

Problems 1-4: Find the Laplace transform or inverse Laplace transform as indicated.

1. $\mathcal{L}\{(3t + 1)\mathcal{U}(t - 1)\}$

2. $\mathcal{L}\{e^{2t}(t - 1)^2\}$

3. $\mathcal{L}^{-1}\left\{\frac{2s + 5}{s^2 + 6s + 34}\right\}$

4. $\mathcal{L}^{-1}\left\{\frac{se^{-\pi s/2}}{s^2 + 4}\right\}$

Problem 5-6: Express the function $f(t)$ in terms of the Heaviside function $\mathcal{U}(t - a)$ and then find the Laplace transform $\mathcal{L}\{f(t)\}$.

5. $f(t) = \begin{cases} \sin t & 0 \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}$

6. $f(t) = \begin{cases} 0 & 0 \leq t < 1 \\ t^2 & 1 \leq t \end{cases}$

Problem 7-9: Use Laplace transforms to solve the initial-value problems.

7. $y' + 2y = f(t)$, $y(0) = 0$, where $f(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}$

8. $y'' + 2y + y = f(t)$, $y(0) = 0$, $y'(0) = 1$, where $f(t) = \begin{cases} 0 & 0 \leq t < 3 \\ 2 & 3 \leq t \end{cases}$

9. $y'' + 4y' + 5y = \delta(t - 2\pi)$, $y(0) = y'(0) = 0$

If you can do these problems easily, you should do well on the exam. If you struggled, you should look up and solve similar problems from your textbook until they become easy.