LASZLO KINDRAT
MATH 527, Sec 04-06
FALL 2015
HW #2 SOLNS.

Partial credit within each group is up to the grader.

1) $2x+y-(x+6y)\frac{dy}{dx}=0$

Let $M(x,y)=2x+y$
$N(x,y)=-(x+6y)$

$\frac{\partial M}{\partial y}=0+1=1$, $\frac{\partial N}{\partial x}=-(1+0)=-1$

Since $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$, the eqn. is $\boxed{\text{not exact.}}$

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2) $2x-1+(3y+7)\frac{dy}{dx}=0$

Let $M(x,y)=2x-1$
$N(x,y)=3y+7$ +5

$\frac{\partial M}{\partial y}=0$, $\frac{\partial N}{\partial x}=0$, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ $\Rightarrow$ the eqn. is exact. +5

$f(x,y)=\int M(x,y)\,dx+g(y)$

$\int M(x,y)\,dx=\int 2x-1\,dx=x^2-x\,(+c)$ +5

$g'(y)=N(x,y)-\frac{\partial}{\partial y}\int M(x,y)\,dx=3y+7-\frac{\partial}{\partial y}(x^2-x)=3y+7$

$g(y)=\int g'(y)\,dy=\int 3y+7\,dy=\frac{3}{2}y^2+7y\,(+c)$ +5

$$\boxed{f(x,y)=x^2-x+\frac{3y^2}{2}+7y=C}$$ +5

Check: $\frac{\partial f}{\partial x}=2x-1\overset{\checkmark}{=}M(x,y)$, $\frac{\partial f}{\partial y}=3y+7\overset{\checkmark}{=}N(x,y)$

3) $5t + 4y + (4t - 8y^3)\frac{dy}{dt} = 0$

Let $M(t,y) = 5t + 4y$

$N(t,y) = 4t - 8y^3$

$\frac{\partial M}{\partial y} = 0 + 4 = 4$, $\frac{\partial N}{\partial t} = 4 - 0 = 4$, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t} \Rightarrow$ eqn is exact

$f(t,y) = \int M(t,y)\,dt + g(y)$

$\int M(t,y)\,dt = \int 5t + 4y\,dt = \frac{5}{2}t^2 + 4ty\,(+C)$

$g'(y) = N(t,y) - \frac{\partial}{\partial y}\int M(t,y)\,dt = 4t - 8y^3 - \frac{\partial}{\partial y}\left(\frac{5}{2}t^2 + 4ty\right) =$

$= 4t - 8y^3 - (0 + 4t) = -8y^3$

$g(y) = \int g'(y)\,dy = \int -8y^3\,dy = -2y^4\,(+C)$

$$\boxed{f(t,y) = \frac{5}{2}t^2 + 4ty - 2y^4 = C}$$

Check: $\frac{\partial f}{\partial t} = 5t + 4y \overset{\checkmark}{=} M(t,y)$, $\frac{\partial f}{\partial y} = 4t - 8y^3 \overset{\checkmark}{=} N(t,y)$

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4) $x^2 - y^2 + (x^2 - 2xy)\frac{dy}{dx} = 0$

Let $M(x,y) = x^2 - y^2$

$N(x,y) = x^2 - 2xy$

$\frac{\partial M}{\partial y} = -2y$, $\frac{\partial N}{\partial x} = 2x - 2y$, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ the eqn is $\boxed{\text{not exact.}}$

5) $t\frac{dy}{dt} = 2te^t - y + 6t^2$

$y - 2te^t - 6t^2 + t\frac{dy}{dt} = 0$

Let $M(t,y) = y - 2te^t - 6t^2$

$N(t,y) = t$

$\frac{\partial M(t,y)}{\partial y} = 1 - 0 - 0 = 1, \frac{\partial N(t,y)}{\partial t} = 1$

