Math 527
Homework #10
Due 12/~~8~~/15

1.) $x' = x + 2y$ $\quad \vec{X}(t) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} X, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$

$y' = 4x + 3y$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}, \quad \det(A - \lambda I) = (1-\lambda)(3-\lambda) - 8 = 0$$

$$= 3 - 4\lambda + \lambda^2 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

$$(\lambda - 5)(\lambda + 1) = 0$$

$$\lambda_1 = 5, \quad \lambda_2 = -1$$

$$A - 5I = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix}, \quad \left[\begin{array}{cc|c} -4 & 2 & 0 \\ 4 & -2 & 0 \end{array}\right] \xrightarrow{R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} -4 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A + I = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}, \quad \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 4 & 4 & 0 \end{array}\right] \xrightarrow{-2R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Thus, $x(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$

2.) $x' = -4x + 2y$ $\quad \vec{x}(t) = \begin{bmatrix} -4 & 2 \\ -\frac{5}{2} & 2 \end{bmatrix} \vec{x}$

$y' = -\frac{5}{2}x + 2y$

$$A - \lambda I = \begin{bmatrix} -4-\lambda & 2 \\ -5/2 & 2-\lambda \end{bmatrix} \quad \det(A - \lambda I) = (-4-\lambda)(2-\lambda) + 5 = 0$$

$$= -8 + 2\lambda + \lambda^2 + 5 = 0$$

$$\lambda^2 + 2\lambda - 3 = 0$$

$$(\lambda + 3)(\lambda - 1) = 0$$

$$\lambda_1 = -3, \quad \lambda_2 = 1$$

$$A + 3I = \begin{bmatrix} -1 & 2 \\ -5/2 & 5 \end{bmatrix} \to \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} \Rightarrow V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A - I = \begin{bmatrix} -5 & 2 \\ -5/2 & 1 \end{bmatrix} \to \begin{bmatrix} -5 & 2 \\ 0 & 0 \end{bmatrix} \Rightarrow V_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Thus, $x(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-3t} + c_2 \begin{bmatrix} 2 \\ 5 \end{bmatrix} e^{t}$

3.) $x' = x+y$ $\quad \vec{X}(t) = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \vec{X}$

$y' = -2x-y$

$A-\lambda I = \begin{bmatrix} 1-\lambda & 1 \\ -2 & -1-\lambda \end{bmatrix}$ $\quad \det(A-\lambda I) = (1-\lambda)(-1-\lambda)+2=0$

$-1+\lambda^2+2=0$

$\lambda^2+1=0$

$\lambda^2=-1$

$\lambda = \pm i$

$A-iI = \begin{bmatrix} 1-i & 1 \\ -2 & -1-i \end{bmatrix}$

$(1-i)k_1 + k_2 = 0$ $\quad k_2 = -(1-ik_1)$ $\quad$ let $k_1 = 1$

$-2k_1 + (-1-i)k_2 = 0$

$k_2 = -1+i$ $\quad \vec{V}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + i\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$A+iI = \begin{bmatrix} 1+i & 1 \\ -2 & -1+i \end{bmatrix}$

$(1+i)k_1 + k_2 = 0$ $\quad k_2 = -(1+i)k_1$ $\quad$ let $k_1 = 1$

$-2k_1 + (-1+i)k_2 = 0$ $\quad k_2 = -1-i$

$\vec{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + i\begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$x(t) = c_1 \begin{bmatrix} 1 \\ -1+i \end{bmatrix} e^{it} + c_2 \begin{bmatrix} 1 \\ -1-i \end{bmatrix} e^{-it}$

$x(t) = c_1 \begin{bmatrix} 1 \\ -1+i \end{bmatrix} \overset{X_1}{(\cos t + i\sin t)} + c_2 \begin{bmatrix} 1 \\ -1-i \end{bmatrix} \overset{X_2}{(\cos t - i\sin t)}$

$x(t) = c_1\cos t + c_1 i\sin t + c_2\cos t - c_2 i\sin t$

$y(t)$ $-c_1\cos t + ic_1\cos t - c_1 i\sin t - c_1\sin t - c_2\cos t - c_2\sin t - ic_2\cos t + c_2 i\sin t$

