Homework #9 **Math 527, UNH fall 2011**

Practice problems for exam #3, do not turn in.

Problem 1: Use Laplace transforms to solve the initial value problem

$$(a)\quad y'' + y' + y = 1 + e^{-t}, \quad y(0) = 3, \quad y'(0) = -5$$

$$(b)\quad y'' + 2y' + y = 3\,\delta(t-1), \quad y(0) = y'(0) = 0$$

$$(c)\quad y'' + 4y = f(t) = \begin{cases} \cos t & \text{for } 0 \le t < \pi/2 \\ 0 & \text{for } \pi/2 \le t \end{cases} \qquad y'(0) = y(0) = 0$$

Problem 2: Determine the Laplace transform or inverse Laplace transform

$$(a)\quad \mathscr{L}^{-1}\left\{\frac{1}{(s-a)^n}\right\} = \qquad \text{(for positive integer } n\text{)}$$

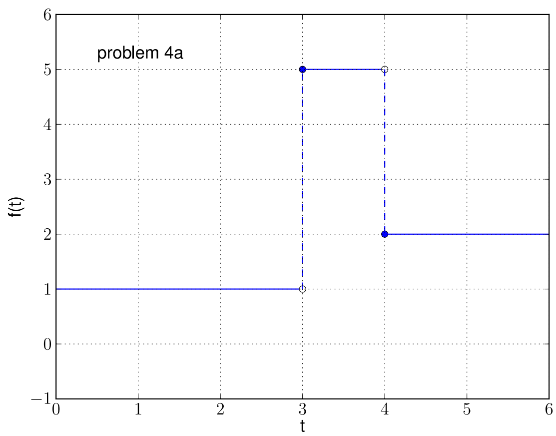
$$(b)\quad \mathscr{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} =$$

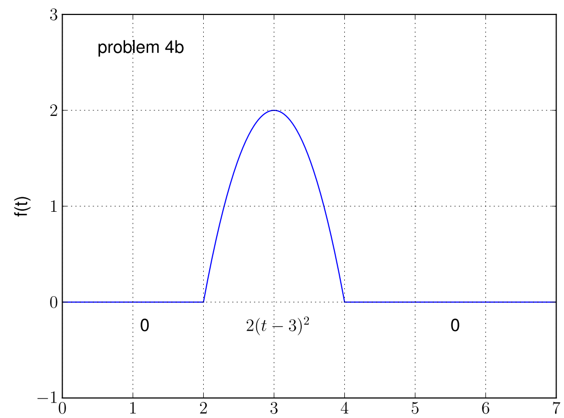
$$(c)\quad \mathscr{L}^{-1}\left\{e^{-as}\frac{1}{(s-b)^2 + k^2}\right\} =$$

$$(d)\quad \mathscr{L}\{tf(t)\} =$$

Problem 3: Use the definition of the Laplace transform $\mathscr{L}$ to show that

$$\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$$





Problem 4: For each of the figures above, re-express $f(t)$ in terms of Heaviside functions and then determine the Laplace transform $\mathscr{L}\{f(t)\}$.

Problem 5: Starting from

$$\mathscr{L}\{U(t-a)f(t-a)\} = e^{-as}\mathscr{L}\{f(t)\}$$

show that

$$\mathscr{L}^{-1}\{e^{-as}F(s)\} = U(t-a)\mathscr{L}^{-1}\{F(s)\}|_{t\to t-a}$$

where $F(s) = \mathscr{L}\{f(t)\}$.