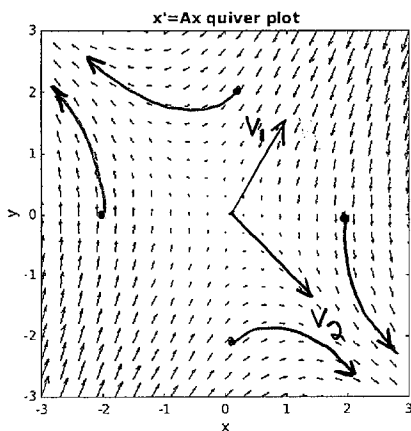


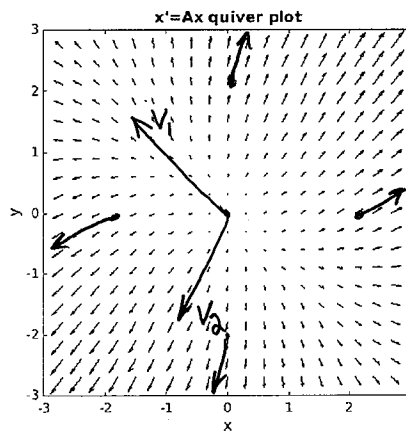
**Problems 1-6.** The following plots show arrows  $x' = dx/dt$  for linear systems.  $x' = Ax$ . In each case,  $A$  has distinct eigenvalues and two eigenvectors.

- Say whether the eigenvalues are real or complex.
- If the eigenvalues are real, draw the eigenvectors, label them  $v_1$  and  $v_2$ , and specify the signs of  $\lambda_1$  and  $\lambda_2$  (e.g  $\text{Re } \lambda_1 < 0$ ,  $\text{Re } \lambda_2 > 0$ ).
- If the eigenvalues are complex, specify the sign of their real part (e.g  $\text{Re } \lambda < 0$ ).
- Draw trajectories starting from the points  $(x, y) = (2, 0)$ ,  $(0, 2)$ ,  $(-2, 0)$ , and  $(0, -2)$ .
- If  $x(t) \rightarrow 0$  for all choices of initial conditions, then the system  $x' = Ax$  is said to be *stable*. If  $x(t) \rightarrow \infty$  for all but very special choices of initial conditions (such as  $x(0)$  lying exactly on an eigenvector), then the system is said to be *unstable*. Specify whether the system is stable or unstable.



(1)

real eigenvalues  
 $\lambda_1 < 0$ ,  $\lambda_2 > 0$   
 unstable



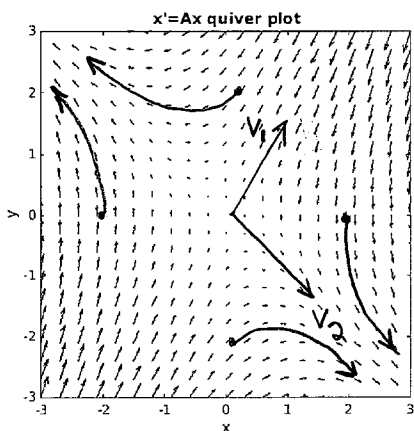
(2)

real eigenvalues  
 $\lambda_1 > 0$ ,  $\lambda_2 > 0$   
 unstable

Note: The subscript labels are arbitrary + interchangeable  
 What's important is that the eigvec pointing up+right has a negative eigenval, and the other has a positive eigenval.

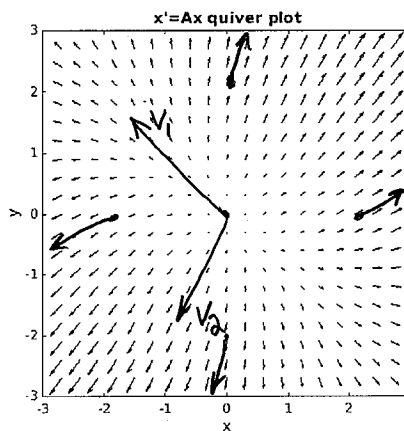
**Problems 1-6.** The following plots show arrows  $\mathbf{x}' = d\mathbf{x}/dt$  for linear systems.  $\mathbf{x}' = A\mathbf{x}$ . In each case,  $A$  has distinct eigenvalues and two eigenvectors.