$c_1 + c_2 = \tilde{c}_1$

$(c_1 - c_2)i = \tilde{c}_2$

$x(t) = \tilde{c}_1 \cos t + \tilde{c}_2 \sin t$

$y(t) = -\tilde{c}_1 \cos t + \tilde{c}_2 \cos t - \tilde{c}_2 \sin t - \tilde{c}_1 \sin t$

$X(t) = \tilde{c}_1 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin t\right] + \tilde{c}_2 \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \sin t\right]$

$X(t) = \tilde{c}_1 \begin{bmatrix} \cos t \\ -\cos t - \sin t \end{bmatrix} + \tilde{c}_2 \begin{bmatrix} \sin t \\ \cos t - \sin t \end{bmatrix}$

4.) $x' = 5x + y$ $\qquad \vec{X}(t) = \begin{bmatrix} 5 & 1 \\ -2 & 3 \end{bmatrix} \vec{X}$, $A - \lambda I = \begin{bmatrix} 5-\lambda & 1 \\ -2 & 3-\lambda \end{bmatrix}$

$y' = -2x + 3y$

$$\det(A - \lambda I) = (5-\lambda)(3-\lambda) + 2 = 0 \qquad \lambda = \frac{8 \pm \sqrt{64-68}}{2} = \frac{8 \pm \sqrt{-4}}{2} = \frac{8 \pm 2i}{2} = 4 \pm i$$

$$15 - 8\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 8\lambda + 17 = 0$$

$\lambda_1 = 4+i \Rightarrow \begin{bmatrix} 5-(4+i) & 1 \\ -2 & 3-(4+i) \end{bmatrix} = \begin{bmatrix} 1-i & 1 \\ -2 & -1-i \end{bmatrix}$ $\qquad (1-i)k_1 + k_2 = 0 \Rightarrow k_2 = -(1-i)k_1$

Let $k_1 = 1$, $k_2 = -1+i \Rightarrow \vec{V}_1 = \begin{bmatrix} 1 \\ -1+i \end{bmatrix}$

$\lambda_2 = 4-i \Rightarrow \begin{bmatrix} 5-(4-i) & 1 \\ -2 & 3-(4-i) \end{bmatrix} = \begin{bmatrix} 1+i & 1 \\ -2 & -1+i \end{bmatrix}$ $\qquad (1+i)k_1 + k_2 = 0 \Rightarrow k_2 = -(1+i)k_1$

Let $k_1 = 1$, $k_2 = -1-i \Rightarrow \vec{V}_2 = \begin{bmatrix} 1 \\ -1-i \end{bmatrix}$

$$x(t) = c_1 \begin{bmatrix} 1 \\ -1+i \end{bmatrix} e^{(4+i)t} + c_2 \begin{bmatrix} 1 \\ -1-i \end{bmatrix} e^{(4-i)t}$$

$$x(t) = c_1 \begin{bmatrix} 1 \\ -1+i \end{bmatrix} e^{4t}(\cos t + i\sin t) + c_2 \begin{bmatrix} 1 \\ -1-i \end{bmatrix} e^{4t}(\cos t - i\sin t)$$

$$x(t) = c_1 \begin{bmatrix} \cos t + i\sin t \\ -\cos t - i\sin t + i\cos t - \sin t \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} \cos t - i\sin t \\ -\cos t + i\sin t - i\cos t - \sin t \end{bmatrix} e^{4t}$$

$c_1 + c_2 = \tilde{c}_1$

$(c_1 - c_2)i = \tilde{c}_2$

$c_1 \cos t + c_1 i\sin t + c_2 \cos t - c_2 i\sin t \Rightarrow \tilde{c}_1 \cos t + \tilde{c}_2 \sin t$

$-c_1\cancel{\cos t} - ic_1\sin t + c_1\cancel{i\cos t} - c_1\cancel{\sin t} - c_2\cancel{\cos t} + c_2 i\sin t - c_2\cancel{i\cos t} - \cancel{c_2\sin t}$