$f(t,y) = \int N(t,y)\,dy + g(t)$

$\int N(t,y)\,dy = ty\,(+c)$

$g'(t) = M(t,y) - \frac{\partial}{\partial t}\int N(t,y)\,dy = y - 2te^t - 6t^2 - \frac{\partial}{\partial t}(ty) =$

$= y - 2te^t - 6t^2 - y = -2te^t - 6t^2$

$g(t) = \int g'(t)\,dt = \int -2te^t - 6t^2\,dt = -2\int te^t\,dt - 6\int t^2\,dt$

$u = t \Rightarrow du = dt$

$dv = e^t dt \Rightarrow v = e^t$

$g(t) = -2\left[te^t - \int e^t\,dt\right] - 2\int 3t^2\,dt$

$= -2te^t + 2e^t - 2t^3\,(+c)$

$f(t,y) = ty + 2(1-t)e^t - 2t^3 = C$

check: $\frac{\partial f}{\partial t} = y + 2(1-t)e^t - 2e^t - 6t^2 = y - 2te^t - 6t^2 = M(t,y)$ ✓

$\frac{\partial f}{\partial y} = t + 0 + 0 = t = N(t,y)$ ✓

6) $(x+y)^2+(2xy+x^2-1)\frac{dy}{dx}=0$

Let $M(x,y)=(x+y)^2=x^2+2xy+y^2$
$N(x,y)=2xy+x^2-1$ +5

$\frac{\partial M}{\partial y}=2x+2y^{+0}$, $\frac{\partial N}{\partial x}=2y+2x+0$, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}\Rightarrow$ eqn is exact.

+5

$f(x,y)=\int M(x,y)\,dx+g(y)$

$\int M(x,y)\,dx=\int x^2+2xy+y^2\,dx=\frac{x^3}{3}+x^2y+y^2x\;(+C)$ +5

$g'(y)=N(x,y)-\frac{\partial}{\partial y}\int M(x,y)\,dx=2xy+x^2-1-\frac{\partial}{\partial y}\left(\frac{x^3}{3}+x^2y+y^2x\right)$

$=2xy+x^2-1-x^2-2xy=-1$

$g(y)=\int g'(y)\,dy=\int -1\,dy=-y\;(+C)$ +5

$f(x,y)=\frac{x^3}{3}+x^2y+y^2x-y=C$ +5

Check:

$\cancel{\frac{\partial M}{\partial x}}\;\frac{\partial f}{\partial x}=x^2+2xy+y^2=M(x,y)$ ✓

$\frac{\partial f}{\partial y}=x^2+2xy-1=N(x,y)$ ✓

7) $\sin y - y\sin x + (\cos x + x\cos y - y)\dfrac{dy}{dx} = 0$

Let $M(x,y) = \sin y - y\sin x$

$N(x,y) = \cos x + x\cos y - y$

$$\frac{\partial M}{\partial y} = \cos y - \sin x, \quad \frac{\partial N}{\partial x} = -\sin x + \cos y - 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{eqn. is exact}$$

$$f(x,y) = \int M(x,y)\,dx + g(y)$$

$$\int M(x,y)\,dx = \int \sin y - y\sin x \,dx = x\sin y + y\cos x \;(+c)$$

$$g'(y) = N(x,y) - \frac{\partial}{\partial y}\int M(x,y)\,dx$$

$$= \cos x + x\cos y - y - \frac{\partial}{\partial y}\left(x\sin y + y\cos x\right)$$

$$= \cos x + x\cos y - y - x\cos y - \cos x = -y$$

$$g(y) = \int g'(y)\,dy = \int -y\,dy = -\frac{y^2}{2} \;(+c)$$

$$f(x,y) = x\sin y + y\cos x - \frac{y^2}{2} = C$$

Check:

$$\frac{\partial f}{\partial x} = \sin y - y\sin x = M(x,y) \checkmark$$

$$\frac{\partial f}{\partial y} = x\cos y + \cos x - y = N(x,y) \checkmark$$