Homework #1 **IAM 961, UNH fall 2012**
Due Monday, Sept 17th in lecture

1. Prove that any linear map $\mathcal{L} : \mathbb{C}^n \to \mathbb{C}^m$ can written as an $m \times n$ matrix. You can assume the existence of the canonical basis set $\{e_1, e_2, \ldots, e_m\}$ where $e_1 = [1\ 0\ 0 \ldots 0]^*$, etc.

2. Prove that $A \in \mathbb{C}^{m \times n}$ with $m \geq n$ has full rank iff $Ax \neq Ay\ \forall x, y \in \mathbb{C}^n$, using just the basic definition of rank.

3. Prove that $\|AB\|_p \leq \|A\|_p \|B\|_p$.

4. Prove that $\|A\|_1 = \max_{1 \leq j \leq n} \|a_j\|_1$ where $\{a_j\}$ are the columns of the $A \in \mathbb{C}^{m \times n}$.

5. If u and v are m-vectors the matrix $A = I + uv^*$ is known as a *rank-one perturbation of the indentity*. Show that if A is nonsingular, then its inverse has the form $A^{-1} = I + \alpha uv^*$ for some scalar α, and give an expression for α. For what u and v is A singular? If it is singular, what is null(A)? (Trefethen exercise 2.6).

These problems are intended to develop or exercise your understanding of basis sets, matrix norms, and singular versus nonsingular matrices. Problems 1, 3, and 5 are representative of the kind of analysis we'll need for the development of our main numerical linear algebra algorithms.

Note that a couple of these are proofs are outlined verbally in the text. You can look at these proofs and follow the general strategy. But see if you can improve on the presentation.