- Say whether the eigenvalues are real or complex.
- If the eigenvalues are real, draw the eigenvectors, label them  $\mathbf{v}_1$  and  $\mathbf{v}_2$ , and specify the signs of  $\lambda_1$  and  $\lambda_2$  (e.g  $\text{Re } \lambda_1 < 0, \text{Re } \lambda_2 > 0$ ).
- If the eigenvalues are complex, specify the sign of their real part (e.g  $\text{Re } \lambda < 0$ ).
- Draw trajectories starting from the points  $(x, y) = (2, 0), (0, 2), (-2, 0)$ , and  $(0, -2)$ .
- If  $\mathbf{x}(t) \rightarrow 0$  for all choices of initial conditions, then the system  $\mathbf{x}' = A\mathbf{x}$  is said to be *stable*. If  $\mathbf{x}(t) \rightarrow \infty$  for all but very special choices of initial conditions (such as  $\mathbf{x}(0)$  lying exactly on an eigenvector), then the system is said to be *unstable*. Specify whether the system is stable or unstable.



(1)

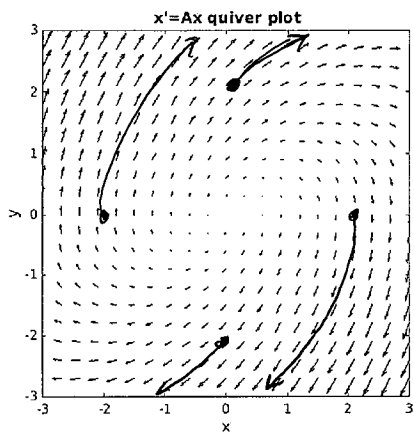
real eigenvalues  
 $\lambda_1 < 0, \lambda_2 > 0$   
 unstable



(2)

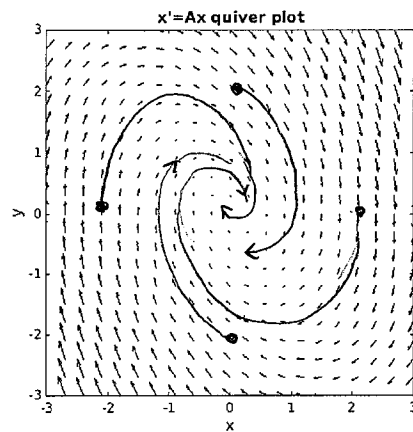
real eigenvalues  
 $\lambda_1 > 0, \lambda_2 > 0$   
 unstable

Note: The subscript labels are arbitrary + interchangeable  
 What's important is that the eigvec pointing up+right has a negative eigenval, and the other has a positive eigenval.



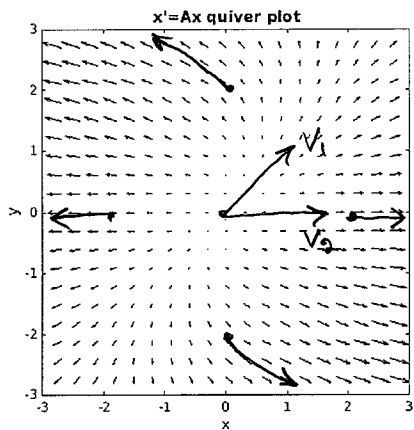
(3)

complex eigvals  
 $\text{Re } \lambda > 0$   
 unstable



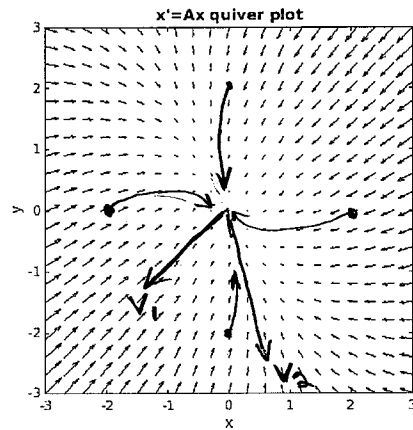
(4)

complex eigenvalues  
 $\text{Re } \lambda < 0$   
 stable



(5)

real eigenvalues  
 $\lambda_1 > 0, \lambda_2 > 0$   
 unstable



(6)

real eigenvalues  
 $\lambda_1 < 0, \lambda_2 < 0$   
 stable