$\Rightarrow -\tilde{c}_1 \cos t + \tilde{c}_2 \cos t - \tilde{c}_1 \sin t - \tilde{c}_2 \sin t$

$$x(t) = \tilde{c}_1 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin t \right] e^{4t} + \tilde{c}_2 \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \sin t \right] e^{4t}$$

$$x(t) = \tilde{c}_1 \begin{bmatrix} \cos t \\ -\cos t - \sin t \end{bmatrix} e^{4t} + \tilde{c}_2 \begin{bmatrix} \sin t \\ \cos t - \sin t \end{bmatrix} e^{4t}$$

5.) $x' = -x + 3y,\ y' = -3x + 5y, \quad \vec{x}(t) = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \vec{X}$

$A - \lambda I = \begin{bmatrix} -1-\lambda & 3 \\ -3 & 5-\lambda \end{bmatrix}, \ \det(A - \lambda I) = (-1-\lambda)(5-\lambda) + 9 = 0$

$= \lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 2)^2 = 0$

$\lambda_1 = 2$ repeated root

$A - 2I = \begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix} \Rightarrow V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$(A - 2I) P = \begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$-3p_1 + 3p_2 = 1$, let $p_2 = 0$, then $p_1 = -\frac{1}{3}$

Thus, $x(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \left[\begin{bmatrix} 1 \\ 1 \end{bmatrix} te^{2t} + \begin{bmatrix} -\frac{1}{3} \\ 0 \end{bmatrix} e^{2t} \right]$

6.) $x' = 12x - 9y,\ y' = 4x, \quad \vec{x}(t) = \begin{bmatrix} 12 & -9 \\ 4 & 0 \end{bmatrix} \vec{X}$

$A - \lambda I = \begin{bmatrix} 12-\lambda & -9 \\ 4 & -\lambda \end{bmatrix}, \ \det(A - \lambda I) = (12-\lambda)(-\lambda) + 36 = 0$

$\lambda^2 - 12\lambda + 36 = 0$

$(\lambda - 6)^2 = 0$

$\lambda = 6$

$A - 6I = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix} \Rightarrow \vec{V}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$(A - 6I) P = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \Rightarrow 6p_1 - 9p_2 = 3$ let $p_2 = 0 \Rightarrow p_1 = \frac{1}{2} \Rightarrow \vec{V}_2 = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$

Thus, $x(t) = c_1 \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{6t} \right) + c_2 \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} te^{6t} + \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} e^{6t} \right)$

7.) $x' = -3x - y,\ y' = 9x - 3y, \ X(0) = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad \vec{X}(t) = \begin{bmatrix} -3 & -1 \\ 9 & -3 \end{bmatrix} \vec{X}$

$(A - \lambda I) = \begin{bmatrix} -3-\lambda & -1 \\ 9 & -3-\lambda \end{bmatrix}, \ \det(A - \lambda I) = (-3-\lambda)(-3-\lambda) + 9 = 0$

$\lambda^2 + 6\lambda + 18 = 0$

$\lambda = \frac{-6 \pm \sqrt{36 - 72}}{2} = -3 \pm 3i$

$A - (-3+3i)I = \begin{bmatrix} -3i & -1 \\ 9 & -3i \end{bmatrix} \Rightarrow \vec{V}_1 = \begin{bmatrix} 1 \\ -3i \end{bmatrix}, \ A - (-3-3i)I = \begin{bmatrix} 3i & -1 \\ 9 & 3i \end{bmatrix} \Rightarrow \vec{V}_2 = \begin{bmatrix} 1 \\ 3i \end{bmatrix}$

$x(t) = c_1 \begin{bmatrix} 1 \\ -3i \end{bmatrix} e^{(-3+3i)t} + c_2 \begin{bmatrix} 1 \\ 3i \end{bmatrix} e^{(-3-3i)t} = c_1 \begin{bmatrix} 1 \\ -3i \end{bmatrix} e^{-3t}(\cos 3t + i \sin 3t) + c_2 \begin{bmatrix} 1 \\ 3i \end{bmatrix} e^{-3t}(\cos 3t - i \sin 3t)$

$x(0) = c_1 \begin{bmatrix} 1 \\ -3i \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 3i \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \Rightarrow c_1 + c_2 = 3 \Rightarrow c_1 = 3 - c_2$

