Homework #7 (version 2, 2011-10-12) **Math 527, UNH fall 2011**
Due Friday, Oct. 14 in lecture.

Problem 1: Use the definition of the Laplace transform $\mathscr{L}$ to show that, for positive integer n,

$$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

Problem 2: Use the definition of the Laplace transform $\mathscr{L}$ to show that

$$\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$$

Problem 3: The *hyperbolic sine* function is defined as $\sinh t = (e^t - e^{-t})/2$. Use the result of Problem 2 and the linearity of the Laplace transform to show that

$$\mathscr{L}\{\sinh kt\} = \frac{k}{s^2 - k^2}$$

Problem 4: (optional) The *hyperbolic cosine* function is defined as $\cosh t = (e^t + e^{-t})/2$. Following the method of Problem 3, show that

$$\mathscr{L}\{\cosh kt\} = \frac{s}{s^2 - k^2}$$

Problem 5: Solve the initial value problem two ways: first using the ansatz $y = e^{\lambda t}$ and then the method of Laplace transforms.

$$y'' - y' - 2y = 0, \quad y(0) = 1, y'(0) = 0.$$

Both methods should give the same answer.