1.) Classify each differential as separable, 1st order linear, exact, homogeneous, or Bernoulli. Some equations may be more than one kind. Do not solve the equation.

(a) $\frac{dy}{dx} = \frac{x-y}{x}$ 1st Order Linear, Homogeneous, Exact

Partial credit within each group is up to the grader

(b) $\frac{dy}{dx} = \frac{1}{y-x}$ 1st Order Linear in X.

(c) $(x+1)\frac{dy}{dx} = -y+10$ 1st Order Linear, separable, exact

(d) $\frac{dy}{dx} = \frac{1}{x(x-y)}$ Bernoulli in X.

(e) $\frac{dy}{dx} = \frac{y^2+y}{x^2+x}$ Separable

(f) $\frac{dy}{dx} = 5y+y^2$ Separable, Bernoulli

(g) $y = (y-xy^2)\frac{dy}{dx}$ 1st Order Linear in X

(h) $x\frac{dy}{dx} = ye^{x/y} - x$ Homogeneous

(i) $xy\frac{dy}{dx} + y^2 = 2x$ Bernoulli

(j) $2xy\frac{dy}{dx} + y^2 = 2x^2$ Exact, Bernoulli, Homogeneous

Problems 2-5. Solve the differential equation.

2.) $y^2+1 = y\sec^2 x\frac{dy}{dx} \Rightarrow \frac{1}{\sec^2 x}dx = \frac{y}{y^2+1}dy$

$\int \frac{1}{\sec^2 x}dx = \int \frac{y}{y^2+1}dy = \int \cos^2 x\, dx$

$u = y^2+1$
$du = 2y\,dy$
$\frac{1}{2}du = y\,dy$

$\frac{1}{2}\int \frac{1}{u}du = \int \cos^2 x\, dx$

$\frac{1}{2}\ln(u) = \frac{1}{2}(x+\sin x\cos x)+C$

$\ln|y^2+1| = x+\sin x\cos x + C$ Exponentiate

$y^2+1 = Ke^{x+\sin x\cos x}$

$y^2 = Ke^{x+\sin x\cos x} - 1$

$\boxed{y = \pm\sqrt{Ke^{x+\sin x\cos x}-1}}$ Note: $\sin x\cos x = \frac{1}{2}\sin(2x)$

3.) $(6x+1)y^2 \frac{dy}{dx} + 3x^2 + 2y^3 = 0$

$(3x^2 + 2y^3)dx + (6xy^2 + y^2)dy = 0$

$M_y = 6y^2 \qquad N_x = 6y^2 \quad \checkmark$ Exact!

$F_p = \int 3x^2 + 2y^3 \, dx = x^3 + 2xy^3 + h(y)$

$\frac{\partial F_p}{\partial y} = 0 + 6xy^2 + h'(y)$, set $\frac{\partial F_p}{\partial y} = \frac{\partial f}{\partial y}$

$6xy^2 + h'(y) = 6xy^2 + y^2$

$\Rightarrow h'(y) = y^2$

$\Rightarrow h(y) = \frac{y^3}{3} + C$

Thus, $f(x,y) = 0 \Rightarrow \boxed{x^3 + 2xy^3 + \frac{y^3}{3} = C}$

If another general form is used remaining steps may differ

4.) $t\frac{dQ}{dt} + Q = t^4 \ln(t)$

1st Order Linear

$\left[\frac{dQ}{dt} + \frac{1}{t}Q = t^3 \ln(t)\right]$ general form +5

$\left[\begin{array}{l} p(t) = \frac{1}{t} \\ P(t) = \ln(t) \\ I = e^{\ln(t)} = t \end{array}\right]$ +5 find correct integrating factor

$t\left(\frac{dQ}{dt} + \frac{1}{t}Q\right) = t(t^3 \ln(t))$

$t\frac{dQ}{dt} + Q = t^4 \ln(t)$

$\int \frac{d}{dt}(tQ)\,dt = \int t^4 \ln(t)\,dt$

proper implementation of integrating factor +5

Integrate $\quad u = \ln(t) \quad v = \frac{t^5}{5}$

$du = \frac{1}{t} \quad dv = t^4 dt$

$\left[tQ = \frac{t^5}{5}\ln(t) - \int \frac{t^4}{5} dt\right]$ +5 proper integration

$tQ = \frac{t^5}{5}\ln(t) - \frac{1}{25}t^5 + C$

$\left[\boxed{Q = \frac{1}{5}t^4 \ln(t) - \frac{1}{25}t^4 + \frac{C}{t}}\right]$ +5