$-3ic_1 + 3ic_2 = 5 \Rightarrow -3i(3 - c_2) + 3ic_2 = 5$

7 continued) $-9i + 6ic_2 = 5 \Rightarrow 6ic_2 = 5+9i \Rightarrow c_2 = \frac{5+9i}{6i} = \frac{9-5i}{6}$

$c_1 = 3 - \frac{9-5i}{6} = \frac{9+5i}{6}$

$x(t) = \left(\frac{9+5i}{6}\right)\begin{bmatrix}1\\-3i\end{bmatrix}e^{(-3+3i)t} + \left(\frac{9-5i}{6}\right)\begin{bmatrix}1\\3i\end{bmatrix}e^{(-3-3i)t}$

Or: $x(t) = c_1\begin{bmatrix}\cos 3t + i\sin 3t\\ -3i\cos 3t + 3\sin 3t\end{bmatrix}e^{-3t} + c_2\begin{bmatrix}\cos 3t - i\sin 3t\\ 3i\cos 3t + 3\sin 3t\end{bmatrix}e^{-3t}$

$c_1\cos 3t + c_1 i\sin 3t + c_2\cos 3t - c_2 i\sin 3t \Rightarrow \tilde{c}_1\cos 3t + \tilde{c}_2\sin 3t$ $\quad \tilde{c}_1 = c_1 + c_2$

$-3c_1 i\cos 3t + 3c_1\sin 3t + 3ic_2\cos 3t + 3c_2\sin 3t \Rightarrow 3\tilde{c}_1\sin 3t - 3\tilde{c}_2\cos 3t$ $\quad \tilde{c}_2 = i(c_1 - c_2)$

$x(t) = \tilde{c}_1\left[\begin{pmatrix}1\\0\end{pmatrix}\cos 3t + \begin{pmatrix}0\\3\end{pmatrix}\sin 3t\right]e^{-3t} + \tilde{c}_2\left[\begin{pmatrix}0\\-3\end{pmatrix}\cos 3t + \begin{pmatrix}1\\0\end{pmatrix}\sin 3t\right]e^{-3t}$

$x(t) = \tilde{c}_1\begin{bmatrix}\cos 3t\\ 3\sin 3t\end{bmatrix}e^{-3t} + \tilde{c}_2\begin{bmatrix}\sin 3t\\ -3\cos 3t\end{bmatrix}e^{-3t}$

$x(0) = \tilde{c}_1\begin{bmatrix}1\\0\end{bmatrix} + \tilde{c}_2\begin{bmatrix}0\\-3\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix} \Rightarrow c_1 = 3$
$\Rightarrow c_2 = -\frac{5}{3}$

$x(t) = 3\begin{bmatrix}\cos 3t\\ 3\sin 3t\end{bmatrix}e^{-3t} - \frac{5}{3}\begin{bmatrix}\sin 3t\\ -3\cos 3t\end{bmatrix}e^{-3t}$

8.) Find the general solution

$x' = 2x + 4y + 4z$

$y' = -x - 2y$

$z' = -x - 2z$

$\vec{x}(t) = \begin{bmatrix}2 & 4 & 4\\ -1 & -2 & 0\\ -1 & 0 & -2\end{bmatrix}\vec{x}$

$A - \lambda I = \begin{bmatrix}2-\lambda & 4 & 4\\ -1 & -2-\lambda & 0\\ -1 & 0 & -2-\lambda\end{bmatrix}$ $\quad \det(A-\lambda I) = -1\begin{vmatrix}4 & 4\\ -2-\lambda & 0\end{vmatrix} + (-2-\lambda)\begin{vmatrix}2-\lambda & 4\\ -1 & -2-\lambda\end{vmatrix} = 0$

$\det(A-\lambda I) = -1(8+4\lambda) + (-2-\lambda)\left[(2-\lambda)(-2-\lambda)+4\right] = 0$

$= -8 - 4\lambda + (-2-\lambda)\left[(-4+\lambda^2)+4\right] = 0$

$= -8 - 4\lambda - 2\lambda^2 - \lambda^3 = 0$

$= (\lambda+2)(-\lambda^2 - 4) = 0 \Rightarrow \lambda + 2 = 0,\ -\lambda^2 - 4 = 0$

