Homework #6 **Math 527, UNH fall 2011**
Will not be collected; do as practice for exam.

Problem 1:
Show that the general solution $y(t)$ of $y'' + \omega^2 y = 0$ can be written in two equivalent forms,

$$y(t) = c_1 e^{i\omega t} + c_2 e^{-i\omega t}$$
$$y(t) = c_3 \cos \omega t + c_4 \sin \omega t,$$

by deriving formulae for c_3 and c_4 in terms of c_1 and c_2, and vice versa. Note that real-valued solutions $y(t)$ are obtained by setting c_3 and c_4 to real-valued constants. This derivation allows you, in practice, to go straight from $\lambda = \pm i\omega$ to the form involving sines and cosines, completely skipping the complex exponentials.

Problem 2:
Here we will generalize the results of problem 1. Consider the ODE $y'' + 2\mu y' + \omega_0^2 y = 0$. Assuming $\mu^2 - \omega_0^2 < 0$, show that the ansatz $y = e^{\lambda t}$ results $\lambda = -\mu \pm i\omega$, where $\omega = \sqrt{\omega_0^2 - \mu^2}$, and that the general solution $y(t)$ can be written in either of these forms

$$y(t) = c_1\, e^{-\mu t} e^{i\omega t} + c_2\, e^{-\mu t} e^{-i\omega t}$$
$$y(t) = c_3\, e^{-\mu t} \cos\, \omega t + c_4\, e^{-\mu t} \sin\, \omega t,$$

This derivation allows you, in practice, to go straight from $\lambda = -\mu \pm i\omega$ to the form involving sines and cosines, completely skipping the complex exponentials.

Problem 3:
Find the solution of the forced mass-spring system $my'' + ky = F_0 \sin(\sqrt{k/m}\, t)$ with initial conditions $y(0) = y'(0) = 0$. Note that the solution grows without bound as $t \to \infty$. This is called *resonant forcing*: since the forcing frequency is the same as the frequency of natural oscillation, the pushing is always in synch with the motion, and the oscillations always grow in time.

Problems 4-9: Find the general solution of the linear nonhomogeneous ODE. If you solve a problem with judicious guessing, try solving it again with variation of parameters, just for practice.

Problem 4: $y'' - 2y' + 5y = e^x \cos x$

Problem 5: $y'' - 4y = (x^2 - 3)\sin 2x$

Problem 6: $y''' - 2y'' - 4y' + 8y = 6xe^{2x}$

Problem 7: $y'' + 2y' + y = e^{-t}\ln t$

Problem 8: $y'' + y = \cos^2 t$

Problem 9: $3y'' - 6y' + 6y = e^x \sec x$