y=-3$
$\boxed{y=1}$

$x+(1)-2(-5)=14$
$x+11=14$
$\boxed{x=3}$

Thus, $\mathbf{X}=\begin{bmatrix}3\\1\\-5\end{bmatrix}$ +5 correct X vector

10pts 2.) $5x-2y+4z=10$
$x+y+z=9$
$4x-3y+3z=1$

$$\overset{A}{\begin{bmatrix}1&1&1\\5&-2&4\\4&-3&3\end{bmatrix}}\overset{X}{\begin{bmatrix}x\\y\\z\end{bmatrix}}=\overset{b}{\begin{bmatrix}9\\10\\1\end{bmatrix}}$$

$$\left[\begin{array}{ccc|c}1&1&1&9\\5&-2&4&10\\4&-3&3&1\end{array}\right]\xrightarrow[-4R_1+R_3\to R_3]{-5R_1+R_2\to R_2}\left[\begin{array}{ccc|c}1&1&1&9\\0&-7&-1&-35\\0&-7&-1&-35\end{array}\right]\xrightarrow{-R_2+R_3\to R_3}\left[\begin{array}{ccc|c}1&1&1&9\\0&-7&-1&-35\\0&0&0&0\end{array}\right]\Rightarrow$$

$x+y+z=9$
$-7y-z=-35$
$0=0$

Notice $0=0$ is always true so there are infinitely many solutions. +10

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Pt 2

10pts 3.)

$$\begin{aligned} 5x + 4y - 16z &= -10 \\ y + z &= -5 \\ x - y - 5z &= 7 \end{aligned} \qquad \overset{A}{\begin{bmatrix} 1 & -1 & -5 \\ 0 & 1 & 1 \\ 5 & 4 & -16 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{b}{\begin{bmatrix} 7 \\ -5 \\ -10 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -5 & 7 \\ 0 & 1 & 1 & -5 \\ 5 & 4 & -16 & -10 \end{array}\right] \xrightarrow{-5R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & -5 & 7 \\ 0 & 1 & 1 & -5 \\ 0 & 9 & 9 & -45 \end{array}\right] \xrightarrow{-9R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & -5 & 7 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Notice the last equation is always true. Thus, there are infinitely many solutions to the system. +10

Problems 4-6: For the given matrix A, find all solutions X to the equation $AX = 0$. First calculate $\det A$. If $\det A = 0$, then there are infinitely many solutions X. If $\det A \neq 0$, then the only solution is $X = 0$.

10pts 4.)

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & -2 & 0 \end{pmatrix} \qquad \det A = 3 \begin{vmatrix} 2 & 1 \\ -1 & -2 \end{vmatrix} = 3(-4+1) = -9$$

As $\det A \neq 0$, the only solution is $X = 0$. +10

10pts 5.)

$$A = \begin{pmatrix} 2 & 4 & -2 \\ 4 & 2 & -2 \\ 8 & 10 & -6 \end{pmatrix} \qquad \det A = 2\begin{vmatrix} 2 & -2 \\ 10 & -6 \end{vmatrix} - 4\begin{vmatrix} 4 & -2 \\ 8 & -6 \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ 8 & 10 \end{vmatrix}$$

$$\det A = 2(-12+20) - 4(-24+16) - 2(40-16)$$

$$\det A = 16 + 32 - 48 = 0$$

As $\det A = 0$, there are infinitely many solutions X. +10

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 4 & 2 & -2 & 0 \\ 8 & 10 & -6 & 0 \end{array}\right] \xrightarrow[-4R_1 + R_3 \to R_3]{-2R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & -6 & 2 & 0 \\ 0 & -6 & 2 & 0 \end{array}\right] \xrightarrow{-R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & -6 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Thus, $\vec{X} = \begin{bmatrix} 2x + 4y - 2z \\ -6y + 2z \\ z \end{bmatrix}$ ← Gaussian Elimination to general solution for $\vec{X}$. Not required, but beneficial for students to know.

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Pt 3

10 pts 6.)

$$A = \begin{pmatrix} -1 & 3 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \qquad \det A = -1 \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$$

$$\det A = -1(-5) - 3(2) = -1$$

As $\det A \neq 0$, the only solution is $\mathbf{X} = 0$

+10 pts