$\lambda = -2,\ \lambda^2 = -4 \Rightarrow \lambda = \pm 2i$

Thus, $\lambda_1 = -2,\ \lambda_2 = -2i,\ \lambda_3 = 2i$

$$\lambda_1 : A + 2I = \begin{bmatrix} 4 & 4 & 4 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v}_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda_2 : A + 2iI = \begin{bmatrix} 2+2i & 4 & 4 \\ -1 & -2+2i & 0 \\ -1 & 0 & -2+2i \end{bmatrix} \Rightarrow \begin{aligned} (2+2i)k_1 + 4k_2 + 4k_3 &= 0 \\ -k_1 + (-2+2i)k_2 &= 0 \\ -k_1 + (-2+2i)k_3 &= 0 \end{aligned}$$

$$k_2 = \frac{k_1}{(-2+2i)}, \quad k_3 = \frac{k_1}{(-2+2i)}$$

$$(2+2i)k_1 + 4\left(\frac{k_1}{-2+2i}\right) + 4\left(\frac{k_1}{-2+2i}\right) = 0$$

$$(2+2i)k_1 + 2\left(\frac{k_1(-1-i)}{1+1}\right) + 2\left(\frac{k_1(-1-i)}{2}\right) = 0$$

$$(2+2i)k_1 - 2k_1 - 2ik_1 = 0 \Rightarrow 0 = 0$$

let $k_1 = 4$, $k_2 = \frac{4}{(-2+2i)} = \frac{2}{-1+i} = \frac{2(-1-i)}{1+1} = -1-i \Rightarrow k_3 = -1-i$

Thus, $\vec{v}_2 = \begin{bmatrix} 4 \\ -1-i \\ -1-i \end{bmatrix}$ or more simply $\vec{v}_2 = \begin{bmatrix} -2+2i \\ 1 \\ 1 \end{bmatrix}$

$$\lambda_3 : A - 2iI = \begin{bmatrix} 2-2i & 4 & 4 \\ -1 & -2-2i & 0 \\ -1 & 0 & -2-2i \end{bmatrix} \Rightarrow \vec{v}_3 = \begin{bmatrix} -2-2i \\ 1 \\ 1 \end{bmatrix}$$

Thus, $x(t) = c_1 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} -2+2i \\ 1 \\ 1 \end{bmatrix} e^{-2it} + c_3 \begin{bmatrix} -2-2i \\ 1 \\ 1 \end{bmatrix} e^{2it}$

$e^{-2it} = \cos 2t - i\sin 2t$, $\quad e^{2it} = \cos 2t + i\sin 2t$

$$x(t) = c_1 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} -2+2i \\ 1 \\ 1 \end{bmatrix} (\cos 2t - i\sin 2t) + c_3 \begin{bmatrix} -2-2i \\ 1 \\ 1 \end{bmatrix} \cos 2t + i\sin 2t$$

$(c_2 + c_3) = \tilde{c}_2$

$i(c_2 - c_3) = \tilde{c}_3$

1) $-2c_2\cos 2t + 2ic_2\sin 2t + 2ic_2\cos 2t + 2c_2\sin 2t - 2c_3\cos 2t - 2ic_3\sin 2t - 2ic_3\cos 2t + 2c_3\sin 2t$

$\Rightarrow \quad 2\tilde{c}_2\cos 2t + 2\tilde{c}_2\sin 2t + 2\tilde{c}_3\cos 2t + 2\tilde{c}_3\sin 2t$

2),3) $c_2\cos 2t - c_2 i\sin 2t + c_3\cos 2t + c_3 i\sin 2t \Rightarrow \tilde{c}_2\cos 2t - \tilde{c}_3\sin 2t$

$$x(t) = c_1 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} e^{-2t} + \tilde{c}_2 \left[\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cos 2t + \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \sin 2t \right] + \tilde{c}_3 \left[\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cos 2t + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \sin 2t \right]$$

$$x(t) = c_1 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} e^{-2t} + \tilde{c}_2 \begin{bmatrix} 2\cos 2t + 2\sin 2t \\ \cos 2t - \sin 2t \\ \cos 2t - \sin 2t \end{bmatrix} + \tilde{c}_3 \begin{bmatrix} 2\cos 2t + 2\sin 2t \\ \cos 2t + \sin 2t \\ \cos 2t + \sin 2t \end{bmatrix}$$