5.) $(2x+y+1)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2x+y+1} \Rightarrow \frac{dx}{dy} = 2x+y+1$

$\frac{dx}{dy} - 2x = y+1$

$p(y) = -2$

$P(y) = -2y$

$I = e^{-2y}$

$e^{-2y}\left(\frac{dx}{dy} - 2x\right) = e^{-2y}(y+1)$

$e^{-2y}\frac{dx}{dy} - 2xe^{-2y} = e^{-2y}y + e^{-2y}$

$\int \frac{d}{dy}(e^{-2y}x)\,dy = \int e^{-2y}y + e^{-2y}\,dy$

Integrate $\quad u = y \quad v = \frac{e^{-2y}}{-2}$

$du = dy \quad dv = e^{-2y}dy$

$e^{-2y}x = -\frac{1}{2}ye^{-2y} + \frac{1}{2}\int e^{-2y}dy \; - \frac{1}{2}e^{-2y}$

$e^{2y}(e^{-2y}x) = \left(-\frac{1}{2}ye^{-2y} - \frac{1}{4}e^{-2y} - \frac{1}{2}e^{-2y} + C\right)e^{2y}$

$\boxed{x = ce^{2y} - \frac{1}{2}y - \frac{3}{4}}$

x is dependent

y is independent

$\frac{dy}{dx} = \frac{1}{2x+y+1}$

$\frac{du}{dx} - 2 = \frac{1}{u}$

let $u = 2x+y+1$

$y = u - 2x - 1$

$\frac{dy}{dx} = \frac{du}{dx} - 2$

$\frac{du}{dx} = \frac{1}{u} + 2$

6.) $\sin x \frac{dy}{dx} + (\cos x) y = 0, \quad y\left(\frac{7\pi}{6}\right) = -2$

Identify the classification of differential equation to solve

$$\frac{dy}{dx} + (\cot x) y = 0$$

$$\frac{dy}{dx} = -(\cot x) y$$

+5 ← This is using separable equation classification, if another form is used, remaining steps may differ.

Proper Integration

$$\int \frac{1}{y}\,dy = -\int (\cot x)\,dx$$

+5 Integrate

$$\ln(y) = -\ln|\sin x| + C$$

Exponentiate

$$y = e^{-\ln|\sin x| + C}$$

$$y = C_1 \frac{1}{\sin x} = C_1 \csc x$$

Use initial condition:

Solve for C

$$-2 = C_1 \csc\left(\tfrac{7\pi}{6}\right) \Rightarrow \frac{-2}{\csc\left(\frac{7\pi}{6}\right)} = C_1 \Rightarrow -2\sin\left(\tfrac{7\pi}{6}\right) = C_1$$

$$\Rightarrow -2\left(-\tfrac{1}{2}\right) = C_1$$

$$\Rightarrow 1 = C_1$$

+5

family of solutions

Thus, $\boxed{y = \csc x}$ +5

$\pi < x < 2\pi$

$\boxed{(\pi, 2\pi) \quad \text{interval of solution}}$ +5 interval of solution given initial condition $y\left(\frac{7\pi}{6}\right) = -2$

7.) $\frac{dy}{dt} + 2(t+1)y^2 = 0, \quad y(0) = -1/8$

$$\frac{dy}{dt} = -2(t+1)y^2$$

$$\frac{dy}{y^2} = -2(t+1)\,dt$$

Integrate

$$\int \frac{1}{y^2}\,dy = -2\int (t+1)\,dt$$

$$-y^{-1} = -2\left(\tfrac{1}{2}t^2 + t\right) + C$$

$$-y^{-1} = -t^2 - 2t + C$$

$$\frac{1}{y} = t^2 + 2t + C_1$$

$$y = \frac{1}{t^2 + 2t + C_1}$$

Use initial condition:

$$-\frac{1}{8} = \frac{1}{0 + 0 + C_1} \Rightarrow -\frac{1}{8} = \frac{1}{C_1} \Rightarrow -8 = C_1$$

Thus, $y = \dfrac{1}{t^2 + 2t - 8}$

Check for dividing by zero:

$$t^2 + 2t - 8 = 0$$

$$(t+4)(t-2) = 0$$

$$t = -4, 2$$

Thus, $t \neq -4, 2$ and $(-4, 2)$ is the interval of the solution.